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Diffractive Dissociation in $pp \to \Delta^{*}\pi^{-}p$. I. Slope-Mass-Cos $\theta^{G.J.}$ Correlation

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Abstract We have made the complete calculation for $pp \to \Delta^{++}\pi^{-}p$ diffractive dissociation reaction at high energy in the framework of the Three Components Deck Model. This calculation suffers from some difficulties originated by the $(3/2^{+}, 3/2^{+}, 1^{-})$ vertex that appears in one of the components. We give the main technical details and so this paper remains essentially technical. Our conclusion, based on the results obtained, is that the structures of "zeros" or dips predicted by the Model can not be analytically seen because of the complexity of the formulae involved. But we have performed numerical calculations for several dlstributions. A strong interference among the threecomponents may appear according to a particular choice of the parameters.

1. INTRODUCTION

In the last years, the Diffractive Dissociation Reaction (DDR) in strong interactions was almost exhaustively studied from a phenomenological point of view[1]. At high energy, these reactions have peculiar characteristics and are intrinsically associated to the vacuum of the Regge Theory (the Pomeron-(**P**)). More recently there has been an increasing interest in Quantum Chromodynamics, or in general, in the results of gauge theories. As a consequence experimental and phenomenological works show a clear orientation in the senseof testing such theories. But (DDR) and, in general, diffractive aspects of hard scattering exhist and dominate high energy regime and many problems are undefined. We think that in the next future, a solid bridge must be constructed between Quantum Chromodynamics and Regge behaviour.

Perhaps the Dual Topological Unitarity[2] is an adequate approach, for it adds dual interesting aspects. In this sence, the vacuum of several theories and different points of view must be unified.

In this paper we have concentrated in a particular (DDR)

$$pp \to \Delta^{++}\pi^{-}p \qquad (1)$$

or the spin parity structure

$$(1/2)^{+}\ (1/2)^{+} \to (3/2)^{+}\ (0)^{-}\ (1/2)^{+} \qquad (2)$$

in the scope of the Three Components Deck Model (T C D M)[3]. This model has been very successful in describing experimental results. Its main success consists in the description of mass-slope-angular correlation phenomena that appears differently in each particular reaction. These effects are not understood if we do not consider the three components contributions. In this sence the net slope given by (T C D M) is a consequence of the interference of these components dictated by spin-parity structures for each reaction.

We have performed many calculations[4] for several reactions. Our results have indeed confirmed the expectations of (T C D M). It has been shown[3a] that it is essential to consider duality in (T C D M) because it is the only correct treatment for the three channel and gives results that are comparable with experiments. In order to apply in all cases the main ideas of this model, in the present case we have first examined the model without dualization for reaction (1). This means that we are presently interested only in the interference mechanism at the level of the helicity amplitudes with the Born terms.

In next section we treat each particular vertex involved in interaction (1). The particular Delta-Delta-Pomeron ($\Delta\Delta\mathbb{P}$) vertex with spin -parity (J^P) structure ($3/2^+$, $3/2^+$, 1^-) gives a very complicated contribution to our amplitude, and this calculation is one of the main features of this paper. To take into account all difficulties and therefore completely solve the problem, we will be obliged to do a lot of detailed calculations, making this paper a very technical one. But the solution of this case is very important to complete the systematics of the (T C D M) for all (DDR). Further we calculate the ($\Delta\Delta\mathbb{P}$) vertex from a general hypothetical reaction A-hadron elastic with $\mathbb{P}$-exchange.

The principal hypothesis used to obtain the most simple tensor structure of the $C^{\mu\nu\alpha}$ that represent the vertex $(\Delta\Delta\mathbb{P})$, is the s-channel-helicity conservation (SCHC) for $(\mathbb{P})$-exchange, and the Photon - $\mathbb{P}$ - analogy ($\gamma\mathbb{P}A$) or vectorial coupling hypothesis (VCH)[5]. To obtain the quantitative value of $g_{\Delta\Delta\mathbb{P}}$ coupling constant we use the optical theorem and $g_{\Delta\Delta\mathbb{P}}$ depends only on the experimental asymptotic cross-section. In section 3 we describe the (T C D M) and its application to $pp \to \Delta^{++}\pi^{-}p$ reaction. We also give the currents used in each component of the (T C D M) which are the a-exchange or t_1-channel, the A-exchange or u_1 -channel, and p-direct-pole-exchange or s_1 channel components, respectively referred here as T, U and S-components. These three components are coherantly added to give the total amplitude A. In section 4 we define the helicity amplitudes with their high energy approximations (HEA). The helicity amplitudes are given in the end of this section 4 as a function of the invariants, masses, angles and known coupling constants only.

Finally, section 5 is devoted to the discussion of the main points and to our final conclusions about this application of the (T C D M). A set of technical appendices (A,B,C,D and E) with the detailed calculations is given. In order to read the paper without calculations we can follow the text and forget these appendices. The kinematical notations aredefined in appendix A, the 3/2-wave functions is given in appendix B, currents and couplings in appendix C, the behaviour of currents at high-energy conditions in appendix D and tables of useful formulas in appendix E

In a next paper we will present a partial wave expansion of the helicity amplitudes. This is very important in order to study the interferences and, consequently, the slopes ineachpartial wave as appears in several experiments.

2. THE HELICITY CONSERVING $\Delta\Delta\mathbb{P}$ COUPLING

As we emphasize in the introduction, the vertex (A@)-$(3/2^+, 3/2^+, 1^-)$ $(\equiv J^P)$ is the principal complication of this paper. Then, we decide to begin with this point and explicitly calculate the structure of this tensorial vertex $C^{\mu\nu\alpha}$. The complete formof $C^{\mu\nu\alpha}$ can be found in reference 6. But, with some hypothesis - see below - wecan arrive at the simplest form for $C^{\mu\nu\alpha}$ which is more adapted to our problem. This form will be applied to (T C D M) when treating the $pp \to \Delta^{++}\pi^{-}p$ (DDR), as we will see in the next section. It is well known that the (SCHC)[7] for $\mathbb{P}$-exchange is experimentally verified for high energy phenomena. The current associated to diffractive subreaction $Ap \to Ap$ (see fig. A1b) is obtained in appendix C (C-12), through the (V C H) for the $\mathbb{P}$omeron. Therefore we must impose to it the (SCHC). To do this, let us consider a hypothetical diffractive elastic reaction, $\Delta(p) + h(g) \to \Delta(p') + h(q')$, where h can be a pion or a nucleon and the p, q, p' and q' stand for the associated 4-vectors. In all cases the current connected to h-$\mathbb{P}$-h vertex, following eqs. (C.3) and (D.10), is given by

$$J^{\beta}_{hh}(q',q) = 2g_{hh}Q^{\beta}$$

where

$$Q = \frac{1}{2}(q + q') \tag{3}$$

The helicity amplitudes for this reaction, $A_{\Delta h}(\lambda',\lambda)$ are proportional to the currents scalar product, $J^{\beta}_{\Delta\Delta}(\lambda',A)\ J_{hh\beta}$. We use this fact, spatial reflexion (SR) and time reversal symmetries in order to establish the helicity states independently of the particular form of equations (C-12). So we obtain the following relations between different amplitudes

$$A_{\Delta h}(-\lambda',-\lambda) = (-1)^{(\lambda'-\lambda)}A_{\Delta h}(\lambda',\lambda) \tag{4}$$

$$A_{\Delta h}(\lambda,\lambda') = (-1)^{(\lambda'-\lambda)}A_{\Delta h}(\lambda',h) \tag{5}$$

Looking to the current $J^{\beta}_{\Delta\Delta}(\lambda',\lambda)$ and taking into account relations (4) and (5), we have a substantial reduction of the number of amplitudes, i.e., from sixteen to six independent ones only. Using the subsidiary condition (8-2), $p^{\mu}\ \psi^{(+)}_{\mu}(p,\lambda) = 0$, the $\Delta\mathbb{P}\Delta$ coupling can be written as

$$C^{\mu\beta\nu} = g_1 g^{\mu\nu}\gamma^{\beta} + G^{\mu\beta\nu} \tag{6}$$

where

$$G^{\mu\beta\nu} = g_2 g^{\mu\nu}P^{\beta} + \frac{1}{2}g_3(p^{\mu}g^{\beta\nu} + g^{\mu\beta}\ p'^{\nu}) + \frac{1}{4}g_4 p^{\mu}P^{\beta}p'^{\nu} \tag{7}$$

and $P = (p+p')/2$.

Now, from the explicit Rarita-Schwinger wave functions in the helicity and momentum representation (B-21) and the coupling (6), we may write the independent currents as

$$J^{\beta}(3/2,3/2) = \bar{\varepsilon}_{\mu+}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{+}u_{+} + g_1\bar{\varepsilon}_{\mu+}\varepsilon^{\mu}_{+}\bar{u}_{+}\gamma^{\beta}u_{+} \tag{8a}$$

$$\begin{aligned} J^{\beta}(1/2,3/2) = {} & (1/3)^{1/2}\left(\bar{\varepsilon}_{\mu+}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{-}u_{+} + g_1\bar{\varepsilon}_{\mu+}\varepsilon^{\mu}_{+}\bar{u}_{-}\gamma^{\beta}u_{+}\right) \\ & + (2/3)^{1/2}\left(\bar{\varepsilon}_{\mu0}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{+}u_{+} + g_1\bar{\varepsilon}_{\mu0}\varepsilon^{\mu}_{+}\bar{u}_{+}\gamma^{\beta}u_{+}\right) \end{aligned} \tag{8b}$$

$$\begin{aligned} J^{\beta}(-1/2,3/2) = {} & (1/3)^{1/2}\left(\bar{\varepsilon}_{\mu-}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{+}u_{+} + g_1\bar{\varepsilon}_{\mu-}\varepsilon^{\mu}_{+}\bar{u}_{+}\gamma^{\beta}u_{+}\right) \\ & + (2/3)^{1/2}\left(\bar{\varepsilon}_{\mu0}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{-}u_{+} + g_1\varepsilon_{\mu0}\varepsilon^{\mu}_{+}\bar{u}_{-}\gamma^{\beta}u_{+}\right) \end{aligned} \tag{8c}$$

$$J^{\beta}(-3/2,3/2) = \bar{\varepsilon}_{\mu-}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{-}u_{+} + g_1\bar{\varepsilon}_{\mu-}\varepsilon^{\mu}_{+}\bar{u}_{-}\gamma^{\beta}u_{+} \tag{8d}$$

$$\begin{aligned} J^{\beta}(1/2,1/2) = {} & (1/3)\left(\bar{\varepsilon}_{\mu+}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{-}u_{-} + g_1\bar{\varepsilon}_{\mu+}\varepsilon^{\mu}_{+}\bar{u}_{-}\gamma^{\beta}u_{-}\right) \\ & + (2/3)^{1/2}\left(\bar{\varepsilon}_{\mu+}G^{\mu\beta\nu}\varepsilon_{\nu0}\bar{u}_{-}u_{+} + g_1\bar{\varepsilon}_{\mu+}\varepsilon^{\mu}_{0}\bar{u}_{-}\gamma^{\beta}u_{+}\right) \\ & + (2/3)^{1/2}\left(\bar{\varepsilon}_{\mu0}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{+}u_{-} + g_1\bar{\varepsilon}_{\mu0}\varepsilon^{\mu}_{+}\bar{u}_{+}\gamma^{\beta}u_{-}\right) \\ & + (2/3)\left(\bar{\varepsilon}_{\mu0}G^{\mu\beta\nu}\varepsilon_{\nu0}\bar{u}_{+}u_{+} + g_1\bar{\varepsilon}_{\mu0}\varepsilon^{\mu}_{0}\bar{u}_{+}\gamma^{\beta}u_{+}\right) \end{aligned} \tag{8e}$$

$$\begin{aligned} J^{\beta}(-1/2,1/2) = {} & (1/3)\left(\bar{\varepsilon}_{\mu-}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{+}u_{-} + g_1\bar{\varepsilon}_{\mu-}\varepsilon^{\mu}_{+}\bar{u}_{+}\gamma^{\beta}u_{-}\right) \\ & + (2/3)^{1/2}\left(\bar{\varepsilon}_{\mu-}G^{\mu\beta\nu}\varepsilon_{\nu0}\bar{u}_{+}u_{+} + g_1\bar{\varepsilon}_{\mu-}\varepsilon^{\mu}_{0}\bar{u}_{+}\gamma^{\beta}u_{+}\right. \\ & \left. + \bar{\varepsilon}_{\mu0}G^{\mu\beta\nu}\varepsilon_{\nu+}\bar{u}_{-}u_{-} + g_1\bar{\varepsilon}_{\mu0}\varepsilon^{\mu}_{+}\bar{u}_{-}\gamma^{\beta}u_{-}\right) \\ & + (2/3)\left(\bar{\varepsilon}_{\mu0}G^{\mu\beta\nu}\varepsilon_{\nu0}\bar{u}_{-}u_{+} + g_1\bar{\varepsilon}_{\mu0}\varepsilon^{\mu}_{0}\bar{u}_{-}\gamma^{\beta}u_{+}\right) \end{aligned} \tag{8f}$$

where, for the sake of shortness, we have used the following notation for wave functions: $u(p,\pm1/2) = u_{\pm}$ $\bar{u}(p',\pm1/2) = \bar{u}_{\pm}$, $\varepsilon(p,\pm1) = \varepsilon_{\pm}$, $\varepsilon(p,0) = \varepsilon_{0}$, $\varepsilon^{*}(p',\pm1) = \bar{\varepsilon}_{\pm}$ and $\varepsilon^{*}(p',0) = \bar{\varepsilon}_{0}$.

To obtain these expressions in the diffractive limit we make use of the (H E A) (eqs.D-6,7,8) and (D-13). These approximations makes it possible to neglect terms containing $\varepsilon_{\pm}^{\beta}$ or $\bar{\varepsilon}_{+}^{\beta}$ in face of P^{β}, since when they are contracted with Q_{β}, we have $\varepsilon^{\beta}(p,\pm 1)Q_{\beta} = \varepsilon^{\beta *}(p',\mp 1)Q_{\beta} \simeq i(-t/2)^{1/2}/2$ and $P^{\beta}Q_{\beta} \simeq (s/2)$. Then, in these conditions, the currents (8) can be written as

$$J^{\beta}(\lambda',\lambda) = (-t)^{\frac{1}{2}|\lambda'-\lambda|} V(\lambda',\lambda)\, P^{\beta} \qquad (9)$$

where $V(\lambda',\lambda)$, for each pair (λ',λ), is given by

$$V(3/2,3/2) = -2g_1 - 2mg_2 + (mt/4)\, g_4$$

$$V(1/2,3/2) = (1/3)^{1/2}\left\{\frac{2}{m}\, g_1 + 3g_2 + g_3 - \frac{3}{8}\, tg_4\right\}$$

$$V(-1/2,3/2) = (1/3)^{1/2}\left\{-\frac{1}{m}\, g_2 - \frac{1}{2m}\, g_3 + \frac{1}{4}\left(m + \frac{t}{m}\right) g_4\right\} \qquad (10)$$

$$V(-3/2,3/2) = -\, g_4/8$$

$$V(1/2,1/2) = (1/3)\left\{-\left(6 + \frac{t}{2m^2}\right)g_1 - 2\left(3m + \frac{2t}{m}\right)g_2 - \frac{3t}{m}\, g_3 + \frac{t}{4}\left(m + \frac{2t}{m}\right)g_4\right\}$$

$$V(-1/2,1/2) = (1/3)\left\{\frac{4}{m}\, g_1 + 2\left(3 + \frac{t}{2m^2}\right)g_2 + 2\left(1 + \frac{t}{2m^2}\right)g_3 - \frac{t}{8}\left(5 + \frac{t}{m^2}\right) g_4\right\}$$

and m = m_{Δ}. We take now these expressions to impose the helicity conservation, annihilating the currents which do not exhibit it explicitly in the region defined by $|t| \ll m^2$. This procedure results in

$$g_4 = 0,\quad g_3 = \frac{4}{m}\, g_1 \quad \text{and} \quad g_2 = -\frac{2}{m}\, g_1 \qquad (11)$$

And with these values we can rewrite expression (9) as

$$J^{\beta}(\lambda',\lambda) \simeq 2g_1 P^{\beta}\, \delta_{\lambda'\lambda} \qquad (12)$$

(Note that only $V(-1/2,1/2)$ has a neglegible term $\sim 2t/3m^3$ giving a little helicity violation. But in the limit considered above, this is a good approximation). Now if we take eq.(11) into account in Eq.(C-12), the corresponding coupling (as eq.(6)) with helicity wnservation reads

$$C^{\mu\beta\nu} = g_1\left\{g^{\mu\nu}\gamma^{\beta} - (2/m)\, g^{\mu\nu}P^{\beta} + \frac{4}{m}\left(P^{\mu}g^{\beta\nu} + g^{\mu\beta}P^{\nu}\right)\right\} \qquad (13)$$

We will see in the followlng how we can estimate the value of this constant.

Let us consider now a general elastic reaction like

$$a(p,\lambda_a) + b(q,\lambda_b) \to a(p',\lambda'_a) + b(q',\lambda'_b) \qquad (14)$$

From optical theorem for unpolarized reaction we have

$$\frac{1}{(2s_a+1)(2s_b+1)} \sum_{\hat{a}\,\chi_b} \text{Im}\, A(s,t=0;(\lambda_{a'bi}\ A)\ (A_{a'bf}\ A)) =$$

$$= \lambda^{1/2}(s,m_a^2,m_b^2)\sigma_{Tot}(s) \qquad (15)$$

It is well known that at high energy and particularly for the diffractive region, where $s \gg m_a^2$, m_b^2, t, the differential cross çection behaves as a strongly durnped exponential. Using the habitual parametrization,we have

$$\frac{d\sigma}{dt} = \text{cte.}\, e^{bt} \qquad (16)$$

where this constant is $d\sigma/dt|_{t=0}$. In that physical region the scattering amplitudes for hadronic elastic reaction are mainly purely imaglnary and can be described by $\mathbb{P}$omeron exchange[1]. Here we have considered the $(\gamma\mathbb{P}A)^5$, i.e., the $\mathbb{P}$omeron couples to the hadrons as a spin one objecit, so that the corresponding hadronic currents are vectorial. The corresponding helicity amplitudes for reactions (14) have the following form

$$A(s,t;\lambda_a,\lambda_b;\lambda'_a,\lambda'_b) = \frac{i}{2} f(t)\ J_\beta(\lambda'_a,\lambda_a) J^\beta(\lambda'_b,\lambda_b) \qquad (17)$$

where $f(t)$ is a function which contains all information about $\mathbb{P}$-exchange, and the two currents in the diffractive region are

$$\begin{aligned} J_\beta(\lambda'_a,\lambda_a) &= 2\, g_{aa\mathbb{P}}\, P_\beta\, \delta_{\lambda'_a\lambda_a} \\ J_\beta(\lambda'_b,\lambda_b) &= 2\, g_{bb\mathbb{P}}\, Q_\beta\, \delta_{\lambda'_b\lambda_b} \end{aligned} \qquad (18)$$

where $g_{aa\mathbb{P}}$, $g_{bb\mathbb{P}}$ are the $\mathbb{P}$omeron coupling constants with the hadrons a and b, $P = \frac{1}{2}(p+p')$ and $Q = \frac{1}{2}(q+q')$. As in (HEA), $P.Q \simeq s/2$ and $\lambda(s,m_a^2,m_b^2) \simeq s^2$, from eq. (17) we have

$$A(s,t;\lambda_a,\lambda_b;\lambda'_a,\lambda'_b) \simeq i\, g_{aa\mathbb{P}}\, g_{bb\mathbb{P}}\, f(t)\, s\, \delta_{\lambda'_a\lambda_a}\, \delta_{\lambda'_b\lambda_b} \qquad (19)$$

and putting this expression into eq. (15) we obtain

$$g_{aa\mathbb{P}}\, g_{bb\mathbb{P}}\, f(0) = \sigma^{\infty}_{Tot} \qquad (20)$$

Now, replacing the amplitude (19) in the general expression (A-45), and by comparison with eq. (16), we obtain

$$f(t) = f(0)\, e^{bt/2} \qquad (21)$$

Choosing $f(0) = 1$ the amplitudes (17) become

$$A(s,t;\lambda_a,\lambda_b;\lambda'_a,\lambda'_b) = \frac{i}{2}\, e^{bt/2}\, J_\beta(\lambda'_a,\lambda_a)\, J^\beta(\lambda'_b,\lambda_b) \qquad (22)$$

with

$$g_{aa\mathbb{P}}\, g_{bb\mathbb{P}} = \sigma^{\infty}_{Tot} \qquad (23)$$

As we said above, to determine the constant $g_1 = g_{\Lambda\Lambda\mathbb{P}}$ we use relation (23). In our case we have the reaction $\Delta p \to \Delta p$, then $g_{aa\mathbb{P}} = g_{\Delta\Delta\mathbb{P}}$ and $g_{bb\mathbb{P}} = g_{NN\mathbb{P}}$ and, if we know σ^{∞}_{Tot}, s_a and s_b we have the desired value of $g_{\Lambda\Lambda\mathbb{P}}$. Other reactions can be found by comparison and, with other relations, we can estimate the best value.

3. THE THREE COMPONENTS DECK MODEL FOR $pp \to \Delta^{++}\pi^- p$ REACTIONS

In former calculations[3] the existence of the "zeros" or "dips" in the (TCDM) amplitudes as a consequence of the interference between the three components of this model, was displayed in complete agreement with the experimental results[8]. Each amplitude has its slope behaviour. But the interference among the three terms gives an effective slope and a mass-$\cos\theta^{G.J.}$-slope correlation. On the other hand, these interferences produce a minimum ("zeros") in the amplitude and consequently in the differential cross-sections.

It is experimentally known that the structure of the interferences changes for each reaction. In our model these changes proceed directly from different structures of spin-parity of the particles involved. There are some reactions where these interferences are stronger, e.g. $pp\to n\pi p$ [3a] and other reactions, where we only see the "zeros" or a dip in some partial waves. This is the case in $KN \to K\rho N$, $K^*\pi N$ [9]. The main Interest of the present calculations for $pp \to \Delta^{++}\pi^- p$ taking into account their spin structure, is to verify how these complications coming from the spins affect the interferences above mentioned. So it is very natural to extend this study for $pp \to \Delta^{++}\pi^- p$ diffractive dissociation reaction by the (T C D M). To complete the test of this model and for a future generalization, we need these calculations with all difficulties coming from the $\Delta\Delta\mathbb{P}$-vertex. The three graphs representing the (T C D M) are given in fig. A1. In this model the total amplitude results from the addition of each component corresponding to the graphs of fig. A1 (a), (b) and (c). Besides, each term is constructed from the Born term of the $\mathbb{P}p\to\Delta\pi$ subreaction times a term representing the $\mathbb{P}$-exchange for the diffractive part, i.e., the correspondent elastic off-mass-shell subreaction. It is evident that we must take into account the spin of all particles involved. In fact, this point is very important in the context of this paper, because the existence of the "zeros" proceeding from the interferences of the three terms, very dependent of spin structure of the amplitudes, is not evident. We adopt the form commonly used for the Pomeron amplitude, $\mathbb{P} = = is\sigma_T^\infty e^{bt_2/2}$. The two "parameters" σ_T^∞ and b correspond to rhe asymptotic cross section and the slope of the $d\sigma/dt_2$ distribution for each elastic diffractive subreaction related to the three terms. The subreactions are off-shell, but in the kinematical region of interest, i.e., near the poles in t_1 (π-exchange pole), in u_1 (A-exchange pole) and s_1 (p-direct pole), they can be approximated by on-shell ones.

It is important to call the attention on the limit of validity of the model in the kinematical region where it was constructed, i.e., near the poles in the s_1, t_1 and u_1 channels. Any extrapolation for other physical regions far from the threshold of the resonances in s_1, would implicate a Dual-Reggeized amplitude[3]. From fig. A1 we see that there are four different vertices, the $(p\mathbb{P}p)$, $(\pi\mathbb{P}\pi)$, $(\Delta\mathbb{P}\Delta)$ and $(p\Delta\pi)$. On the $(p\mathbb{P}p)$ vertex the nucleons are on shell in all three components, corresponding to the "diffractive vertex". In (HEA) this vertex behaves as $(\pi\mathbb{P}\pi)$ one (equations (C-3) and (D-10)). So, it does not contribute to the helicity structure of the amplitude. In the three other vertex there is always one off-shell particle. The correction off-on-shell is introduced by the σ_{iT}^{∞} and b_i parameters referred above. This procedure avoids the introduction of additional form-factors functions that certainly would destroy any interference mechanism.

Due to the common vertex $(p\mathbb{P}p)$ in the three components we have a current

$$J_\beta^{NN}(\lambda_3,\lambda_b) = g_{\mathbb{P}NN}\, \bar{u}(p_3,\lambda_3)\gamma_\beta\, u(p_b,\lambda_b) \tag{24}$$

obtained in appendix D. Its (H E A) is gíven by eq. (D-10) and using notation (A-2)

$$J_\beta^{NN}(\lambda_3,\lambda_b) \simeq 2\, g_{\mathbb{P}NN}\, R_\beta\, \delta_{\lambda_3\lambda_b} \tag{25}$$

And for $(\pi\mathbb{P}\pi)$ vertex, following eqs. (C-3) and (A-2) we have the current

$$J_\beta^{\pi\pi} = 2g_{\mathbb{P}\pi\pi}\, Q_\beta \tag{26}$$

and for $(p\pi\Delta)$, following eq.(C-2), we have

$$J^{(N\Delta\pi)}(\lambda_1,\lambda_a) = g_{N\pi\Delta}\, \bar{\psi}_\mu(p_1,\lambda_1)p_a^{\mu}\, u(p_a,\lambda_a) \tag{27}$$

WIth these currents, the amplitude for t_1-channel or T-component can be written (using eq. (23)) as

$$T = 2\delta_{\lambda_b\lambda_3}\, T\, R.Q\, \bar{\psi}_\mu(p_1,\lambda_1)p_a^{\mu}\, u(p_a,\lambda_a) \tag{28}$$

where

$$T = ig_{N\pi\Delta}\, \frac{g^t(t_2)}{t_1 - m_2^2} = i\, T' \tag{29}$$

and

$$g^t(t_2) = \sigma_{Tot}^{\infty}\,(\pi N)\; e^{\frac{1}{2}\, b_{\pi N} t_2} \tag{30}$$

For the s_1-channel, the $(p\mathbb{P}p)$ vertex gives a current similar to eq.(24). And to $(\Delta p\pi)$ vertex corresponds a current

$$J^{N\Delta\pi}(\lambda_1,\lambda) = g_{N\pi\Delta}\, \bar{\psi}_\mu(p_1,\lambda_1)p_2^{\mu}\, u(p,\lambda) \tag{31}$$

The corresponding amplitude, following eqs. (23) and (A-2) can be written as

$$S = \partial_{\lambda_b \lambda_3} S \quad \bar{\psi}_\mu(p_1,\lambda_1) p_2^{\ \mu} (\not{p} + m_a) \not{R} u(p_a,\lambda_a) \tag{32}$$

where

$$S = ig_{N\pi\Delta} \frac{g^s(t_2)}{s_1 - m_a^2} = iS' \tag{33}$$

and

$$g^s(t_2) = \sigma_{\text{Tot}}^{\infty} \quad (NN) \; e^{\frac{1}{2} b_{NN} t_2} \tag{34}$$

Finally, the component coming from u_1-channel contains the A-propagator (see eq.B-14) and the $(\Delta \mathbf{P} \Delta)$ vertex, (calculated in the previous section 2) so

$$J_\beta^{\Delta\Delta}(\lambda_1,\lambda) = g_{\Delta \mathbf{P} \Delta} \; \bar{\psi}^\mu(p_1,\lambda_1) \Gamma_{\mu\beta\nu} \; \psi^\nu(k,\lambda) \tag{35}$$

where

$$\Gamma_{\mu\beta\nu} = g_{\mu\nu}(\gamma_\beta - \frac{2}{m_1} P_\beta) + \frac{4}{m_1} (g_{\mu\beta} P_\nu + P_\mu g_{\beta\nu}) \tag{36}$$

For the $(p\Delta\pi)$ vertex we have

$$J^{N\Delta\pi}(\lambda,\lambda_a) = g_{N\pi\Delta} \; \bar{\psi}_\sigma(k,\lambda) \; p_2^\sigma \; u(p_a,\lambda_a) \tag{37}$$

Now with the variables defined in appendix A we write the u_1-channel or U-component amplitude

$$U = - \delta_{\lambda_b \lambda_3} \; R_\beta \, U \bar{\psi}_\mu(p_1,\lambda_1) \Gamma^{\mu\beta\nu} \; (m_1 + \not{k}) \Lambda_{\nu\sigma}(k) p_2^\sigma \; u(p_a,\lambda_a) \tag{38}$$

where

$$U = ig_{N\pi\Delta} \frac{g^u(t_2)}{u_1 - m_1^2} = iU' \tag{39}$$

$$g^u(t_2) = \sigma_{\text{Tot}}^{\infty} \; (N\Delta) \quad e^{\frac{1}{2} b_{N\Delta} t_2} \tag{40}$$

and

$$\Lambda_{\nu\sigma}(k) = g_{\nu\sigma} - \frac{2}{3m_1^2} k_\nu k_\sigma - \frac{1}{3} \gamma_\nu \gamma_\sigma + \frac{1}{3m_1} (k_\nu \gamma_\sigma - k_\sigma \gamma_\nu) \tag{41}$$

Then, the total amplitude of the (TCDM) is obtained adding the three components (eqs. 28, 32 and 38) above

$$A = S + T + U \tag{42}$$

As we can see, the non-diffractive part (or the $p\mathbb{P} \to \Delta^{++}\pi^{-}$) is not affected by the spin structure coming from $(p\mathbb{P}p)$ "diffractive vertex" represented by $\delta_{\lambda_b\lambda_3}$. So we can pass over this factor in the helicity amplitudes, which is equivalent to neglect the spin effects of the particles b and 3 in (H E A).

4. APPROXIMATIONS ON THE HELICITY AMPLITUDES IN THE DIFFRACTIVE REGION

In this section we present explicit forms for the helicity amplitudes in the (H E A). The amplitude (28) with the approximation (A-17) can be written as

$$T \simeq s_2 T \; \bar{\psi}_\mu(p_1,\lambda_1)p_a^{\ \mu} \; u(p_a,\lambda_a) \tag{43}$$

In the equation (32) for s_1-channel helicity amplitude, we have $(\not{p}+m_a)\not{R} = 2R.p + \not{R}(m_a-\not{p})$ where $R.p = R.p_a+R.(p_b-p_3)$. But as R. $(p_b-p_3) = 0$, then $2R.p$ = R. $(p+p_a)$ = 2R.K. The amplitude then becomes

$$S = S \; \bar{\psi}_\mu(p_1,\lambda_1)p_2^{\ \mu} \left[2R.K + \not{R}(m_a-\not{p})\right]u(p_a,\lambda_a) \tag{44}$$

With Dirac equation and the energy-momentum conservation we obtain $(m_a-\not{p})u(p_a) = (\not{p}_a-\not{p})u(p_a) = (\not{p}_3-\not{p}_b)u(p_a)$; then, with the subsidiary condition (B-2) together with eqs. (A-18) and (E-4) we obtain

$$S = s\,S \; \bar{\psi}_\mu(p_1,\lambda_1)p^\mu\{1+i\frac{|\vec{p}_{\bar{a}}|}{2\sqrt{s_1}} \sin\alpha\left[\sigma^{31} + \sigma^{03} \sin\alpha - \sigma^{01}\cos\alpha\right]\} \; (p_a,\lambda_a) \tag{45}$$

For compactness we write now the helicity amplitude (38) as

$$U = - U \; \bar{\psi}_\mu(p_1,\lambda_1)\Omega^\mu \; u(p_a,\lambda_a) \tag{46}$$

where

$$\Omega^\mu = R_\beta \; \Gamma^{\mu\beta\nu}(m_1+\not{k})\Lambda_{\nu\sigma}(k)p_2^{\ \sigma} \tag{47}$$

Using expressions (36,41) together with the Dirac and Rarita- Schwinger equations (B-8,1) and the subsidiary conditions (B-2,3), we obtain (see

Table E-1 where the r_i, v_i and θ_i are defined)

$$\Omega^\mu = p_a^\mu\{2P.R\, r_6 + 2r_8 R.p_2 - \frac{2R.p_2}{m_1}\not{p}_1\} + p^\mu\{-4r_6 P.R - 2r_8 R.p_2 + \frac{2}{m_1}(P.R + R.p_2)\not{p}\} + \frac{(p^\mu - p_a^\mu)}{m_1}(u_1 + 2k.p_2 + m_1^2 + 2m_1 m_a)\not{R} + \frac{1}{3}(r_6 p_a^\mu + r_9 p^\mu)(\not{p}_b\not{p}_3 - p_b.p_3) + \frac{2}{3}R^\mu\{r_1 m_1 - m_2^2 + 2p_a.p_2 + r_3 k.p_2 + 2r_8(3P.p_2 - \frac{2k.p_2}{m_1^2}p.k) - 2P.k\, r_7 + \frac{2P.k}{m_1^2}(2p_a.p_2 - m_2^2) - \frac{(p_1+k)^2}{m_1^2}(k.p_2 - m_1^2) + [-r_1 - \frac{k.p_2}{m_1} - \frac{2}{m_1}(3P.p_2 - \frac{2k.p_2}{m_2^2}p.k) + 2\frac{P.k}{m_1}r_7 - \frac{(p_1+k)^2}{m_1}]\not{p}\} \quad (48)$$

From the results (A-9) to (A-16) with the approximations (A-17) to (A-20) and (E-4)

$$\Omega^\mu = s_3\{r_6(p_a^\mu - 2p^\mu) + \frac{1}{m_1}p^\mu\not{p}\} + s_2\{r_8(p_a^\mu - p^\mu) + \frac{1}{m_1}(p^\mu - p_a^\mu)\not{p}\} + (p^\mu - p_a^\mu)\not{R}r_{10} + is\frac{|\vec{p}_a|}{6\sqrt{s_1}}\sin\alpha[r_6 p_a^\mu + r_9 p^\mu](\sigma^{31} + \sigma^{03}\sin\alpha - \sigma^{01}\cos\alpha) + R^\mu(F_1 + \frac{F_2}{m_1}\not{p}) \quad (49)$$

$$F_1 = (1/3)\{3r_8(s_1 - u_1 - r_1 r_2 - 2m_2^2) + 2(m_1 r_1 + m_a^2 - u_1) + r_3(m_a^2 - m_2^2 - u_1) - (r_8/m_1^2)(m_a^2 - m_2^2 - u_1)(3u_1 - t_2 + m_1^2) + (1/m_1^2)(3u_1 - t_2 + m_1^2)(m_a r_1 - m_1^2 - u_1) - (1/m_1^2)(2u_1 - t_2 + 2m_1^2)(m_a^2 - 2m_1^2 - m_2^2 - u_1)\}$$

$$F_2 = (1/3)\{2t_2 - 3s_1 - 2m_1 m_a + 5m_2^2 - 4m_a - 3m_1^2 + (1/m_1^2)(3u_1 - t_2 + m_1^2)(m_1^2 + m_a^2 - m_1 m_a - m_2^2 - u_1)\} \quad (50)$$

$$U = s_3\, U\, \bar{\psi}_\mu(p_1\lambda_1)\{r_6(2p^\mu - p_a^\mu) - \frac{\sqrt{s_1}}{m_1} p^\mu \gamma^0\} u(p_a,\lambda_a)$$

$$+ s_2\, U\, \bar{\psi}_\mu(p_1,\lambda_1)(p^\mu - p_a^\mu)(r_8 - \frac{\sqrt{s_1}}{m_1}\gamma^0) u(p_a,\lambda_a)$$

$$- U\, r_{10}\, \bar{\psi}(p_1\lambda_1)(p^\mu - p_a^\mu)\not{R}\, u(p_a\lambda_a) - U\bar{\psi}(p_1\lambda_1)R^\mu(F_1 + \frac{\sqrt{s_1}}{m_1} F_2\gamma^0) u(p_a,\lambda_a)$$

$$- i\, \frac{s|\vec{p}_a|}{6\sqrt{s_1}} \sin\alpha\, U\, \bar{\psi}(p_1\lambda_1)(r_6 p_a^\mu + r_9 p^\mu)(\sigma^{31} + \sigma^{03}\sin\alpha - \sigma^{01}\cos\alpha) u(p_a,\lambda_a)$$

(51)

Following eq.(42) with the components (43,45,51), the helicity amplitudes read

$$A_{\lambda_1\lambda_a} = \bar{\psi}_\mu(p_1,\lambda_1)p^\mu(sS + s_2T + s_3U)u(p_a,\lambda_a) - s_2(T - r_8U)\bar{\psi}_\mu(p_1\lambda_1)(p^\mu - p_a^\mu)u(p_a,\lambda_a)$$

$$- s_3\, U\, \bar{\psi}_\mu(p_1,\lambda_1)\{r_6 p_a^\mu + p^\mu\, (\frac{\sqrt{s_1}}{m_1}\gamma^0 - r_6 - r_3)\} u(p_a,\lambda_a)$$

$$- s_2\, U \frac{\sqrt{s_1}}{m_1} \bar{\psi}_\mu(p_1,\lambda_1)(p^\mu - p_a^\mu)\gamma^0 u(p_a,\lambda_a)$$

$$- r_{10}\, U\, \bar{\psi}_\mu(p_1,\lambda_1)(p^\mu - p_a^\mu)\not{R}\, u(p_a,\lambda_a) - U\, \bar{\psi}_\mu(p_1,\lambda_1)R^\mu(F_1 + \frac{\sqrt{s_1}}{m_1} F_2\gamma^0)u(p_a,\lambda_a)$$

$$- i\, \frac{s|\vec{p}_a|}{6\sqrt{s_1}} \sin\alpha\, \bar{\psi}_\mu(p_1,\lambda_1)\left[r_6 U\, p_a^\mu\right.$$

$$\left. + (r_9U - 3S)\, p^\mu\right]\, (\sigma^{31} + \sigma^{03}\sin\alpha - \sigma^{01}\cos\alpha)u(p_a,\lambda_a)$$

(52)

With expression **(E-10)** we obtain

$$\not{R} = (s/2\sqrt{s_1})(\gamma^0 - \sin\alpha\gamma^1 - \cos\alpha\gamma^3) \qquad (53)$$

and the amplitudes may be written as

$$A_{\lambda_1\lambda_a} = \bar{\psi}_\mu(p_1,\lambda_1)E^\mu\{\left[\sqrt{s_1}\, sS + \sqrt{s_1}\, s_2\, T + r_6 s_3 v_4\, U\right] - \frac{s_1}{m_1} s_3 U\gamma^0\}u(p_a,\lambda_a)$$

$$- r_6|\vec{p}_a|s_3\, U\bar{\psi}_\mu(p_1,\lambda_1)Z^\mu u(p_a,\lambda_a) + s_2\, \bar{\psi}_\mu(p_1,\lambda_1)\left[v_3E^\mu - |\vec{p}_a|Z^\mu\right]\{(r_8U - T)$$

$$- \frac{\sqrt{s_1}}{m_1} U\, \gamma^0\}u(p_a,\lambda_a) - \frac{r_{10}}{2\sqrt{s_1}} s\, U\bar{\psi}_\mu(p_1,\lambda_1)\left[v_3E^\mu - |\vec{p}_a|Z^\mu\right]$$

$$\times \quad (\gamma^0 - \gamma^1 \sin\alpha - \gamma^3 \cos\alpha) u(p_a,\lambda_a)$$

$$- \frac{s}{2} U \bar{\psi}_\mu(p_1,\lambda_1)(E^\mu + X^\mu \sin\alpha + Z^\mu \cos\alpha)\left(\frac{F_1}{\sqrt{s_1}} + \frac{F_2}{m_1}\gamma^0\right) u(p_a,\lambda_a)$$

$$- \frac{s|\vec{p}_a|}{6\sqrt{s_1}} \sin\alpha \, \bar{\psi}_\mu(p_1,\lambda_1)\{[(r_9 s_1 + r_1 E_a) U - 3\sqrt{s_1}\, S]E^\mu$$

$$+ U r_1 |\vec{p}_a| Z^\mu\} i(\sigma^{31} + \sigma^{03}\sin\alpha - \sigma^{01}\cos\alpha) u(p_a,\lambda_a) \tag{54}$$

These amplitudes may be calculated explicitly using the results (E-6,7) and (B-21) and results,

$$A_{(\pm 3/2,\pm 1/2)}(\theta,\phi) = \pm \frac{e^{\mp i\phi}}{\sqrt{2}} \{\mp \mathrm{Re}(\pm 3/2,\pm 1/2) + i\, \mathrm{Im}(\pm 3/2,\pm 1/2)\} \tag{55}$$

$$A_{(\mp 3/2,\pm 1/2)}(\theta,\phi) = \frac{e^{\pm 2i\phi}}{\sqrt{2}} \{\pm \mathrm{Re}(\mp 3/2,\pm 1/2) + i\, \mathrm{Im}(\mp 3/2,\pm 1/2)\} \tag{56}$$

$$A_{(\pm 1/2,\pm 1/2)}(\theta,\phi) = \{\pm \mathrm{Re}(\pm 1/2,\pm 1/2) + i\, \mathrm{Im}(\pm 1/2,\pm 1/2)\} \tag{57}$$

$$A_{(\mp 1/2,\pm 1/2)}(\theta,\phi) = \mp e^{\pm i\phi}\{\pm \mathrm{Re}(\mp 1/2,\pm 1/2) + i\, \mathrm{Im}(\mp 1/2,\pm 1/2)\} \tag{58}$$

where

$$\mathrm{Re}(\pm 3/2, \pm 1/2) = \frac{s}{2} U' \sin\alpha \sin\phi \left[v_5 \cos\left(\frac{\theta}{2}\right) - \theta_2 |\vec{p}_a| \left(\frac{r_{10} G_+}{\sqrt{s_1}} - \frac{r_6 |\vec{p}_a| v_7}{3}\right)\right] \tag{55-a}$$

$$\mathrm{Im}(\pm 3/2, \pm 1/2) = \mathrm{Im}_1 + s_2\, \mathrm{Im}_2 + s_3\, \mathrm{Im}_3 + \mathrm{Im}_4 \cos\phi \tag{55-b}$$

$$\mathrm{Re}(\mp 3/2, \pm 1/2) = \frac{sU'}{2} \sin\alpha \sin\phi \left[v_6 \sin\left(\frac{\theta}{2}\right) - \theta_1 |\vec{p}_a| \left(\frac{r_{10} G_-}{\sqrt{s_1}} - \frac{r_6 |\vec{p}_a| v_8}{3}\right)\right] \tag{56-a}$$

$$\mathrm{Im}(\mp 3/2, \pm 1/2) = \mathrm{Im}_5 + s_2\, \mathrm{Im}_6 + s_3\, \mathrm{Im}_7 + \mathrm{Im}_8 \cos\phi \tag{56-b}$$

$$\mathrm{Re}(\pm 1/2,\pm 1/2) = s\ \sin\alpha\ \sin\phi\ \sqrt{2/3}\left\{\frac{|\vec{p}_\alpha|}{6} v_7 \left[\left[(r_9 U' - 3S')\ \sqrt{s_1}\right.\right.\right.$$

$$\left.\left. + r_6 E_\alpha U'\right] v_1\ \sin(\theta/2) - r_6 v_2 |\vec{p}_\alpha| \theta_4 U'\right] + U' \left[\frac{\theta_1}{4}\left[\frac{r_{10}|\vec{p}_\alpha|}{\sqrt{s_1}} G_- - \frac{r_6 |\vec{p}_\alpha|^2 v_8}{3}\right]\right.$$

$$\left.\left. + \frac{r_{10}}{2\sqrt{s_1}}\left[v_3 v_1\ \sin(\theta/2) + v_2 |\vec{p}_\alpha| \theta_4\right] G_+\ \right] + \frac{U'}{4}\sin(\theta/2)\ v_6\right\} \tag{57-a}$$

$$\mathrm{Im}(\pm 1/2,\pm 1/2) = \mathrm{Im}_9 + s_2\ \mathrm{Im}_{10} + s_3\ \mathrm{Im}_{11} + \mathrm{Im}_{12}\ \cos\phi \tag{57-b}$$

$$\mathrm{Re}(\mp 1/2,\pm 1/2) = \frac{s\ \sin\alpha\ \sin\phi}{\sqrt{6}}\left\{U'\left[\frac{r_{10}|\vec{p}_\alpha|}{2\sqrt{s_1}} G_+\ \theta_2 - \frac{r_6}{6}\ |\vec{p}_\alpha|^2 v_7 \theta_2\right.\right.$$

$$\left. - \frac{r_{10}}{\sqrt{s_1}}\ (v_3 v_1 + v_2 |\vec{p}_\alpha| \cos\theta) G_-\ \cos(\theta/2)\right] - \frac{|\vec{p}_\alpha|}{3}\left[((r_9 U' - 3S')\ \sqrt{s_1}\right.$$

$$\left.\left. + r_6 E_\alpha U') v_1 - r_6 v_2 |\vec{p}_\alpha| U'\right] v_8 \theta_3 + \frac{U'}{2}\cos(\theta/2) v_5\right\} \tag{58-a}$$

$$\mathrm{Im}(\mp 1/2,\pm 1/2) = \mathrm{Im}_{13} + s_2\ \mathrm{Im}_{14} + s_3\ \mathrm{Im}_{15} + \mathrm{Im}_{16}\ \cos\phi \tag{58-b}$$

$$\mathrm{Im}_1 = \frac{s}{2} U'\ \theta_1\left[v_5\ \cos\alpha - r_{10} v_8 |\vec{p}_\alpha| - r_6\ \frac{|\vec{p}_\alpha|}{3\sqrt{s_1}} G_-\ \sin\alpha\right] \tag{55-b1}$$

$$\mathrm{Im}_2 = |\vec{p}_\alpha| \theta_1 (v_{10} U' - E_- T') \tag{55-b2}$$

$$\mathrm{Im}_3 = |\vec{p}_\alpha| \theta_1 r_6 E_- U' \tag{55-b3}$$

$$\mathrm{Im}_4 = \left[\theta_2 |\vec{p}_\alpha| \left(\frac{r_{10} G_+}{\sqrt{s_1}} - \frac{r_6 |\vec{p}_\alpha|}{3} v_7\right) + v_5 \theta_3\right] \frac{sU'}{2}\ \ \sin\alpha \tag{55-b4}$$

$$\mathrm{Im}_5 = \frac{s}{2} U' \theta_2 \left[v_6\ \ \cos\alpha - |\vec{p}_\alpha| \left(r_{10} v_7 + \frac{|\vec{p}_\alpha| r_6}{3\sqrt{s_1}} G_+\ \sin^2\alpha\right)\right] \tag{56-b1}$$

$$\mathrm{Im}_6 = |\vec{p}_\alpha| \theta_2 (v_9 U' - E_+ T') \tag{56-b2}$$

$$\mathrm{Im}_7 = |\vec{p}_\alpha| \theta_2 r_6 E_+ U' \tag{56-b3}$$

$$\text{Im}_8 = \left[v_6\theta_4 - \theta_1|\vec{p}_\alpha| \left(\frac{r_{10}G_-}{\sqrt{s_1}} - \frac{r_6|\vec{p}_\alpha|}{3} v_{,}\right]\right] \frac{s}{2} U' \sin\alpha \qquad (56\text{-b4})$$

$$\text{Im}_9 = \left\{ (2/3)^{1/2} \sqrt{s_1} E_- S' - \frac{U'}{\sqrt{6}} (v_5 + r_{10}v_3v_8) + \frac{|\vec{p}_\alpha|}{6\sqrt{s_1}} (2/3)^{1/2} \left[(r_9U' - 3S')\sqrt{s_1} \right.\right.$$

$$\left.\left. + r_6E_\alpha U'\right] G_- \sin^2\alpha \right\} v_1 s \cos(\theta/2) + \frac{sU'}{\sqrt{6}} \left\{ v_5\theta_3\cos\alpha - r_{10}v_2|\vec{p}_\alpha|\theta_3 v_8 \right.$$

$$\left. + \frac{r_{10}|\vec{p}_\alpha|}{2}\theta_2 v_7 - \frac{|\vec{p}_\alpha|^2}{\sqrt{6}\sqrt{s_1}}(2/3)^{\frac{1}{2}} r_6 v_2 G_- \theta_3 \sin^2\alpha + \frac{r_6|\vec{p}_\alpha|^2}{6\sqrt{s_1}} G_+\theta_2\sin^2\alpha \right\} \qquad (57\text{-b1})$$

$$\text{Im}_{10} = (2/3)^{1/2} \left\{ v_1\left[\sqrt{s_1}E_- T' + v_3\left[v_{10}U' - E_- T'\right]\right]\cos(\theta/2) \right.$$

$$\left. + |\vec{p}_\alpha| v_2\theta_3 \left[v_{10}U' - E_- T'\right] - \frac{|\vec{p}_\alpha|}{2}\left[v_9U' - E_+T'\right]\theta_2 \right\} \qquad (57\text{-b2})$$

$$\text{Im}_{11} = U'(2/3)^{1/2} \left\{ v_1\left[r_6v_4E_- - \frac{s_1E_+}{m_1}\right]\cos(\theta/2) + r_6|\vec{p}_\alpha|\left\{v_2E_-\theta_3 - \frac{E_+\theta_2}{2}\right\}\right\}$$

(57-b3)

$$\text{Im}_{12} = (2/3)^{1/2} s \sin\alpha \left\{ \frac{|\vec{p}_\alpha| v_7}{6} \left[\left[(r_9U' - 3S')\sqrt{s_1} + r_6E_\alpha U'\right] v_1 \sin(\theta/2) \right.\right.$$

$$\left. - r_6v_2|\vec{p}_\alpha|\theta_4 U'\right] + U'\left[\theta_1\left[\frac{r_{10}|\vec{p}_\alpha|G_-}{4\sqrt{s_1}} - \frac{r_6|\vec{p}_\alpha|^2}{12} v_8\right]\right.$$

$$\left.\left. + \frac{r_{10}}{2s_1}\left[v_3v_1\sin(\theta/2) + v_2|\vec{p}_\alpha|\theta_4\right] G_+ - \frac{1}{2}\left[v_5v_2\theta_1 - \frac{v_6}{2}\theta\right] \right\}$$

(57-b4)

$$\text{Im}_{13} = s\,(2/3)^{1/2} \left\{ v_1\sqrt{s_1}E_+ S' \sin(\theta/2) + \frac{|\vec{p}_\alpha|}{6\sqrt{s_1}} \left[((r_9U' - 3S')\sqrt{s_1} \right.\right.$$

$$\left. + r_6E_\alpha U')v_1 - r_6v_2|\vec{p}_\alpha|U'\right] G_+ \theta_4\sin^2\alpha + \frac{U'}{2}\left[(v_6v_2\theta_4 + \frac{v_5\theta_1}{2})\cos\alpha\right.$$

$$- (v_6 + r_{10}v_3v_7)\sin(\theta/2)v_1 - (v_2v_7\theta_4 + \frac{v_8\theta_1}{2})\, r_{10}|\vec{p}_\alpha|$$

$$\left.\left. - \frac{r_6|\vec{p}_\alpha|^2}{6\sqrt{s_1}} G_-\theta_1 \sin^2\alpha\right]\right\} \tag{58-b1}$$

$$Im_{14} = (2/3)^{\frac{1}{2}} \left\{ v_1 \sin(\theta/2) \left[E_+ \sqrt{s_1}\, T' + v_3[v_9 U' + E_+ T']\right]\right.$$

$$\left. + |\vec{p}_\alpha| \left[v_2[v_9 U' - E_+ T']\, \theta_4 + [v_{10} U' - E_- T'] \frac{\theta_1}{2}\right]\right\} \tag{58-b2}$$

$$Im_{15} = (2/3)^{\frac{1}{2}} U' \left[v_1 (r_6v_4E_+ - \frac{s_1E_-}{m_1}) \sin(\theta/2) + r_6|\vec{p}_\alpha|(v_2E_+\theta_4 + \frac{E_-\theta_1}{2})\right] \tag{58-b3}$$

$$Im_{16} = \frac{s \sin\alpha}{\sqrt{6}} \left\{ U' \left[\frac{r_{10}|\vec{p}_\alpha|}{2\sqrt{s_1}} G_+\theta_2 - \frac{r_6|\vec{p}_\alpha|^2}{6} v_7\theta_2\right.\right.$$

$$\left. - \frac{r_{10}}{\sqrt{s_1}} (v_3v_1 + v_2|\vec{p}_\alpha|\cos\theta)G_- \cos(\theta/2)\right]$$

$$- \frac{|\vec{p}_\alpha|}{3} \left[((r_9U' - 3S')\sqrt{s_1} + r_6E_\alpha U')v_1 - r_6v_2|\vec{p}_\alpha|\, U'\right]v_8\theta_3$$

$$\left. + U' \left(\frac{v_5\theta_3}{2} - v_6v_2\theta_2\right)\right\} \tag{58-b4}$$

5. RESULTS AND CONCLUSIONS

This paper contalns many extense calculations. In order to make it easy to read, we have found necessary to describe these calculations in great technical detail. Besides, the case of $pp \to \Delta^{++}\pi^- p$ reaction is not trivially calculated in the framework of (T C D M). So in this case it is not possible to obtain qualitatlve Informations from (T C D M) without performing numerical calculations. This is presented below. On the other hand we need to test whether the general predictlons of the (T C D M) are mantained at least qualitatively in the present case. We recall that these are the existence of the "dips" in $d\sigma/dt_2$ or "zeros" in amplitudes in windows of invariant mass M_{12}, and $\cos\theta^{G.J.}$. This is the slope-mass-$\cos\theta^{G.J.}$ correlation. Or for certain processes slope-mass-partia1 wave correlation, as observed experimentally[9]. The principal hypothesis and approximations used in our model, throughout the calculations presented here, are

(i) concerning the (DDR), it is evident that in general we consider the (H E A) (eq.(A-6)). These approximations, identified as diffractive region for hadron-hadron, appear in all calculations and at all levels, so that the final amplitude obtained is rigorously valid in these conditions which are the experimental conditions for (DDR) data.

(ii) The (T C D M) considers only the three Born terms, corresponding to the graphs presented in Fig.A1. No more components and extra effects such as backgrounds, threshold effects, form factors, resonances eventually produced in physical regions considered, etc., are added to our model. The applicability of the (T C D M) and comparability with experimental data for all distributions are possible when we can dualize it in order to avoid double counting, as it has been done in earlier papers[3]. It is very difficult to say now if it is possible or not to dualize the present case in the present form of the amplitudes. This is an open problem.

(iii) Pomeron factorization. The P-factorization hypothesis seems to be a well established one, and we make use of it extensively. From spin-structure point of view, the diffractive vertex $(p\,\mathbb{P}p)(1/2,1,1/2)$ in our approximations (H E A) behaves as $(0,1,0)$ one. This nice property of this vertex together with the $\mathbb{P}$-factorization hypothesis gives a simpler spin structure amplitude. From our calculations point of view this is very welcome, since the simple structure obtained for the $(p\mathbb{P}p)$ vertex does not interfere with the remaining parts of the amplitude. This may be represented by Fig. 5.1.

(iv) Helicity conservation. This hipothesis is used in the simplificatión of all diffractive vertices. The $C^{\mu\nu\alpha}$ tensor representing the coupling of the $(3/2,3/2,1)$ vertex is treated with this hypothesis to arrive at a final expression with only one constant. On the other hand we recall that there is a reasonable experimental support[7] for helicity conservation.

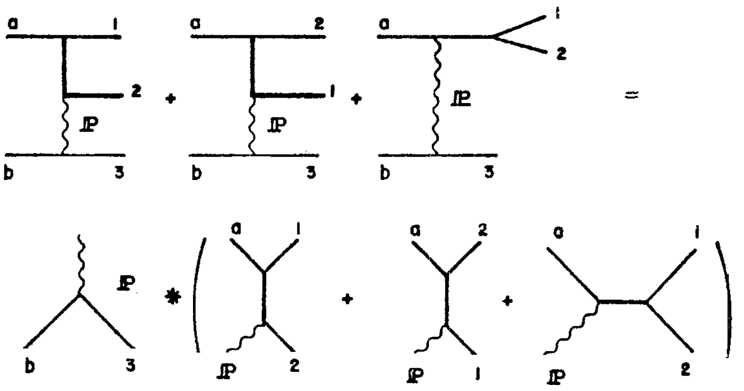

Fig. 5.1 - IP-factorization in the (T C D M)

Summlng up, the maln relevant steps of our calculations are:

(i) First, wemakea complete treatmentof the (ΔIPΔ) vertex, the source of the rnain complications of the present calculations. In sectlon *2* we show how (ΔIPΔ) can be obtained from a hypothetic reaction in order to have an expression containing just one constant $g_,$. The estimation of this constant is made through the Optical theorem and using the (γIPA)[5] hypothesis, which results in $g_1 = g_{\Delta \mathbb{P} \Delta}$. Sn, this is easily obtained just with experimental inputs as total asymptotic cross-section. This procedure has the advantage of giving a result which is fixed by parameters well established only experimentally.

'(ii) Each amplitude is obtained in the framework of (T C D M), as it is shown in section 3. The expressions (28), (32) and (38) below

$$T = 2\delta_{\lambda_b \lambda_3} T\, R.Q\, \overline{\psi}_\mu (p_1, \lambda_1) p_a^\mu\, u(p_a, \lambda_a) \qquad (28)$$

$$S = \delta_{\lambda_b \lambda_3} S\, \overline{\psi}_\mu (p_1, \lambda_1) p_2^\mu (\not{p} + m_a) \not{R}\, u(p_a, \lambda_a) \qquad (32)$$

and

$$U = -\delta_{\lambda_b \lambda_3} R_\beta\, U \overline{\psi}_\mu (p_1, \lambda_1) \Gamma^{\mu\beta\nu} (m_1 + \not{k}) \Lambda_{\nu\sigma}(k) p_2^\sigma\, u(p_a, \lambda_a) \qquad (38)$$

represent the three components from (T C D M) and they were added coherently. This point is very important since the interfererice effects are one of the main features of the modet. So the complete amplitude reads

$$A = S + T + U \tag{42}$$

(iii) Approximations. Next, we write this total amplitude taking into account all (H E A) above referred In terms of the invariantsonly. This Is made In section 5.

Now we study numerically the structures that may arise from the interference mechanism in the amplitude at high energy and low t_2, in particular, the "zeros" that may appear strongly in some physical region.

We show in Fig.5 .2 the tn-distributions $(d\sigma/dt_2)$ which correspond to different values for the elastic subreactions $(\sigma^{\infty}_{\pi N}$, σ^{∞}_{NN}, $\sigma^{\infty}_{\Delta N}$, $b_{\pi N}$, b_{NN} and $b_{\Delta N})$. We may conclude that for a particular choice of these parameters (Fig. 5.2) we can see dIfferent regimes of the t_2-distribution. It is clear that only the experimental results may select the best set of parameters. This procedure may also be useful for the estimation of the off-mass-shell effects in the subreactions considered.

In Fig. 5.3a,b we show the $\cos\theta^{G.J}$ distribution for the same parameters as used in Fig.5.2. The forward and backward enhancements are characteristics of the π and Δ exchange, respectively. In Fig. 5.4 we show the azimuthal distribution with the same parameters. All these curves presented here are the predictlons of our model, since experrimental results does not exist.

In next paper we will present a partia1 wave analysisforthe reaction studied here.

We would like to acknowledge Mrs. Ligia Rodrigues foracritical reading of the manuscript.

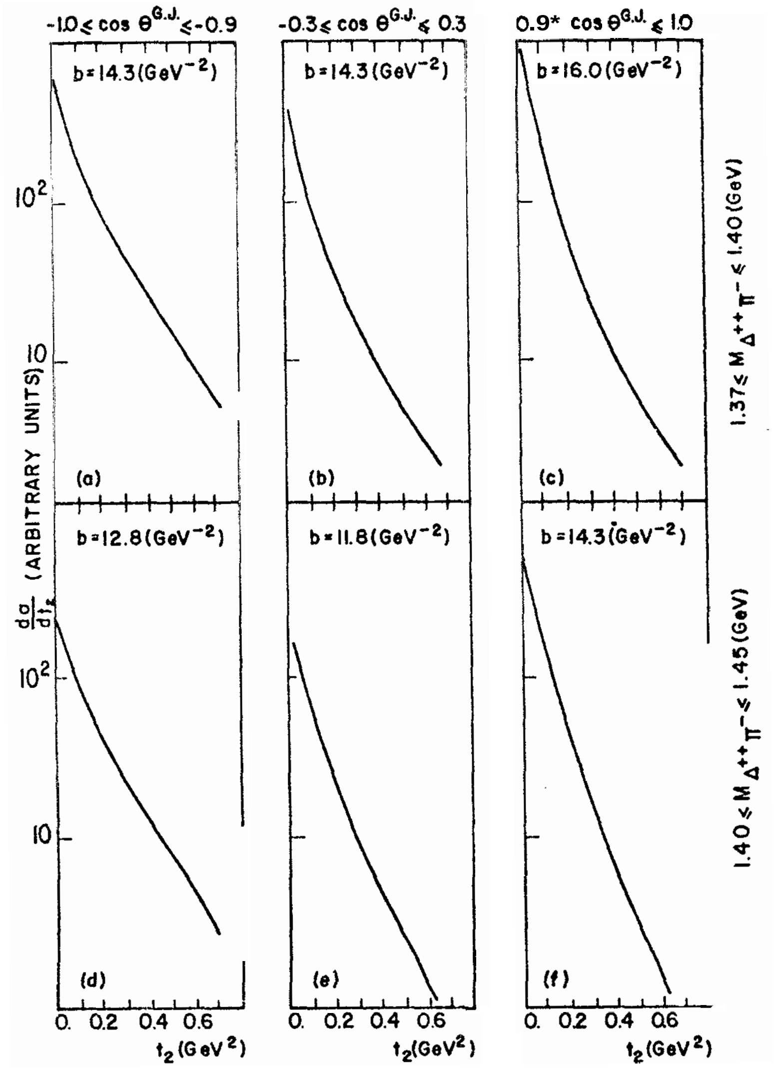

Fig. 5.2 - t_2 distributions integrated in several cosθG.J. and $M_{\Delta\pi}$ regions. The values of the parameters are (see text): $b_{\pi N}$ = 10. (GeV^{-2}), b_{NN} = 9. (GeV^{-2}), $b_{N\Delta}$ = 8. (GeV^{-2}); $\sigma_{Tot}^{\infty}(\pi N)$ = 25.0 (mb), $\sigma_{Tot}^{\infty}(NN)$ = 50 (mb) and $\sigma_{Tot}^{\infty}(\Delta N)$=80 (mb).

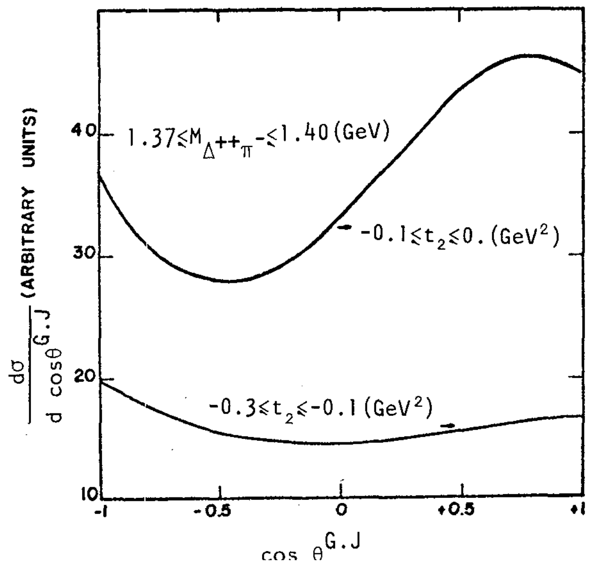

Fig.5.3(a)

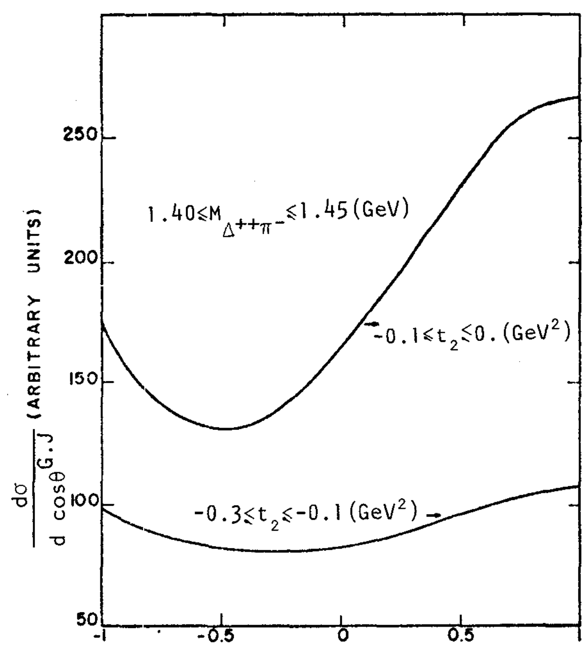

Fig.5.3(b) cos $\theta^{G.J}$

Fig.5.3 - $d\sigma/d\cos\theta^{G.J.}$ distributions for the same set of parameters used in Fig. 5.2. (a) for $1.37 \leqslant M_{\Delta^{++}\pi^-} \leqslant 1.40$ (GeV) and (b) for $1.4 \leqslant M_{\Delta^{++}\pi^-} \leqslant 1.45$ (GeV).

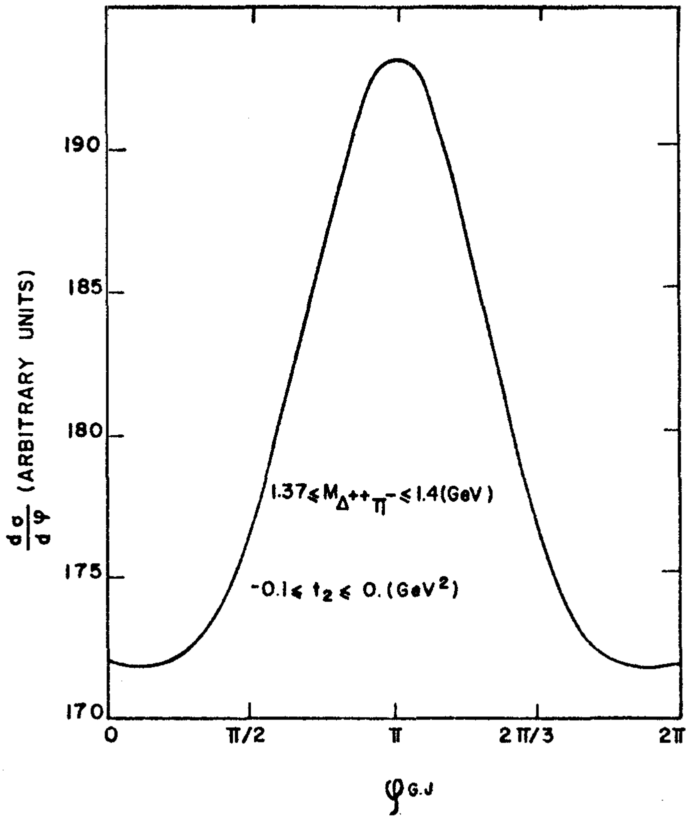

Fig. 5.4 - $\frac{d\sigma}{d\phi^{G.J.}}$ distribution for the same set of parameters used in Fig. 5.2.

APPENDIX A

Kinematics of the pp → $\Delta^{++}\pi^-$p Reactions

In this Appendix we present the main kinematic expressions concerning the present work. This is a simple application of the well known particles kinematics[10].

Fig. A1 shows the three graphs corresponding to the (T C D M) described in section 3. The 4-vectors p_i ($i = a,b$, 1, 2, 3) related by energy-momentum conservation ($p_a+p_b = p_1+p_2+p_3$), represent the external particles of the reaction (1) as is shown in fig. A1. For internal lines we define

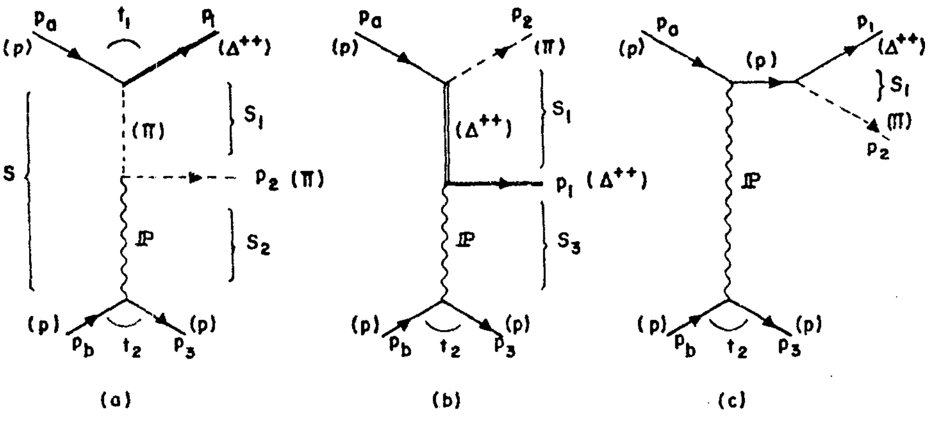

Fig.A1 - These graphs represent the (T C D M) described in the text for $pp \to p\Delta^{++}\pi^-$ reaction. (a) is the π-exchange or the T-component term, (b) the Δ-exchange or the U-component and (c) the "direct"-pole - exchange or the S-component. Here the S, T and U components are referred to the s_1, t and u channels of the $p\mathbb{P} \to \Delta^{++}\pi^-$ subreaction.

$$q = p_a - p_1 \ , \ k = p_a - p_2 \quad \text{and} \quad p = p_1 + p_2 \qquad (A.1)$$

and, for convenience, the particular 4-vectors

$$P = \frac{1}{2}\,(p_1+k) \ ; \ Q = \frac{1}{2}\,(q+p_2) \ ; \ R = \frac{1}{2}\,(p_b+p_3) \quad \text{and} \quad K = \frac{1}{2}\,(p_a+p) \qquad (A.2)$$

With the 4-vectors $p_i (i=0,\dots,3)$ we can define a set of useful invariants as

$$
\begin{aligned}
s &= (p_a+p_b)^2 \\
s_1 &= (p_1+p_2)^2 \\
s_2 &= (p_2+p_3)^2 \\
s_3 &= (p_1+p_3)^2
\end{aligned}
\qquad (A.3)
$$

$$
\begin{aligned}
t_1 &= (p_a-p_1)^2 \\
u_1 &= (p_a-p_2)^2 \\
t_2 &= (p_b-p_3)^2 \\
u_2 &= (p_b-p_2)^2
\end{aligned}
\qquad (A.4)
$$

The mass of the particles involved are denoted by

$$
\begin{aligned}
a &= m_b = m_3 = m_p \\
m_1 &= m_\Delta \\
m_2 &= m_\pi
\end{aligned}
\qquad (A.5)
$$

whose quantitative values we have taken from the Particle Data Group Review[11].

High - Energy - Approximations

Some invariants at very high energy can be simplified by reasonable approximations. These approxirnations for high energy reactions corresponding to physical region of (DDR), are determined by

$$s, s_2, s_3 >> s_1 \ , \ (\text{masses})^2 \ , \ t_1, t_2, u_1 \qquad (A.6)$$

Putting this condition (A.6) in the relation

$$s_2 + t_2 + u_2 = m_b^2 + m_2^2 + m_3^2 + t_1 \qquad (A.7)$$

we obtain

$$s_2 \simeq -u_2 \tag{A.8}$$

and for the scalar products

$$4Q.R = s_2 - u_2 \tag{A.9}$$

$$2K.R = s - \frac{1}{2}\,(s_1+m_a^2+m_b^2+m_3^2-t_2) \tag{A.10}$$

$$2k.R = 2P.R = s_3 - \frac{1}{2}\,(m_b^2+m_1^2+m_3^2+u_1-t_2) \tag{A.11}$$

$$2k.p_2 = m_a^2 - m_2^2 - u_1 \tag{A.12}$$

$$4P.p_2 = s_1 - u_1 + m_a^2 - m_1^2 - 2m_2^2 \tag{A.13}$$

$$4P.k = 3u_1 + m_1^2 - t_2 \tag{A.14}$$

$$4R.p_2 = s - s_3 + s_2 + u_1 - m_a^2 - m_2^2 - m_3^2 \tag{A.15}$$

and

$$(p_1+k)^2 = 2u_1 + 2m_1^2 - t_2 \tag{A.16}$$

By eq. (A.6) we have the approxirnated relations

$$2Q.R \simeq s_2 \tag{A.17}$$

$$2K.R \simeq s \tag{A.18}$$

$$2k.R \simeq 2p.R \simeq s_3 \tag{A.19}$$

$$4R.p \simeq s - s_3 + s_2 \simeq 2(s-s_3) \tag{A.20}$$

The others stay unchanged. The energies (E_i) and momenta (p_i) in the *R12* system, $\vec{p}_1+\vec{p}_2 = 0$, as function of the invariants, are given by

$$E_a = (s_1+m_a^2-t_2)/2\sqrt{s_1} \tag{A.21}$$

$$E_b = (s-m_a^2-m_3^2+t_2)/2\sqrt{s_1} \tag{A.22}$$

$$E_1 = (s_1+m_1^2+m_2^2)/2\sqrt{s_1} \tag{A.23}$$

$$E_2 = (s_1+m_2^2-m_1^2)/2\sqrt{s_1} \tag{A.24}$$

$$E_3 = (s-s_1-m_3^2)/2\sqrt{s_1} \qquad (A.25)$$

$$|\vec{p}_a| = (\lambda^{1/2}(s_1,m_a^2,t_2))/2\sqrt{s_1} \qquad (A.26)$$

$$|\vec{p}_b| = (\lambda^{1/2}(s_1,m_b^2,t_{a3}))/2\sqrt{s_1} \qquad (A.27)$$

$$|\vec{p}_1| = |\vec{p}_2| = (\lambda^{1/2}(s_1,m_1^2,m_2^2))/2\sqrt{s_1} \qquad (A.28)$$

$$|\vec{p}_3| = (\lambda^{1/2}(s_1,m_3^2,s))/2\sqrt{s_1} \qquad (A.29)$$

where

$$t_{a3} + s + t_2 = s_1 + m_a^2 + m_b^2 + m_3^2. \qquad (A.30)$$

Then, in conditions *(A.6)* we have, for relation (A.30)

$$t_{a3} \simeq -s \qquad (A.31)$$

and for energles and momenta

$$E_b = E_3 = |\vec{p}_b| = |\vec{p}_3| \simeq s/2\sqrt{s_1} \qquad (A.32)$$

and the others stay unchanged. $\lambda(x,y,z)$ is defined by

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy+xz+yz) \qquad (A.33)$$

Figure (A2) shows the Gottfried-Jackson system (GJS) for $\vec{p}_1+\vec{p}_2=0$. The z-axis is defined by $\vec{p}_a$ and the y-axis by $\vec{p}_3\times\vec{p}_b$. The orientations of the other vectors are

$$\vec{p}_1 = \vec{p}_1(\theta,\phi) \ ; \ \vec{p}_b = \vec{p}_b(\chi,0) \quad \text{and } \vec{p}_3 = \vec{p}_3(\alpha,0) \qquad (A.34)$$

The angle β between $\vec{p}_3$ and $\vec{p}_1$ satisfies the relation

$$\cos\beta = \cos\alpha \cos\theta + \sin\alpha \sin\theta \cos\phi \qquad (A.35)$$

The angle a is given by

$$\cos\alpha = \frac{1}{2|\vec{p}_a||\vec{p}_3|}\,(2E_aE_3 - 2\sqrt{s_1}\,E_b+s_1+m_b^2-m_a^2-m_3^2) \qquad (A.36)$$

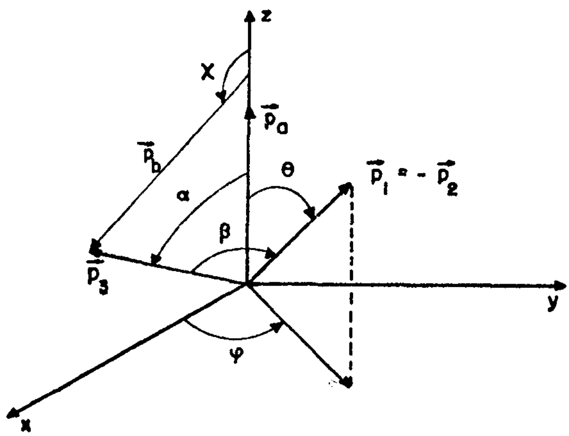

Fig. A2 - Gottfried-Jackson systern for R12 $\equiv \vec{p}_1 + \vec{p}_2 = 0$

and in the (H E A) eq.(A.6), we have

$$\cos\alpha \simeq -(s_1 - m_a^2 + t_2)/\lambda^{1/2}(s_1, m_a^2, t_2) \qquad (A.37)$$

and

$$\sin\alpha \simeq 2\sqrt{s_1}\sqrt{-t_2}/\lambda^{1/2}(s_1, m_a^2, t_2)$$

Expanding in power series of t_2, for $|t_2| << (s_1 - m_a^2)^2/2s_1$, we have

$$\cos\alpha \simeq -1 + O(t_2) \;; \quad \sin\alpha \simeq 2\sqrt{s_1}\sqrt{-t_2}/(s_1 - m_a^2) \qquad (A.38)$$

and relation (A.35) reads

$$\cos\beta = -\cos\theta + \left[2\sqrt{s_1}\sqrt{-t_2}/(s_1 - m_a^2)\right]\sin\theta\cos\phi \qquad (A.39)$$

With eq.(A.6) we have too

$$s_2 \simeq (s/\sqrt{s_1})\ (E_2 + |\vec{p}_1|\cos\beta) \qquad (A.40)$$

and

$$s_3 \simeq (s/\sqrt{s_1})\ (E_1 - |\vec{p}_1|\cos\beta)$$

and by using eq. (A.39) we have

$$s_2 \simeq (s/\sqrt{s_1})(E_2 - |\vec{p}_1|\cos\theta) + \frac{2s|\vec{p}_1|\sqrt{-t_2}}{s_1 - m_a^2} \sin\theta \cos\phi$$

$$s_3 \simeq (s/\sqrt{s_1})(E_1 + |\vec{p}_1|\cos\theta) - \frac{2s|\vec{p}_1|\sqrt{-t_2}}{s_1 - m_a^2} \sin\theta \cos\phi \qquad \text{(A.41)}$$

In the same approximation we derive

$$(t_1 - m_2^2) \simeq - \frac{(s_1 - m_a^2)}{\sqrt{s_1}} (E_2 - |\vec{p}_1|\cos\theta)$$

$$(u_1 - m_1^2) \simeq - \frac{(s_1 - m_a^2)}{\sqrt{s_1}} (E_1 + |\vec{p}_1|\cos\theta) \qquad \text{(A.42)}$$

From eqs. (A.41) and (A.42) we obtain

$$\frac{s_3}{u_1 - m_1^2} \simeq - \frac{s}{s_1 - m_a^2} \left[1 - \frac{2|\vec{p}_1|\sqrt{s_1}\,\sqrt{-t_2}\,\sin\theta\cos\phi}{(s_1 - m_a^2)(E_1 + |\vec{p}_1|\cos\theta)} \right]$$

$$\frac{s_2}{(t_1 - m_2^2)} \simeq - \frac{s}{s_1 - m_a^2} \left[1 + \frac{2|\vec{p}_1|\sqrt{s_1}\,\sqrt{-t_2}\,\sin\theta\cos\phi}{(s_1 - m_a^2)(E_2 - |\vec{p}_1|\cos\theta)} \right] \qquad \text{(A.43)}$$

and in the limit $t_2 = 0$ we have the relation

$$\frac{s_2}{t_1 - m_2^2} \simeq \frac{s_3}{u_1 - m_1^2} \simeq - \frac{s}{s_1 - m_a^2} \qquad \text{(A.44)}$$

The expressions for cross-sections used in the text are defined by the following formulae.

For a reaction 2 → 2 like $a(p,\lambda_a) + b(q,\lambda_b) \to a(p',\lambda_a') + b(q',\lambda_b')$ the differential cross-section for non-polarized beam and target, we have

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{16\pi\lambda(s,m_a^2,m_b^2)} \frac{1}{(2s_a+1)(2s_b+1)} \sum_{\substack{\lambda_a,\lambda_b \\ \lambda_a',\lambda_b'}} |A(s,t;\lambda_a\lambda_b,\lambda_a'\lambda_b')|^2 \qquad \text{(A.45)}$$

where $s=(p+q)^2$, $t=(p-p')^2$, s_a and s_b are the spins of the particles a and b and the h_a and λ_b are the helicities, respectively. (The same notation is used for final particles λ'_a, λ'_b).

For a general reaction $a+b \to 1+2+3$ we use for differential cross-sections

$$d\sigma = c \int \frac{\lambda^{1/2}(s_1,m_1^2,m_2^2)}{s_1} \; ds_1 dt_2 d\cos\theta^{G.J.} d\phi^{G.J.} |A|^2 \tag{A.46}$$

where

$$c = 1/(2^{10}\pi^4 \lambda(s,m_a^2,m_b^2)) \tag{A.47}$$

APPENDIX B

The Spin 3/2 Wave Functions (3/2 SWF)

This Appendix concerns the Rarita-Schwinger wave functions used in this paper. Although it is a well known subject[12] we put here explicitly our conventions and normalizations for a self consistent exposition.

The wave functions for spin 3/2 particles, for positive and negative energies, in the momentum (p) and helicity (λ) representation, $\psi_\mu^{(\pm)}(p,\lambda)$ (where μ is a Lorentz label) satisfy the Rarita-Schwinger equations

$$(\not p \mp m)\, \psi_\mu^{(\pm)}(p,\lambda) = 0 \tag{B.1}$$

where m is the spin 3/2 particle mass. They are subjected to the subsidiary condition

$$p^\mu \psi_\mu^{(\pm)}(p,\lambda) = 0 \tag{B.2}$$

We can also obtain a second subsidiary equation from eq.(B.2) using eq.(B.1) above

$$g^{\mu\nu} p_\mu \psi_\nu^{(\pm)}(p) = (1/2)(\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu) p_\mu \psi_\nu^{(\pm)}(p) = (1/2)(\not p \pm m)\gamma^\nu \psi_\nu^{(\pm)}(p) = 0$$

Then the (3/2 SWF) also satisfies the condition

$$\gamma^\mu \psi_\mu^{(\pm)}(p,h) = 0 \tag{B.3}$$

As a matter of fact, to completely define a (3/2 SWF), $\psi_\mu^{(\pm)}(p,\lambda)$, we need eq. (B.1) and one subsidiary condition that can be eq. (8.2) or (B.3). In momentum and helicity representation, the (3/2 SWF) can be written as a combination of a vector and a Dirac spinor[12] as

$$\psi_\mu^{(+)}(p,\lambda) = \sum_{\substack{\lambda_1,\lambda_2 \\ (\lambda_1+\lambda_2=\lambda)}} C(\lambda_1,\lambda_2,\lambda)\ \varepsilon_\mu(p,\lambda_1)u(p,\lambda_2) \tag{B.4}$$

for positive energies, and

$$\psi_\mu^{(-)}(p,\lambda) = \sum_{\substack{\lambda_1,\lambda_2 \\ (\lambda_1+\lambda_2=\lambda)}} C(\lambda_1,\lambda_2,\lambda)\ \varepsilon_\mu^*(p,\lambda_1)v(p,\lambda_2) \tag{B.5}$$

for negative energies, where $C(\lambda_1,\lambda_2,\lambda)=\langle 1,\lambda_1;1/2,\lambda_2|3/2,\lambda;1,1/2\rangle$ are the Clebsh-Gordon coefficients for $1 \oplus 1/2 = 3/2$ coupling.

The spin 1 fields fulfill the Proca equations

$$(p^2-m^2)\varepsilon_\mu(p,\lambda_1) = 0 \tag{B.6}$$

with the subsldiary equation

$$p^\mu\varepsilon_\mu(p,\lambda_1) = 0 \tag{B.7}$$

(ε^* for negative energies) and the spin 1/2 fields, the Dirac equation

$$(\not{p}-m)u(p,\lambda_2) = 0 \tag{B.8}$$

for positive energies and

$$(\not{p}+m)v(p,\lambda_2) = 0 \tag{B.9}$$

for negative energies.

Normalizations

The fields of spin 1/2 and 1 are normalized following the relation[12c]

$$\bar{u}(p,\lambda')u(p,\lambda) = 2m\ \delta_{\lambda'\lambda} \tag{8.10}$$

$$\bar{v}(p,\lambda')v(p,\lambda) = -2m\ \delta_{\lambda'\lambda} \tag{B.11}$$

and

$$\varepsilon^*_\mu(p,\lambda')\varepsilon^\mu(p,\lambda) = -\ \delta_{\lambda'\lambda} \tag{B.12}$$

so that for spin 3/2 fields we have,

$$\bar{\psi}^{(\pm)}_\mu(p,\lambda')\ \psi^\mu\quad(p,\lambda) = \mp\ 2m\ \delta_{\lambda'\lambda} \tag{B.13}$$

With this normalization the spin 3/2 field projector reads[12,13]

$$P^{\mu\nu(\pm)}(p) = \frac{m \pm \not{p}}{2m}\left[g^{\mu\nu} - \frac{2}{3m^2}p^\mu p^\nu - \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3m}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\right] \tag{B.14}$$

Explicit Form of Rarita-Schwinger Wave Function in the Mornentum and Helicity Representations

The conjugate wave function, for positive energies, of eq. (B.4), is defined by

$$\begin{aligned}\bar{\psi}_\mu(p,\lambda) &\equiv \psi^+(p,\lambda)\gamma^0\\ \bar{u}(p,\lambda) &\equiv u^+(p,\lambda)\gamma^0\end{aligned} \tag{B.15}$$

The Dirac spinors can be written in the momentum and helicity representation[12c] as

$$u(\vec{p},\lambda) = N(p)\begin{bmatrix}(E+m)I\\ \vec{\sigma}.\vec{p}\end{bmatrix}\chi_\lambda(\hat{p}) \tag{B.16}$$

where the normalization factor, from eq. (B.10), is $N(p)=1/\sqrt{E+m}$; I and $\vec{\sigma}$ are the identity and Pauli matrices, respectively. And $\chi_\lambda(\hat{p})$ are the spinors so that for ($\vec{p} = p\hat{z}$)

$$\chi_{+1/2}(\hat{z}) = \begin{pmatrix}1\\0\end{pmatrix} \quad \text{and} \quad \chi_{-1/2}(\hat{z}) = \begin{pmatrix}0\\1\end{pmatrix} \tag{B.17}$$

When the momentum has an arbitrary orientation $\vec{p}=\vec{p}(\theta,\phi)$, where θ and ϕ are the polar and azimuthal angles of the vector $\vec{p}$ in a particular referential, we can write

$$\chi_\lambda(\hat{p}) = D^{(1/2)}(\phi,\theta,-\phi)\chi_\lambda(\hat{z}) \tag{B.18}$$

where[14]

$$D^{(1/2)}(\phi,\theta,-\phi) = e^{-i\sigma_z\phi/2}\, e^{-i\sigma_y\theta/2}\, e^{i\sigma_z\phi/2} \tag{B.19}$$

For spin 1 field with the momentum oriented in an arbitrary direction $\vec{p}(\theta,\phi)$, we have

$$\varepsilon^{\mu}(\vec{p},0) = \frac{1}{m}\begin{pmatrix} p \\ E\sin\theta\cos\phi \\ E\sin\theta\sin\phi \\ E\cos\theta \end{pmatrix}$$

and

$$\varepsilon^{\mu}(\vec{p},\pm 1) = \frac{e^{\pm i\phi}}{\sqrt{2}}\begin{pmatrix} 0 \\ \mp\cos\theta\cos\phi + i\sin\phi \\ \mp\cos\theta\sin\phi - i\cos\phi \\ \pm\sin\theta \end{pmatrix} \tag{B.20}$$

So, the (3/2 SWF) for the helicity states, taking into account the $1 \oplus 1/2$ Clebsh-Gordon coefficients, are

$$\psi_{\mu}(p,\pm 3/2) = \varepsilon_{\mu}(p,\pm 1)u(p,\pm 1/2)$$
$$\psi_{\mu}(p,\pm 1/2) = (2/3)^{1/2}\varepsilon_{\mu}(p,0)u(p,\pm 1/2) + (1/3)^{1/2}\varepsilon_{\mu}(p,\pm 1)u(p,\mp 1/2) \tag{B.21}$$

Now, with the wave functions of spins 1/2, 1 and 3/2 above defined, we obtain the following useful relations

$$\bar{u}(p',\lambda')\, u(p,\lambda) = (\alpha_1 - 4\lambda\lambda'\alpha_2)\chi^{+}_{\lambda'}(\hat{p}')\chi_{\lambda}(\hat{p}) \tag{B.22}$$

$$\bar{u}(p',\lambda')\gamma^0 u(p,\lambda) = (\alpha_1 + 4\lambda\lambda'\alpha_2)\chi^{+}_{\lambda'}(\hat{p}')\chi_{\lambda}(\hat{p}) \tag{B.23}$$

$$\bar{u}(p',\lambda')\vec{\gamma}\, u(p,\lambda) = (2\alpha_3\lambda + 2\lambda'\alpha_4)\chi^{+}_{\lambda'}(\hat{p}')\vec{\sigma}\chi_{\lambda}(\hat{p}) \tag{B.24}$$

where

$$\alpha_1 = \sqrt{E'+m'}\,\sqrt{E+m} \;;\; \alpha_2 = \sqrt{E'-m'}\,\sqrt{E-m}$$

$$\alpha_3 = \sqrt{E'+m'}\,\sqrt{E-m} \;\text{ and }\; \alpha_4 = \sqrt{E'-m'}\,\sqrt{E+m}$$

And using the matrix

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^{\mu},\gamma^{\nu}\right] \tag{B.25}$$

we obtain

$$\bar{u}(p',\lambda')\sigma^{0j}u(p,\lambda) = 2i(\lambda\alpha_3-\lambda'\alpha_4)\chi^{+}_{\lambda'}(\hat{p}')\sigma_j\chi_\lambda(\hat{p}) \tag{B.26}$$

and

$$\bar{u}(p',\lambda')\sigma^{ij}u(p,\lambda) = (\alpha_1-4\lambda'\lambda\alpha_2)\chi^{+}_{\lambda'}(\hat{p}')\sigma_k\chi_\lambda(\hat{p}) \tag{B.27}$$

with cyclic permutations of $i,j,k = 1,2,3$. With the choice $\vec{p} = \vec{p}(0,0)$ and $\vec{p}\,' = \vec{p}\,'(\theta,\phi)$ we obtain

$$\chi_\lambda(\hat{p}) = \chi_\lambda(\hat{z})$$

and

$$\chi^{+}_{\lambda'}(\hat{p}') = \chi^{+}_{\lambda'}(\hat{z})D^{(1/2)^{+}}(\phi,\theta,-\phi) \tag{B.28}$$

Then

$$\chi^{+}_{\lambda'}(\hat{p}')\chi_\lambda(\hat{p}) = e^{-i(\lambda'-\lambda)\phi}\left[\cos(\theta/2)\delta_{\lambda',\lambda}-2\lambda\sin(\theta/2)\delta_{\lambda',-\lambda}\right] \tag{B.29}$$

$$\chi^{+}_{\lambda'}(p')\sigma_x\chi_\lambda(\hat{p}) = e^{-i(\lambda'-\lambda)\phi}\left[\cos(\theta/2)\delta_{\lambda',-\lambda}+2\lambda\sin(\theta/2)\,\delta_{\lambda',\lambda}\right]$$

$$-2i\sin(\phi/2)\;e^{-i\lambda'\phi}\left[2\lambda\cos(\theta/2)\;\delta_{\lambda',-\lambda}+\sin(\theta/2)\delta_{\lambda',\lambda}\right] \tag{B.30}$$

$$\chi^{+}_{\lambda'}(\hat{p}')\sigma_y\chi_\lambda(\hat{p}) = i\,e^{-i(\lambda'-\lambda)\phi}\left[2\lambda\cos(\theta/2)\delta_{\lambda',-\lambda}+\sin(\theta/2)\delta_{\lambda',\lambda}\right]$$

$$+\,2\sin(\phi/2)e^{-i\lambda'\phi}\left[\cos(\theta/2)\delta_{\lambda',-\lambda}+2\lambda\sin(\theta/2)\delta_{\lambda',\lambda}\right] \tag{B.31}$$

$$\chi^{+}_{\lambda}(\hat{p}')\sigma\;\chi_\lambda(\hat{p}) = e^{-i(\lambda'-\lambda)\phi}\left[2\lambda\cos(\theta/2)\delta_{\lambda',\lambda}-\sin(\theta/2)\delta_{\lambda',-\lambda}\right] \tag{B.32}$$

We can now explicitly write the products of Dirac spinors

$$\bar{u}(p',\lambda')u(p,\lambda) = e^{-i(\lambda'-\lambda)\phi}(\alpha_1-4\lambda'\lambda\alpha_2)\left[\cos(\theta/2)\delta_{\lambda',\lambda}-2\lambda\sin(\theta/2)\delta_{\lambda',-\lambda}\right] \tag{B.33}$$

$$\bar{u}(p',\lambda')\gamma^0u(p,\lambda) = e^{-i(\lambda'-\lambda)\phi}(\alpha_1+4\lambda'\lambda\alpha_2)\left[\cos(\theta/2)\delta_{\lambda',\lambda}-2\lambda\sin(\theta/2)\delta_{\lambda',-\lambda}\right] \tag{B.34}$$

$$\bar{u}(p',\lambda')\gamma^1 u(p,\lambda) = 2(\lambda\alpha_3+\lambda'\alpha_4)\{e^{-i(\lambda'-\lambda)\phi}[\cos(\theta/2)\delta_{\lambda',-\lambda}+2\lambda\sin(\theta/2)\delta_{\lambda',\lambda}]$$

$$- 2i\sin(\phi/2)e^{-i\lambda'\phi}[2\lambda\cos(\theta/2)\delta_{\lambda',-\lambda} +\sin(\theta/2)\delta_{\lambda',\lambda}]\} \qquad \text{(B.35)}$$

$$\bar{u}(p',\lambda')\gamma^3 u(p,\lambda) = e^{-i(\lambda'-\lambda)\phi}2(\lambda\alpha_3+\lambda'\alpha_4)[2\lambda\cos(\theta/2)\delta_{\lambda',\lambda}-\sin(\theta/2)\delta_{\lambda',-\lambda}]$$

(B.36)

$$\bar{u}(p',\lambda')i\sigma^{03}u(p,\lambda) = -2e^{-i(\lambda'-\lambda)\phi}(\lambda\alpha_3-\lambda'\alpha_4)[2\lambda\cos(\theta/2)\delta_{\lambda',\lambda}-\sin(\theta/2)\delta_{\lambda',-\lambda}]$$

(B.37)

$$\bar{u}(p',\lambda')i\sigma^{01}u(p,\lambda) = -2(\lambda\alpha_3-\lambda'\alpha_4)\{e^{-i(\lambda'-\lambda)\phi}[\cos(\theta/2)\delta_{\lambda',-\lambda}$$

$$+2\lambda\sin(\theta/2)\delta_{\lambda',\lambda}]-2i\ \sin(\phi/2)e^{-i\lambda'\phi}[2\lambda\cos(\theta/2)\delta_{\lambda',-\lambda}+\sin(\theta/2)\delta_{\lambda',\lambda}]\}$$

(B.38)

$$\bar{u}(p',\lambda')i\sigma^{31}u(p,\lambda) =(\alpha_1-4\lambda'\lambda\alpha_2)\{-e^{-i(\lambda'-\lambda)\phi}[2\lambda\cos(\theta/2)\delta_{\lambda',-\lambda}$$

$$+ \sin(\theta/2)\delta_{\lambda',\lambda}] + 2i\sin(\phi/2)e^{-i\lambda'\phi}[\cos(\theta/2)\delta_{\lambda',-\lambda}+2\lambda\sin(\theta/2)\delta_{\lambda',\lambda}]\}$$

(B.39)

Using the spin one wave functions (8.20) with $\vec{p}\,' = \vec{p}\,'(\theta,\phi)$ and $p_\mu = (E, 0, 0, -p)$, we have

$$\varepsilon^*_\mu(p',0)p^\mu = (1/m')(p'E-E'p\cos\theta)$$

and

$$\varepsilon^*_\mu(p',\pm 1)p^\mu= \mp \frac{e^{\mp i\phi}}{\sqrt{2}}\, p\ \sin\theta \qquad \text{(B.40)}$$

In the particular situation in which $\vec{p}\,'$ is in the xz-plane, i.e., choosing $\phi = 0$, we obtain from eq. (8.24)

$$\bar{u}(p',\lambda')\vec{\gamma}\, u(p,\lambda) = 2(\lambda\alpha_3+\lambda'\alpha_4)\{[(2\lambda\hat{x}+i\hat{y})\sin(\theta/2)$$

$$+ 2\lambda\hat{z}\cos(\theta/2)]\delta_{\lambda',\lambda} +[(\hat{x}+2i\lambda\hat{y})\cos(\theta/2)-\hat{z}\sin(\theta/2)]\delta_{\lambda',-\lambda}\} \qquad \text{(B.41)}$$

And with spin one particle wave function we obtain the very useful relations

$$\varepsilon^*_\mu(p',0)p^\mu = (1/m')(p'E-pE'\cos\theta)$$
$$\varepsilon^*_\mu(p',\pm 1)p^\mu = \mp \frac{p}{\sqrt{2}}\sin\theta \qquad (B.42)$$

$$p'^\mu\varepsilon_\mu(p,0) = (1/m)(pE'-p'E\cos\theta)$$
$$p'^\mu\varepsilon_\mu(p,\pm 1) = \pm \frac{p'}{\sqrt{2}}\sin\theta \qquad (B.43)$$

and the scalar products

$$\varepsilon^*_\mu(p',0)\varepsilon^\mu(p,0) = (1/mm')(pp'-EE'\cos\theta)$$

$$\varepsilon^*_\mu(p',0)\varepsilon^\mu(p,\pm 1) = \frac{\pm E'}{m'\sqrt{2}}\sin\theta$$

$$\varepsilon^*_\mu(p',\pm 1)\varepsilon^\mu(p,0) = \mp \frac{E}{m\sqrt{2}}\sin\theta \qquad (B.44)$$

$$\varepsilon^*_\mu(p',\pm 1)\varepsilon^\mu(p,\pm 1) = -(1/2)(1+\cos\theta)$$

$$\varepsilon^*_\mu(p',\mp 1)\varepsilon^\mu(p,\pm 1) = -(1/2)(1-\cos\theta)$$

APPENDIX C

Currents and Couplings

Fig. Ai presents the graphs corresponding to the (TDCM) whlch contains a set of vertices associated to the a-exchange, A++--exchange and p-dlrect-pole. In thls Appendix we analyse the $(p\pi^-\Delta^{++})$, $(\pi\mathbb{P}\pi)$, $(N\mathbb{P}N)$ and $(\Delta\mathbb{P}\Delta)$ vertices.

$(p\pi^-\Delta^{++})$ Vertex

This vertex has the spin-parity $J^P=(1/2^+,0^-,3/2^+)$. The simplest form[12c] of the current associated to it, compatible with P-invariance, time-reversal and charge ronjugation is

$$J(p',p) = g_{N\pi\Delta}\,\bar{\Psi}_\mu(p')\;q^\mu\,u(p) \tag{C.1}$$

where $g_{N\pi\Delta}$ is the coupling constant associated to it and $q=p'-p$ (see Fig. C1). The subsidiary conditions (B.2) make possible to rewrite eq. (C.1) as

$$J(p',p) = -g_{N\pi\Delta}\,\bar{\Psi}_\mu(p')\;p^\mu\,u(p) \tag{C.2}$$

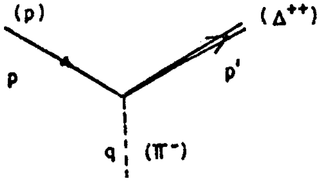

Fig.C1 - Vertex $p\pi^-\Delta^{++}$ with their 4--vectors associated to each particle involved.

In the $\mathbb{P}$-hadrons-coupling calculations for reaction (1), we have taken into account

(i) The vectorial coupling hypothesis or $(\gamma\mathbb{P}A)$[5]

(ii) The (SCHC)[7]

In fact, in this Appendix we consider only condition (i) to obtain the general form for the three vertices ($\pi\mathbb{P}\pi$, NPN and APA) and in the next Appendix we take into account condition (ii) These vertices can be represented by a general notation ($a\mathbb{P}a'$). The momenta associated to these vertex are defined in Fig. C2 by p, q and p', respectively. For convenience we define a useful 4-vectors $P\equiv(p+p')/2$.

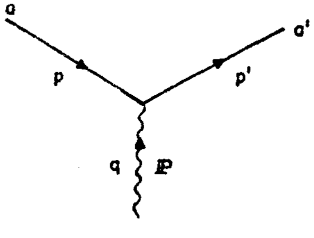

Fig. C2 - General structure of the ($a\mathbb{P}a'$) vertices and their associated momenta.

$(\pi \mathbb{P} \pi)$ or $J^P = (0^-, 1^-, 0^-)$ Vertex

With q and P momenta defined above we can construct the vectorial current

$$J^{\beta}(p',p) = 2g_1 P^{\beta} + g_2 q^{\beta}$$

but, from current conservation we have

$$q_{\beta} J^{\beta}(p',p) = 2g_1 \, P.q + g_2 q^2 = 0$$

As P.q = 0 we have then $g_2 = 0$ and the current is

$$J^{\beta}(p',p) = 2gP^{\beta} \qquad (C.3)$$

where $g = g_1$ is the respective coupling constant. This is the most general vector current compatible with Parity, charge conjugation, and time reversal invariances.

$(N \mathbb{P} N)$ or $J^P = (1/2^+, 1^-, 1/2^+)$ Vertex

To this vertex we can build the vector current from P and q momenta and γ^{β} Dirac matrices, as

$$J^{\beta}(p',p) = \bar{u}(p') \; (g_1 \gamma^{\beta} + g_2 P^{\beta} + g_3 q^{\beta}) u(p) \qquad (C.4)$$

Other terms can be formed from $\gamma_5 = - \gamma_0 \gamma_1 \gamma_2 \gamma_3$ and the antisymmetric tensor $\varepsilon_{\mu\nu\sigma\rho}$ $(\varepsilon_{0123} = +1)$, but these terms can be eliminated from Gordon identities valid for Rarita-Schwinger and Dirac spinors, i.e.

$$i \, \sigma_{\mu\nu} \, q^{\nu} = 2(m\gamma_{\mu} - P_{\mu}) \qquad (C.5)$$

$$i \, \sigma_{\mu\nu} \, P^{\nu} = - \, q_{\mu}/2 \qquad (C.6)$$

$$r_{\mu} = -q^2 \, \gamma_{\mu} + 2mi \, \sigma_{\mu\nu} \, q^{\nu} \qquad (C.7)$$

where $r_{\mu} = -2i \, \varepsilon_{\mu\nu\lambda\sigma} \, P^{\nu} \, q^{\lambda} \, \gamma^{\sigma} \, \gamma_5 = (\gamma_{\mu} \not{P} \not{q} - \not{q} \not{P} \gamma_{\mu})$. Since we have equal masses $(m_a = m_{a'} = m_{proton})$ then $\bar{u}(p') \not{q} \, u(p) = 0$ and P.q = 0. Therefore the current conservation $q_{\beta} J^{\beta}(p',p) = 0$ will be fulfilled if $g_3 = 0$. So the vector current nucleon-nucleon compatible with P, time-reversal and C invariances is

$$J^{\beta}(p',p) = \bar{u}(p') \, (g_1 \gamma^{\beta} + g_2 P^{\beta}) u(p) \qquad (C.8)$$

$(\Delta \mathbb{P} \Delta)$ or $J^P = (3/2^+, 1^-, 3/2^+)$ Vertex

For this vertex the most general vector current, taking into account the subsidiary conditions (B.2,3) and Gordon identities (C.5,6,7), are formed from P, q, γ^β, $g^{\mu\nu}$ as

$$J^\beta(p',p) = \bar{\psi}_\mu(p')\{g_1 g^{\mu\nu}\gamma^\beta + g_2 g^{\mu\nu}q^\beta + g_3 g^{\mu\nu}P^\beta + g_4 P^\mu g^{\beta\nu} + g_5 g^{\mu\beta}P^\nu + g_6 P^\mu\gamma^\beta P^\nu + g_7 P^\mu q^\beta P^\nu + g_8 P^\mu P^\beta P^\nu\}\ \psi_\nu(p) \qquad \text{(C.9)}$$

For equal mass particles we have $\bar{\psi}_\mu(p')\ \not{q}\ \psi_\nu(p) = 0$ and $P.q=0$, and so the current conservation $q_\beta J^\beta(p',p)=0$ gives

$$\bar{\psi}_\mu(p')\{g_2 g^{\mu\nu}q^2 + \frac{1}{2}\, g_4 p^\mu p'^\nu - \frac{1}{2}\, g_5 p^\mu p'^\nu + g_7 P^\mu P^\nu q^2\}\psi_\nu(p) = 0$$

This condition is satisfied when $g_2 = g_7 = 0$ and $g_4 = g_5$. So the current takes the form

$$J^\beta(p',p) = \bar{\psi}_\mu(p')\{g_1 g^{\mu\nu}\gamma^\beta + g_3 g^{\mu\nu}P^\beta + g_4 (P^\mu g^{\beta\nu} + g^{\mu\beta}P^\nu) + g_6 P^\mu\gamma^\beta P^\nu + g_8 P^\mu P^\beta P^\nu\}\ \psi_\nu(p) \qquad \text{(C.10)}$$

But if the vector field is a massless one, we have the following identity for Rarita-Schwinger spinors

$$P^\mu\gamma^\beta P^\nu = \frac{m}{2}\ (P^\mu g^{\beta\nu} + g^{\mu\beta}P^\nu) + \frac{1}{2}\, P^2\ g^{\mu\nu}\gamma^\beta - \frac{m}{2}\, g^{\mu\nu}P^\beta \qquad \text{(C.11)}$$

In this case the current has four independent terms

$$J^\beta(p',p) = \bar{\psi}_\mu(p')\{g_1' g^{\mu\nu}\gamma^\beta + g_2' g^{\mu\nu}P^\beta + g_3'(P^\mu g^{\beta\nu} + g^{\mu\beta}P^\nu) + g_4' P^\mu P^\beta P^\nu\}\psi_\nu(p) \qquad \text{(C.12)}$$

where the $g'_{1,2,3,4}$ are real constants. The above current is then compactible with P, time-reversal and C-parity invariances.

APPENDIX D

Behaviour of Hadronic Currents and Helicity Conservation at High Energy and Low Momentum Transfer

To obtain effective forms of the couplings in each hadronic vertex, we must impose the (SCHC) to the currents of Appendix C. For this, we determined the high energy and low momentum transferbehaviour for those currents. For convenlence we calculate firstly the dlffractive limit from some products of Appendix B. As we know, at high energies the total energy and momentum ($E = E_a+E_b$ and $\vec{p} = \vec{p}_a + \vec{p}_b$) in (C M S) are $E = 2|\vec{p}_a| = \sqrt{s}$. For the 4-momentum $P = (1/2)(p'+p)$ (we recall that we have choeen p and p' in xz-plane as was seen in eq. (B.41)) we have the approximetion

$$P^0 = (\sqrt{s}/2)(1 + O(1/s)$$
$$\vec{P} \simeq \left[\sin(\theta/2)\hat{x} + \cos(\theta/2)\hat{z}\right] (\sqrt{s}/2)\left[1 + (t/2s) + O(1/s^2)\right] \qquad (D.1)$$

and for the momentum transfer $q = (p'-p)$ we have

$$\vec{q} \simeq \left[\cos(\theta/2)\hat{x} - \sin(\theta/2)\hat{z}\right] \sqrt{-t} \qquad (D.2)$$

To evaluate expressions (B.33-41) in (H E A) we have first calculated the expressions

$$\alpha_1 + \alpha_2 \simeq \sqrt{s}\,[1 + O(1/s)]$$
$$\alpha_1 - \alpha_2 \simeq (m'+m)\,[1 + O(1/s)]$$
$$\alpha_3 + \alpha_4 \simeq \sqrt{s}\,[1 + O(1/s)] \qquad (D.3)$$
$$\alpha_3 - \alpha_4 \simeq (m'-m)\,[1 + O(1/s)]$$

From 4-momentum transfer definltion

$$t = (p'-p)^2 = m^2 + m'^2 - 2(EE'-|\vec{p}||\vec{p}'|\cos\theta) \qquad (D.4)$$

and at the (H E A) we have

$$\cos\theta \simeq 1 + (2t/s) + O(1/s^2)$$

$$\sin\theta \simeq 2(-t/s)^{1/2} + O(1/s) \tag{D.5}$$

$$\cos(\theta/2) \simeq 1 + (t/2s) + O(1/s^2)$$

$$\sin(\theta/2) \simeq (-t/s)^{1/2} + O(1/s)$$

Then the products of Dirac spinors (B.33) and (8.34) with $\phi=0$, $m = m'$ and (8.41) at (H E A) are

$$\bar{u}(p',\pm 1/2)u(p,\pm 1/2) \simeq 2m$$

$$\bar{u}(p',\mp 1/2)u(p,\pm 1/2) \simeq \mp \sqrt{-t}$$

$$\bar{u}(p',\pm 1/2)\gamma^{\beta}u(p,\pm 1/2) \simeq 2p^{\beta} \tag{D.6}$$

$$\bar{u}(p',\mp 1/2)\gamma^{\beta}u(p,\pm 1/2) \simeq 0$$

The wave function of spin [eq.(B.20)] with $\vec{p} = \vec{p}(0,0)$ and $p' = \vec{p}'(\theta, 0)$ in the diffractive region and $m = m'$, can be written as

$$\varepsilon^{\beta}(p, 0) = \frac{1}{m} p^{\beta}[1 + O(-t/s)^{1/2}]$$

$$\varepsilon^{\beta}(p,\pm 1) = \frac{1}{\sqrt{2}} [\mp \delta_{\beta 1} - i\delta_{\beta 2}]$$

$$\varepsilon^{\beta *}(p',0) = \frac{1}{m} p'^{\beta} [1 + O(-t/s)^{1/2}] \tag{D.7}$$

$$\varepsilon^{\beta *}(p' 1) = \frac{1}{\sqrt{2}} [\mp \delta_{\beta 1} + i\delta_{\beta 2} \pm 2(-t/s)^{1/2} \delta_{\beta 3}]$$

And expressions (B.42), (B.43) and (B.44) can be approximated by

$$\varepsilon^{*}_{\mu}(p',0)p^{\mu} = (-t/2m)[1 + O(1/s)] \tag{D.8a}$$

$$\varepsilon^{*}_{\mu}(p',\pm 1)p^{\mu} = \mp (-t/2)^{1/2} [1 + O(1/s)] \tag{D.8b}$$

$$p'^{\mu}\varepsilon_{\mu}(p,0) = (-t/2m)[1 + O(1/s)] \tag{D.8c}$$

$$p'^{\mu}\varepsilon_{\mu}(p,\pm 1) = \pm(-t/2)^{1/2}[1 + O(1/s)] \qquad (D.8d)$$

$$\varepsilon^{*}_{\mu}(p',0)\varepsilon^{\mu}(p,0) = -(1 + \frac{t}{2m^2}) + O(1/s^2) \qquad (D.8e)$$

$$\varepsilon^{*}_{\mu}(p',0)\varepsilon^{\mu}(p,\pm 1) = \pm\frac{1}{m}(-t/2)^{1/2}[1+O(1/s)] \qquad (D.8f)$$

$$\varepsilon^{*}_{\mu}(p',\pm 1)\varepsilon^{\mu}(p,0) = \mp\frac{1}{m}(-t/2)^{\frac{1}{2}}[1+O(1/s)] \qquad (D.8g)$$

$$\varepsilon^{*}_{\mu}(p',\pm 1)\varepsilon^{\mu}(p,\pm 1) = -1 + O(1/s) \qquad (D.8h)$$

$$\varepsilon^{*}_{\mu}(p',\mp 1)\varepsilon^{\mu}(p,\pm 1) = O(1/s) \qquad (D.8i)$$

We use those approximatlons to determine the couplings at high energy with (SCHC).

As it is well known[5] the currents which conserve s-channel helicity in the $(N\mathbb{P}N)$ vertex are

$$J_{\beta}^{(NN)}(\lambda',\lambda) = g_1\bar{u}(p',\lambda')\gamma_{\beta}u(p,\lambda) \qquad (D.9)$$

and taking into account eq. (D.6) we have

$$J_{\beta}^{(NN)}(\lambda',\lambda) \simeq 2g_1 P_{\beta}\,\delta_{\lambda'\lambda} \qquad (D.10)$$

where $P = (p+p')/2$

We proceecl now similarly with the diffractive reaction $\Delta(p)+\pi(q)\to\Delta(p') + \pi(q')$. We have the current $J^{\beta}_{\pi\pi}(q',q) = 2g\,Q^{\beta}$ for $(\pi\mathbb{P}\pi)$ vertex where $Q = (q+q')/2$ and for $(\Delta\mathbb{P}\Delta)$ vertex, $J^{\beta}_{\Delta\Delta}(p',\lambda';p,\lambda)$ (eq.C.12) obtained from $(\gamma\mathbb{P}A)$. In section 2, the diffractive limits of this current (C.12) were calculated for several helicity states, in order to obtain the coupling satisfying the (SCHC). For this calculations, it is useful to consider some approximatlons in spin 1 wave function.

In the helicity amplitudes for the reaction above, $A_{(\Delta\pi)}(\lambda',\lambda) \propto J^{\beta}_{(\Delta\Delta)}(\lambda',\lambda)J_{\beta}^{(\pi\pi)}$, there appear expressions like $\varepsilon^{*}_{\beta}(p',\lambda'_1)Q^{\beta}$ and $\varepsilon_{\beta}(p,\lambda_1)Q^{\beta}$. Using eq.(D.7), where $\vec{p} = \vec{p}(0,0)$, and $\vec{p}\,' = \vec{p}\,'(\theta,0)$, we have

$$\varepsilon_\beta(p,0)Q^\beta \simeq \frac{1}{m}\, P.Q/m$$

$$\varepsilon_\beta(p,\pm 1)Q^\beta \simeq (1/\sqrt{2})(\pm Q^1 + iQ^2)$$

$$\varepsilon^*_\beta(p',0)Q^\beta \simeq P'.Q/m \qquad \text{(D.11)}$$

$$\varepsilon^*_\beta(p',\pm 1)Q^\beta \simeq (1/\sqrt{2})(\pm Q^1 - i\, Q^2 \mp 2(t/s)^{1/2}\, Q^3)$$

where $P.Q \simeq s/2$. In the (C M S) we have $\vec{q} = -\vec{p}$ and $\vec{q}\,' = -\vec{p}\,'$, and then

$$Q^1 = -(1/2)\ p'\ \sin\theta \simeq -\ (1/2)(\sqrt{-t}/2)(1 + O(1/s))$$

$$Q^2 = 0 \qquad \text{(D.12)}$$

$$Q^3 = -(1/2)(p'\ \cos\theta + p) \simeq -\ -(\sqrt{s}/2)(1 + O(1/s))$$

And the expressfons above become

$$\varepsilon_\beta(p,0)Q^\beta \simeq s/2m$$

$$\varepsilon_\beta(p,\pm 1)Q^\beta \simeq \mp\ \sqrt{-t}/2\sqrt{2}$$

$$\varepsilon^*_\beta(p',0)Q^\beta \simeq s/2m \qquad \text{(D.13)}$$

$$\varepsilon^*_\beta(p',\pm 1)Q^\beta \simeq \pm\ \sqrt{-t}/2\sqrt{2}$$

APPENDIX E

Useful Formulae and Tables

We put in this Appendix a set of useful formulae and results for simpllficatlon of the text.

In section 4 we need the expression

$$(\not{p}_h + \not{p}_3)(\not{p}_h - \not{p}_3) = m_b^2 - m_3^2 + 2(\not{p}_3\not{p}_b - p_b.p_3) \qquad \text{(E.1)}$$

with the tensor $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ we obtain

$$\not{p}_3\not{p}_b = p_b.p_3 - i\ p_{3\mu}p_{b\nu}\ \sigma^{\mu\nu} \qquad \text{(E.2)}$$

In the (G J S) defined in Appendix A we have,

$$p_{3\mu}\, p_{b\nu}\, \sigma^{\mu\nu} = -|\vec{p}_3|\ \sin\alpha\,(E_3-E_b)\ \sigma^{01} \tag{E.3}$$
$$-\ [(E_3-E_b)\,|\vec{p}_3|\cos\alpha - E_3|\vec{p}_a|]\ \sigma^{03} + |\vec{p}_a|\,|\vec{p}_3|\sin\alpha\ \sigma^{31}$$

Using (A.32) and $E_b-E_3 = \sqrt{s_1} - E_a \simeq -|\vec{p}_a|\ \cos\alpha$, (E.1) becomes

$$(\not{p}_b+\not{p}_3)(\not{p}_b-\not{p}_3) \simeq -i\ s\ \frac{|\vec{p}_a|}{\sqrt{s_1}}\ \sin\alpha[-\cos\alpha\ \sigma^{01} + \sin\alpha\ \sigma^{03} + \sigma^{31}] \tag{E.4}$$

We define now,

$$E = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\ ;\ X = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}\ \text{and}\ \ Z = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{E.5}$$

and using eq. (8.20) in the (G J S) we have,

$$\begin{aligned}
\varepsilon^*(p_1,0)\ .\ E &= \frac{|\vec{p}_1|}{m_1}\\
\varepsilon^*(p_1,\pm1)\ .\ E &= 0\\
\varepsilon^*(p_1,0)\ .\ Z &= -\frac{E_1}{m_1}\ \cos\theta\\
\varepsilon^*(p_1,\pm1)\ .\ Z &= \mp\ \frac{e^{\mp i\phi}}{\sqrt{2}}\ \sin\theta\\
\varepsilon^*(p_1,0)\ .\ X &= \frac{E_1}{m_1}\ \sin\theta\ \cos\phi\\
\varepsilon^*(p_1,\pm1)\ .\ X &= \mp\ \frac{e^{\mp i\phi}}{\sqrt{2}}\ (\cos\theta\ \cos\phi \pm i\ \sin\phi)
\end{aligned} \tag{E.6}$$

In the (G J S) the product of Dirac spinors (B.33) to (B.39) reads

$$\begin{aligned}
\bar{u}(p_1,\pm1/2)u(p_a,\pm1/2) &= E_-\ \cos(\theta/2)\\
\bar{u}(p_1,\mp1/2)u(p_a,\pm1/2) &= \mp\ e^{\pm i\phi}\ E_+\ \sin(\theta/2)\\
\bar{u}(p_1,\pm1/2)\gamma^0 u(p_a,\pm1/2) &= E_+\ \cos(\theta/2)
\end{aligned} \tag{E.7}$$

$$\bar{u}(p_1,\mp 1/2)\gamma^0 u(p_a,\pm 1/2) = \mp e^{\pm i\phi} E_- \sin(\theta/2)$$

$$\bar{u}(p_1,\pm 1/2)\gamma^1 u(p_a,\pm 1/2) = e^{\mp i\phi} G_+ \sin(\theta/2)$$

$$\bar{u}(p_1,\mp 1/2)\gamma^1 u(p_a,\pm 1/2) = \pm G_- \cos(\theta/2)$$

$$\bar{u}(p_1,\pm 1/2)\gamma^3 u(p_a,\pm 1/2) = G_+ \cos(\theta/2)$$

$$\bar{u}(p_1,\mp 1/2)\gamma^3 u(p_a,\pm 1/2) = \mp e^{\pm i\phi} G_- \sin(\theta/2)$$

$$\bar{u}(p_1,\pm 1/2) i\sigma^{03} u(p_a,\pm 1/2) = -G_- \cos(\theta/2) \tag{E.7}$$

$$\bar{u}(p_1,\mp 1/2) i\sigma^{03} u(p_a,\pm 1/2) = \pm e^{\pm i\phi} G_+ \sin(\theta/2)$$

$$\bar{u}(p_1,\pm 1/2) i\sigma^{01} u(p_a,\pm 1/2) = -e^{\mp i\phi} G_- \sin(\theta/2)$$

$$\bar{u}(p_1,\mp 1/2) i\sigma^{01} u(p_a,\pm 1/2) = \mp G_+ \cos(\theta/2)$$

$$\bar{u}(p_1,\pm 1/2) i\sigma^{31} u(p_a,\pm 1/2) = -e^{\mp i\phi} E_- \sin(\theta/2)$$

$$\bar{u}(p_1,\mp 1/2) i\sigma^{31} u(p_a,\pm 1/2) = \mp E_+ \cos(\theta/2)$$

where

$$E_\pm = \sqrt{E_1+m_1}\,\sqrt{E_a+m_a} \pm \sqrt{E_1-m_1}\,\sqrt{E_a-m_a}$$

and (E.8)

$$G_\pm = \sqrt{E_1+m_1}\,\sqrt{E_a-m_a} \pm \sqrt{E_1-m_1}\,\sqrt{E_a+m_a}$$

With the definitions (E.5) we have in the (GJS)

$$p^\mu = \sqrt{s_1}\, E^\mu$$

$$p_a^\mu = E_a E^\mu + |\vec{p}_a|\, Z^\mu \tag{E.9}$$

and at (HEA) eq. (A.6)

$$p_b^\mu \simeq p_3^\mu \simeq \frac{s}{2\sqrt{s_1}}\,(E^\mu + \sin\alpha\, X^\mu + \cos\alpha\, Z^\mu) \tag{E.10}$$

In the limit $t_2 = 0$, sina = 0, cosa = -1 and

$$p_b^\mu \simeq p_3^\mu \simeq \frac{s}{s_1 - m_a^2} (p^\mu - p_a^\mu) \tag{E.II}$$

In table EI we present formulae which are used in the text.

Table E-1 - Useful expressions for simplifying formulae and results given in the text

i	r_i	v_i	θ_i
1	$m_1 + m_a$	$\lvert\vec{p}_1\rvert/m_1$	$\sin\theta \cos(\theta/2)$
2	$m_1 - m_a$	E_1/m_1	$\sin\theta \sin(\theta/2)$
3	m_a/m_1	$\sqrt{s_1} - E_a$	$\cos\theta \cos(\theta/2)$
4	$2m_1 + m_a$	$2\sqrt{s_1} - E_a$	$\cos\theta \sin(\theta/2)$
5	$2m_1 - m_a$	$F_1E_-/\sqrt{s_1} + F_2E_+/m_1$	
6	$1 + m_a/m_1$	$F_1E_+/\sqrt{s_1} + F_2E_-/m_1$	
7	$1 - m_a/m_1$	$(E_- - G_-\cos\alpha)/\sqrt{s_1}$	
8	$2 + m_a/m_1$	$(E_+ - G_+\cos\alpha)/\sqrt{s_1}$	
9	$2 - m_a/m_1$	$r_8E_+ - E_-\sqrt{s_1}/m_1$	
10	$(r_1^2 - m_2^2)/m_1$	$r_8E_- - E_+\sqrt{s_1}/m_1$	
11	$(11m_a^2 - m_1^2 + 3m_1m_a - 8m_2^2) + {}$ $+ r_3(m_a^2 - 2m_1^2 - 2m_1m_a - m_2^2)$	$2E_aE_2 - m_a^2 - m_2^2$	
12	$[(5m_a^2 + 4m_1^2 - 3m_1m_a - 8m_2^2)/3 + {}$ $+ r_3(m_1^2 + m_a^2 - m_2^2)]/m_1^2$	$2\lvert\vec{p}_a\rvert\lvert\vec{p}_2\rvert$	
13	$(5/3 + r_3)/m_1^2$		
14	$m_1^2/3 - m_a^2 + m_1m_a + m_2^2$		
15	$(2m_1^2 - 3m_1m_a + 3m_a^2 - 3m_2^2)/3m_1^2$		
16	$1/m_1^2$		

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Resumo

O cálculo completo para a reação de dissociaçáo difrativa $pp \to \Delta^{++}\pi^- p$ foi feito no contexto do Modelo Deck a Três Componentes. Este cálculo apresenta algumas dificuldades originadas pelo vértice $(3/2^+, 3/2^+, 1^-)$ que aparece em uma das componentes. Nós damos os prin-

cipais detalhes do cálculo o que faz este trabalho ser essencialmente técnico. Nossa conclusão, baseada nos resultados obtidos, é de que, estruturas de "zeros" ou mínimos preditos pelo Modelo não podem ser vistos analiticamente devido à complexidade das fórmulas envolvidas. Foram calculadas numericamente varias distribuições, e uma forte interferências entre as três componentes foi comprovada para uma escolha particular dos parâmetros.