Revista Brasileira de Física, Volume 13, n? *2, 1983*

Study of α-^{16}O Scattering by Orthogonality Condition Models

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Recebido em 17 de fevereiro de 1983

Abstract The use of approximate microscopic theories in α-^{16}O scattering is investigated. The Orthogonality Condition Model (OCM) with both the direct potential of the Resonating Group Method (RGM) and withan effective local potential, V_{eff}, derived from Kernels of Generator Coordinate Method (GCM) is employed to study collisions at CM energies up to 30 MeV, for all relevant partial waves. Although the predictions of the OCM are consistent with "exact" RGM results in both cases, the nuclear phase-shifts obtained with the effective potential are better. We notice the presence of ambiguities in the derivation of V_{eff}. The nature of such ambiguities is discussed.

1. INTRODUCTION

In the last decade the effects of Pauli Principle in collisions of light heavy ions have been studied by several authors[*]. These effects can be exactly accounted for in microscopic theories as the Resonating Group Method[1,2] (RGM) or the basically equivalent Generator Coordinate Method[1,3] (GCM), which have the additional advantage of allowing a description of the nucleus-nucleus potential in termsofnucleon-nucleon interactions. The RGM leads to a Schrödinger-type equation for the relative motion with a local folding potential plus a highly complicated non-local potential in which the effects of the Pauli Principle are fully contained. Although the application of these methods to pairs of very light particies (n,a,He^3,H^3) is relatively easy, it becomes increasingly more cornplicated for collisions between heavier nuclei.

As a simple alternative for such "exact" theories Saito proposed the Orthogonality Condition Model[4] (OCM). In the OCM the non-local part of the RGM Hamiltonian is approximated by a simpler separable potential, derived exclusively from the local RGM potential, V_D, with the help of the projector onto the subspace of states satisfying the Pauli Principle. Saito's model was sucessfully ãpplied to collision

* Work supported in part by the CNPq and the FINEP - Brasil

between very light nuclei, as a-particles, but it proved a poor approximation[5] to calculations based on some commonly used nucleon-nucleon interactions or to collisions between heavier systems, where it would be most useful.

The validity of the OCM can, however, be extended[6] if it is based on conveniently chosen effective local potentials, rather than on the local part of the RGM potential. In ref. 5 a prescription for deriving such effective potentials was introduced. This prescription was used to study[5] a-α and ^{16}O-^{16}O scattering and also[7] the collision of α-particles with n, ^3H and ^3He. The results obtained for the very light systems were quite good, including those of calculations with nucleon-nucleon interactions for which the original version of the OCM, with the potential V_D, failed. On the other hand, the results for ^{16}O-^{16}O scattering were unsatisfactory, in spite of being better than those of the OCM with the RGM potential V_D.

The present paper is concerned with the derivation of effective potentials for the OCM. The technique of ref. 5 is used to study the collision α-^{16}O, a problem between a-a scattering, where it proved successful, and ^{16}O-^{16}O scattering, where it did not work very well.

In section 2 we present the basic results of the RGM, the GCM and the OCM and show how effective potentials for Saito's equation can be derived from GCM kernels. In section 3 we apply the OCM to α-^{16}O scattering. The results are compared to "exact" results of the RGM and the details of the derivation of effective potentials are critically discussed. Finally, in section 4 we summarize the conclusions of the present work.

2. MICROSCOPIC THEORIES FOR SCATTERING – GCM, RGM, OCM

2.1. The Generator Coordinate Method

In the GCM the many-body wave function is assumed to have the form[1,3]

$$\psi^{GC} = \int \Phi_\alpha(\vec{x}_1, \ldots \vec{x}_A)\, f(\alpha)\, d\alpha$$

The generator function $\Phi_\alpha(\vec{x}_1, \ldots \vec{x}_A)$ is a function of single particle coordinates $\vec{x}_1, \ldots \vec{x}_A$ of the nucleons, and a set of parameters, the generator coordinates, represented by a. The weight function $f(\alpha)$, which

plays the role of a wave function in the GCM, is determined from the dynamic condition

$$\delta\langle\psi^{GC}|H_T - E|\psi^{GC}\rangle = 0 \qquad (2)$$

for variations of ψ^{GC} through changes in $f(\alpha)$. In eq. (2) H_T is the many-body Hamiltonian and E is the total energy of the system.

The variational Principle expressed in eq. (2) leads to the Griffin-Hill-Wheeler (GHW) equation for the weight function

$$\int \left[\tilde{H}(\alpha,\alpha') - E\,\tilde{I}(\alpha,\alpha')\right] f(\alpha')\,d\alpha' \equiv \int \tilde{K}(\alpha,\alpha') f(\alpha')\,d\alpha' = 0 \qquad (3)$$

where $\tilde{I}(\alpha,\alpha')$ and $\tilde{H}(\alpha,\alpha')$ are respectively the overlap and Hamiltonian Kernels given by

$$\begin{Bmatrix} \tilde{I}(\alpha,\alpha') \\ \tilde{H}(\alpha,\alpha') \end{Bmatrix} = \langle \Phi_\alpha | \{ \begin{matrix} 1 \\ H_T \end{matrix} \} | \Phi_{\alpha'} \rangle \qquad (4)$$

The GCM provides good approximations for the many-body problem when the solution sought is nearly contained in the subspace spanned by the generator functions (Eq. (1)). The success of the GCM depends drastically, therefore, on the choice of the generator coordinates a.

We are concerned here with the application of the GCM to scattering problems. In the collision of two nuclei with mass numbers A, and A_2 the generator coordinates are chosen to be the components of a vector $\vec{\alpha}$ separating the centers of two potential wells to which A, and A, nucleons, respectively, are assumed to be bound. The generator function is written in the form

$$\Phi_{\vec{\alpha}}(\vec{x}_1,\ldots\vec{x}_{A_1+A_2}) = \mathcal{A}\{\phi^{(1)}_{\vec{\alpha}}(\vec{x}_1,\ldots\vec{x}_{A_1})\ \phi^{(2)}_{\vec{\alpha}}(\vec{x}_{A_1+1},\ldots\vec{x}_{A_1+A_2})\} \qquad (5)$$

where $\phi^{(1)}_{\vec{\alpha}}$ and $\phi^{(2)}_{\vec{\alpha}}$ are Slater determinants describing single particle motions of A_1 and A_2 nucleons, respectively. $\mathcal{A}$ is an operator which takes care of the antisymmetrization between nucleons ofdifferent fragments. In most practical situations the wave functions $\phi^{(1)}_{\vec{\alpha}}$ and $\phi^{(2)}_{\vec{\alpha}}$ are bullt with harmonic oscillator orbitals with the same oscillator lenght. In such cases the generator function factorizes as

$$\Phi_{\vec{\alpha}} = \Phi_{CM}(\vec{x}_{CM})\ \mathcal{A}\{\Gamma(\vec{x},\vec{\alpha})\ \phi^{(1)}_{int}(\xi_1)\ \phi^{(2)}_{int}(\xi_2)\} \qquad (6)$$

In eq. (6) Φ_{CM} is the center of mass wave function and $\phi^{(1)}_{int}(\xi_1)$ and $\phi^{(2)}_{int}(\xi_2)$ are functions of internal coordinates ξ_1 and ξ_2, describing the motion of the nucleons in each fragment with respect to its center of mass. The function $\Gamma(\vec{x},\vec{\alpha})$ is a gausslan packet

$$\Gamma(\vec{x},\vec{\alpha}) = (\beta\sqrt{\pi})^{-3/2} \exp\left[-(\vec{x}-\vec{\alpha})^2/2\beta^2\right] \tag{7}$$

with the width β expressed in terms of the oscillator length b as

$$\beta^2 = b^2(A_1 + A_2)/(A_1 \cdot A_2) \tag{8}$$

For practical purposes it is convenient to work with the relative Hamiltonian H_r, obtained by extracting the internal energies of the fragments from the many-body Hamiltonian H_T. The scattering energy ε replaces then the total energy E and the GCM kernel $\tilde{K}(\vec{\alpha},\vec{\alpha}')$ takes the form

$$\tilde{K}(\vec{\alpha},\vec{\alpha}') = \langle \Phi_{\vec{\alpha}} | H_r - \varepsilon | \Phi_{\vec{\alpha}'} \rangle \equiv$$

$$\equiv \langle \Phi^{(1)}_{int}\, \Phi^{(2)}_{int}\, \Gamma(\vec{x},\vec{\alpha}) \,|\, (H_r-\varepsilon)\mathcal{A} \,|\, \Phi^{(1)}_{int}\, \Phi^{(2)}_{int}\, \Gamma(\vec{x},\vec{\alpha}') \rangle \tag{9}$$

As it frequently happens in low-energy nuclear physics it is convenient to make partial wave expansions and solve the resulting equations for each R value. With the expansions

$$f(\vec{\alpha}) = \frac{1}{\alpha} \sum f_\ell(\alpha)\, Y_{\ell 0}(\Omega_{\vec{\alpha}}) \tag{10a}$$

$$\tilde{K}(\vec{\alpha},\vec{\alpha}') = \frac{1}{\alpha\,\alpha'} \sum_{\ell m} \tilde{K}_\ell(\alpha,\alpha')\, Y_{\ell m}(\Omega_{\vec{a}})\, Y_{\ell m}(\Omega_{\vec{\alpha}'}) \tag{10b}$$

the scattering problem is reduced to solving a set of angular momentum projected GHW equations

$$\int \tilde{K}_\ell(\alpha,\alpha')\, f_\ell(\alpha')\, d\alpha' = 0 \tag{11}$$

The first step in applying the GCM to nuclear scattering is the calculation of the kernels $\tilde{K}_\ell(\alpha,\alpha')$. This calculation can be done more easily if the generator functions of eq. (9) are put in the form of a single (A_1+A_2)-dimensional Slater determinant. With this procedure $\tilde{K}_\ell(\alpha,\alpha')$ can be expressed in terms of matrix-elements of one-body and

two-body operators in a basis of Slater determinants which can be handled[8] by commonly used shell model techniques. The GHW equation should then be solved with scattering boundary condition. This is usually done on the basis of the cornection (eqs. (21) and (22) of sec. 2.2) between the GCM and the RGM.

2. The Resonating Group Method

The Resonating Group Method was introduced by Wheeler[9] as a molecular description of clustering phenomena in nuclei. If the nucleus has the tendency to group in two clusters having mass numbers A, and A, the many-body wave function is approximated as

$$\psi^{RG} = \Phi_{CM}(\vec{x}_{CM}) \quad \mathcal{A}\{g(\vec{r})\ \phi^{(1)}_{int}(\xi_1)\ \phi^{(2)}_{int}(\xi_2)\} \qquad (12a)$$

In the above expression $g(\vec{r})$ describes the relative motionof two clusters of masses A, and A,, separated by the distance $\vec{r}$, and $\phi^{(1)}_{int}$, $\phi^{(2)}_{int}$, Φ_{CM} and $\mathcal{A}$ have the same meaning as in the previous section.

The relative wave function is then determined by a variational principle as that of eq. (3), for variations of ψ^{RG} through changes in $g(\vec{r})$. It is cunvenient to rewrite the "Ansatz" of eq. (12a) in the form

$$\psi^{RG} = \Phi_{CM}(\vec{x}_{CM}) \quad \int d^3\vec{x}\ g(\vec{x})\ \mathcal{A}\{\phi^{(1)}_{int}(\xi_1)\ \phi^{(2)}(\xi_2).\delta(\vec{x}-\vec{r})\} \qquad (12b)$$

Using eq. (12b) in the variational condition of eq. (3) one obtains the integral form of the RGM equation

$$\int\left[H(\vec{x},\vec{x}') - \varepsilon A(\vec{x},\vec{x}')\right] g(\vec{x}')d^3\vec{x}' \equiv \int K(\vec{x},\vec{x}')g(\vec{x}')d^3\vec{x}' = 0 \qquad (13)$$

where $A(\vec{x},\vec{x}')$ and $H(\vec{x},\vec{x}')$ are, respectively, the overlap and the Hamiltonian RGM kernels,

$$\begin{Bmatrix} A(\vec{x},\vec{x}') \\ H(\vec{x},\vec{x}') \end{Bmatrix} = \langle \phi^{(1)}_{int}\ \phi^{(2)}_{int}\ \delta(\vec{x}-\vec{r}) \left| \begin{Bmatrix} 1 \\ H_r \end{Bmatrix} \mathcal{A} \right| \phi^{(1)}_{int}\ \phi^{(2)}_{int}\ \delta(\vec{x}'-\vec{r}) \rangle \qquad (14)$$

In order to get the usual integro-differential form of the RGM equation the identity 1 is extracted from the antisymmetrizer

$$\mathcal{A} = 1 - \Theta \qquad (15)$$

When the nucleon-nucleon interaction v_{ij} in the many-body Hamiltonian depends locally on nucleon separations the identity ineq. (15) gives rise to a local part in $K(\vec{x},\vec{x}')$ and the RGM equations takes the integro-differential form

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2_{\vec{x}} + V_D(\vec{x}) - \varepsilon\right] g(\vec{x}) + \int S(\vec{x},\vec{x}')g(\vec{x}')\, d^3\vec{x}' = 0 \qquad (16)$$

where μ is the reduced mass and

$$V_D(\vec{x})\delta(\vec{x}-\vec{x}') = \langle \phi^{(1)}_{int}\, \phi^{(2)}_{int}\, \delta(\vec{x}-\vec{r}) \left| \sum_{\substack{i\to 1 \\ j\to 2}} v_{ij} \right| \phi^{(1)}_{int}\, \phi^{(2)}_{int}\, \delta(\vec{x}'-\vec{r}) \rangle \qquad (17a)$$

$$S(\vec{x},\vec{x}') = -\langle \phi^{(1)}_{int}\, \phi^{(2)}_{int}\, \delta(\vec{x}-\vec{r}) \left| (H_r - \varepsilon)\,\Theta \right| \phi^{(1)}_{int}\, \phi^{(2)}_{int}\, \delta(\vec{x}'-\vec{r}) \rangle \qquad (17b)$$

Equation (16) is a Schrödinger[11] equation with a folding model type local potential V_D plus a short-range* non-local potential $S(\vec{x},\vec{x}')$, resulting from the exchange operator Θ.

With partial waves expansion of $S(\vec{x},\vec{x}')$ and $g(\vec{x})$, analogous to eqs. (10a) and (10b), the angular momentum projected equation

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dx^2} - \frac{\ell(\ell+1)}{x^2}\right) + V_D(x) - \varepsilon\right] g_\ell(x) + \int S_\ell(x,x')g_\ell(x')\, dx' = 0 \qquad (18)$$

is obtained.

In applications of the RGM to nuclear scattering the main difficulty resides in the calculation of the non-local kernel $S(x,x')$. This kernel, which is much more complicated than those of the GCM, is usually obtained by the unfolding of eq. (22).

It will prove convenient to introduce the eigenfunctions of the RGM overlap kernel, defined by the condition

$$\int A(\vec{x},\vec{x}')\varphi_i(\vec{x}')d^3\vec{x}' = \mu_i\, \varphi_i(\vec{x}) \qquad (19)$$

The operator A and its eigenfunctions will play a relevant role in section 2.3. A special class of such eigenfunctions are the redundant states φ^0_i, which have eigenvalues equal to zero. These eigenfunctions[4] are trivial solutions of the RGM equation. They violate Pauli Principle

*For identical fragments[10] an additional long range potential proportional to $\delta(\vec{x}+\vec{x}')$ is contained in $S(\vec{x},\vec{x}')$.

and lead to identically vanishing RGM many-body wave function (eq.12b). The redundant states give rise to ambiguities in the wave function $g(\vec{x})$ since new solutions can be obtained by the addition of arbitrary combinations of redundant states. To overcome this difficulty one defines a redundancy free RGM wave function* $\tilde{g}$

$$|\tilde{g}\rangle = \Lambda\,|g\rangle\ ; \qquad \Lambda \equiv 1 - \sum_i |\varphi_i^0\rangle\langle\varphi_i^0| \tag{20}$$

with the help of the projector Λ that eliminates the cornponents of g in the subspace spanned by the redundant states.

2.2.1 - The equivalence between RGM and GCM

As long as the GCM generator function factorizes in the form of eq. (6) the RGM and the GCM are equivalent methods. Using the factorized $\Phi_{\vec{a}}$ in eq. (1) and comparing to eq. (12b) one gets the relation

$$g(\vec{x}) = \int \Gamma(\vec{x},\vec{\alpha})\ f(\vec{\alpha})\ d^3\vec{\alpha} \tag{21}$$

Following the same procedure it is straightforward to show that GCM kernels and RGM kernels are connected by the double folding relation

$$\tilde{K}(\vec{\alpha},\vec{\alpha}') = \int \Gamma(\vec{\alpha},\vec{x})\ K(\vec{x},\vec{x}')\ \Gamma(\vec{x}',\vec{\alpha}')\, d^3\vec{x}\ d^3\vec{x}' \tag{22}$$

with analogous expressions for $\tilde{H}(\vec{\alpha},\vec{\alpha}')$ and $\bar{I}(\vec{\alpha},\vec{\alpha}')$. Expressions (21) and (22) can be used for each partial wave with the replacement of $\Gamma(\vec{x},\vec{\alpha})$ by $\Gamma_\ell(x,\alpha)$ [10] and d^3x by dx.

In actual applications of the RGM or the GCM it is of utmost importance that the relations (21) and (22) hold. For this reason Harmonic Oscillator orbitals with the same oscillator lenght are commonly used in the description of the interna1 motion of the fragments.

2.3. Approximate treatment of the Pauli Principle - The OCM

The GCM and the RGM are microscopic theories in which antisymmetrization effects are fully accounted for. These methods have been applied to collisions involving few nucleons and to collisions between heavier doubly closed shell nuclei with $A \lesssim 40$. For other cases, however, the calculation of kernels is prohibitively complicated and the application of such theories is difficult. This situation led to simplified

* For compactness, vector notation will be used.

microscopic theories, like the OCM, in which the Pauli Principle is treated approximately and non-localities are more easily handled.

The OCM is an approximate version of the Resonating Group Method based on the assumption that the RGM kernels satisfy the conditions

$$A \simeq \Lambda \tag{23a}$$

$$H \simeq \Lambda(T+V_D)\Lambda \tag{23b}$$

In eqs. (23), written compactly in terms of operators, $T = -\hbar^2 \nabla^2_{\vec{X}} / 2\mu$ is the $\vec{x}$-space kinetic energy operator.

It can be easily shown that eq. (23a) is exactly satisfiedwhen all non-zero eigenvalues of A are equal to one. In collisions between very light nuclei, as a-particles, one is very close to this idealized situation and the approximation in eq. (23a) is fully justified.

It could be argued that the sirnplest approximation for H would be $H \simeq T+V_D$, completely neglecting antysymmetrization effects in the hamiltonian kernel. This would not, however, be good enough. Antisymrnetrizatiori effects should be considered at least to the extent that the hamiltonian kernel keeps the property of annihilating redundant states. The simplest Hermitian approximation according to this criteria is that of eq. (23b).

Using expressions (23a) and (23b) in the RGM equation one gets

$$\Lambda(T+V_D)\,\Lambda\,|u\rangle = \varepsilon\,\Lambda\,|u\rangle \tag{24a}$$

or

$$\Lambda(T+V_D)\,|\tilde{u}\rangle = E\,|\tilde{u}\rangle \tag{24b}$$

with

$$|\tilde{u}\rangle = \Lambda\,|u\rangle \simeq |\tilde{g}\rangle \tag{25}$$

Eq. (24b) is the Saito OCM equation.

It happens frequently, however, that conditions (23a) and (23b) are not met. In such cases it is necessary to go beyond theconventional OCM and look for a better approximation to the RGM non-locality. For this purpose one starts by defining effective potentials V_{eff} by the condition

$$A^{1/2}(T + V_{eff})A^{1/2} = H \tag{26}$$

In eq. (26) $A^{1/2}$ is the square root of the RGM overlap operator, given by

$$A^{1/2} = \sum_i |\varphi_i\rangle \sqrt{\mu_i} \langle\varphi_i| \tag{27}$$

where $|\varphi_i\rangle$ and μ_i have been defined in eq. (19). The RGM equation can then be written[6]

$$\Lambda(T + V_{eff})|\chi\rangle = \varepsilon |\chi\rangle \tag{28}$$

where $|\chi\rangle$ are the renormalized RGM states

$$|\chi\rangle = A^{1/2} |g\rangle \tag{29}$$

The problem of finding approximations to the RGM is then reduced to finding simple potentials for which eq. (26) is nearly satisfied. It can be easily shown that the potential V_D has such a feature when the conditions for applying the OCM (eq. 23a,b) hold. On the other hand, when this is not the case, a simple approximation for V_{eff} can still be sought. This problem was studied in ref.5. Since Gaussiansare convenient for calculational purposes the potential waç parametrized as a sum of M Gaussians,

$$V_{eff}(r) = \sum_{m=1}^{M} V_m \exp\left[-r^2/a_m^2\right] \tag{30}$$

To avoid dealing with highly complicated RGM kernels, eq. (26) was replaced by an equivalent relation[5] involving GCM kernels

$$\Gamma A^{1/2}(T + V_{eff})A^{1/2}\Gamma = \tilde{H} \tag{31}$$

The parameters of the potential (eq. 30) should then be chosen by fitting a selected set of GCM kernels $\tilde{H}(\alpha_i,\alpha_j)$ with the corresponding matrix elements of eq. (31). For the sake of keeping the calculation simple the parameters $\{a_m,V_m\}$ were adjusted as to reproduce the GCM energy surface $E(\vec{\alpha}_i)$

$$E(\vec{\alpha}_i) = \frac{\tilde{H}(\vec{\alpha}_i,\vec{\alpha}_i)}{\tilde{I}(\vec{\alpha}_i,\vec{\alpha}_i)} \tag{32}$$

with the matrix-elements

$$\langle\Gamma(\vec{x},\vec{\alpha}_i)|A^{1/2}(T + V_{eff})A^{1/2}|\Gamma(\vec{x},\vec{\alpha}_i)\rangle / \tilde{I}(\vec{\alpha}_i,\vec{\alpha}_i) \tag{33}$$

at N equally spaced values of the generator coordinate $\vec{\alpha}_i$, taken along the z-axis. The division by $\tilde{I}(\vec{\alpha}_i,\vec{\alpha}_i)$ is to emphasize the fact that the GCM hamiltonian kernels should be calculated with normalized generator functions $\Phi(\vec{\alpha}_i)$.

A trivial improvement of the method outlined above is the introduction of parity dependence. In this case the potential is written

$$V_{\text{eff}} = V_{\text{eff}}^{(+)} P + V_{\text{eff}}^{(-)} (1-P) \tag{34}$$

where P is the projector on the space of positive parity states, and eq. (31) becomes

$$\tilde{H}^{(\pm)} = \Gamma^{(\pm)} A^{(\pm)1/2} (T + V_{\text{eff}}^{(\pm)}) A^{(\pm)1/2} \Gamma^{(\pm)} \tag{35}$$

with

$$A^{(\pm)}(\vec{x},\vec{x}') = A(\vec{x},\vec{x}') \pm A(\vec{x},-\vec{x}') \tag{36a}$$

$$\Gamma^{(\pm)}(\vec{x},\vec{\alpha}) = \Gamma(\vec{x},\vec{\alpha}) \pm \Gamma(\vec{x},-\vec{\alpha}) \tag{36b}$$

$$\tilde{H}^{(\pm)}(\vec{\alpha},\vec{\alpha}') = \tilde{H}(\vec{\alpha},\vec{\alpha}') \pm \tilde{H}(\vec{\alpha},-\vec{\alpha}') \tag{36c}$$

3. APPLICATION OF OCM TO $\alpha - {}^{16}O$ SCATTERING

3.1. Effective potentials

In this section we apply the OCM to a-""O scattering using both the potential V_D and the effective potential of eqs. (30) and (31).

In our calculations the many-body nuclear potential was expressed in terms of Volkov V2 [11] nucleon-nucleon interactions with the forrn

$$v(r_{ij}) = (1 - M P_M)(v_1 e^{-r_{ij}^2/a_1^2} + v_2 e^{-r_{ij}^2/a_2^2}) \tag{37a}$$

where P_M is the Majorana exchange operator and the parameters M, a_1, a_2, v_1, v_2 have the values

$$a_1 = 1.8 \text{ fm} \quad v_1 = -60.65 \text{ MeV}$$
$$a_2 = 1.01 \text{ fm} \quad v_2 = 61.14 \text{ MeV} \quad M = 0.621 \tag{37b}$$

Intrinsec wave functions were described by harmonic oscillators orbitals with the length $b = 1.62$ fm ($\hbar\omega = 15.81$ MeV). In this case the eigenstates of A (eq. (19)) are harmonic oscillator states with width $\beta = 0.91$ fin (eq. (8)) having the eigenvalues[12] represented in fig. 1. The eight lowest states with N = 0,1,...,7 are redundant.

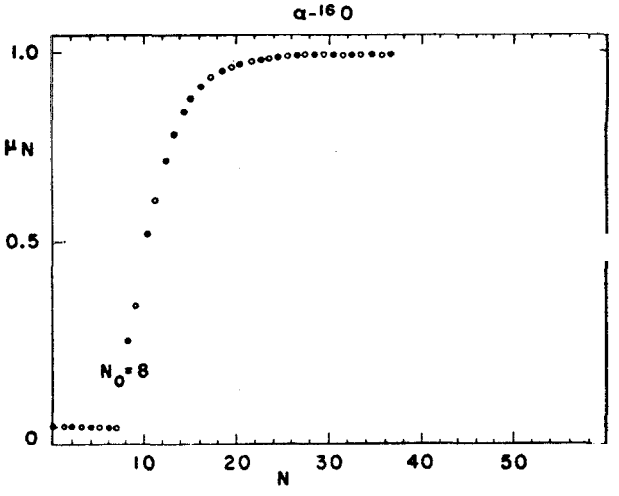

Fig.1 - Eigenvalues μ_N for the α-^{16}O system. $N = N_x+N_y+N_z \equiv 2n+\ell$ is the total number of phonons in the oscillator w.f. Full circles and open circles correspond to even and odd parity states, respectively.

The local part of the RGM potential for α-^{16}O scattering is a sum of two terms. A nuclear term (r which is given analytically in ref. 13 and pictured in fig. 3., and the Coulomb term[13]

$$V_D^{(c)}(r) = \frac{16e^2}{r}\,\mathrm{erf}\left(\sqrt{\frac{16}{27}}\,\frac{r}{b}\right) - \left(\frac{16}{27}\right)^{3/2}\frac{8e^2}{b\sqrt{\pi}}\exp\left[-\frac{16}{27}\left(\frac{r}{b}\right)^2\right] \qquad (38)$$

erf (x) being the usual error function.

The effective potential is also written as a sum of two terms,

$$V_{\mathrm{eff}}(r) = V_{\mathrm{eff}}^{(N)}(r) + V_{\mathrm{eff}}^{(c)}(r) \qquad (39)$$

The Coulomb term $V_{\mathrm{eff}}^{(c)}(r)$ is approximated by $V_D^{(c)}(r)$ (eq. (38)) while $V_{\mathrm{eff}}^{(N)}$ is derived from the GCM energy surface — excluding the Coulomb part — by the technique outlined in the previous section. $V_{\mathrm{eff}}^{(N)}$ is expanded as a sum of four Gaussians (eq. (30)) and the parameters $\{V_1, \ldots, V_4, a_1, \ldots a_4\}$ are determined by comparing the quantities (32) and (33) at 20 equidistant points a = 0.5, 1.0, ...,10.0 fm and minimizing the rms deviation. By this procedure we obtain the parameters listed in table 1, with an rms deviation A = 0.05 MeV.

Fig. 2 illustrates the dependence of the fit on the number of Gaussiäns used. The exact energy surface is compared to values fitted with one (A = 0.4 MeV), two (A = 0.2 MeV) ãnd four Gaussians. The com-

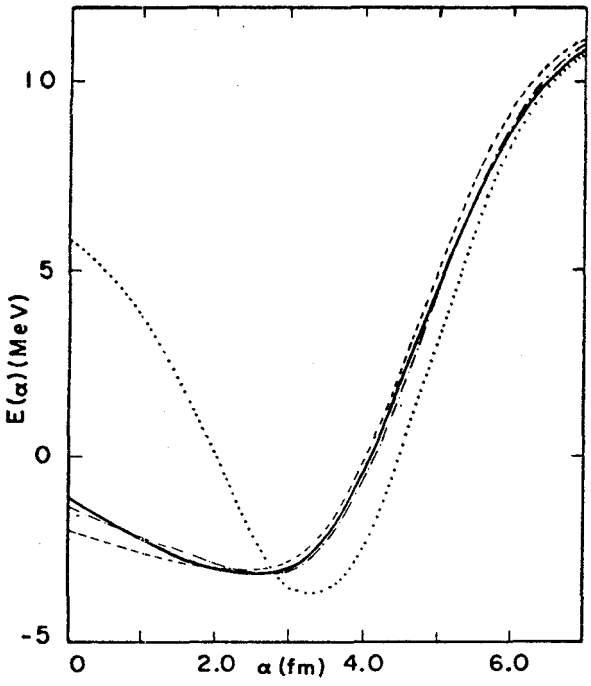

Fig.2 - The α-^{16}O energy surface (without Coulomb contribution). The solid line is the microscopically calculated expression (32) and the others are obtained by inserting local potentials into the expression (33). The dotted line is obtained with V_D and the dashed and dot-dashed lines areobtained with effective potentials fitted with one and two gaussians, respectively. The fit with four gaussians is indistinguishable from the exact kernel.

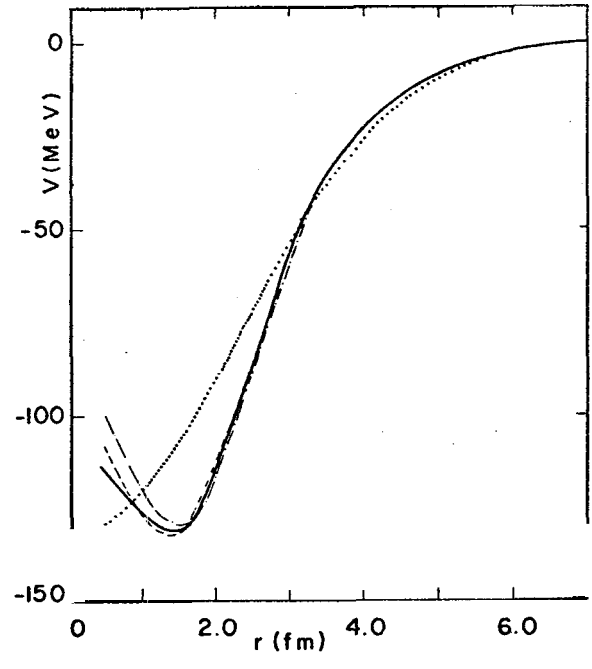

Fig.3 - Potentials for the OCM. The solid line is V_{eff}; the dashed and dot-dashed lines are the parity projected effective potentials V^{+}_{eff} and V^{-}_{eff}, respectively, and the dotted line is the local part V_D of the RGM.

parlson shows that values obtained with one or two Gaussians areclearly poorer.

The procedure described above is also followed to calculate effective potentials $V_{eff}^{(\pm)}$ for even and odd waves. These potentials were derived from parity projested energy surfaces $E^{(\pm)}$ obtained from eq. (36c), with an analogous definition for $\tilde{I}(\vec{\alpha},\vec{\alpha})$. Using four Gaussians in expansion (30) we fit $E^{(\pm)}$ with rms dcviations of the same order as that for unprojected potentials.

The effective potentials V_{eff}, V_{eff}^{+} and V_{eff}^{-} obtained in the present work are shown in fig. 3. For cornparison the potential V_D is aiso shown. The first point to be noticed is that the parity projection has a very weak effect. The potentials for odd and even waves, $V_{eff}^{(-)}$ and $V_{eff}^{(+)}$, are very close to V_{eff}, except at small separations $r \lesssim 1$ fm where these potentials differ slightly. The direct RGM potentials V_D, on the other hand, is not so close to V_{eff}. Although both potentials have the same tail there is some discrepancy between 2.5 and 5 fm and considerable differences at $r < 2.5$ fm.

Table 1 - Parameters of the four gaussians effective potential.

m	1	2	3	4
V_m (MeV)	281	-408	222	-202
a_m (fm)	1.5	2.0	2.5	3.0

3.2. Phase-shifts

The α-^{16}O phase-shifts obtained by solving the OCM equation with the potential V_D and V_{eff} are shown in figs. 4 and 5, together with "exact" RGM values[14]. The calculations were performed at C.M. energies up to 30 MeV for the relevant partial waves $\ell = 0,1,\ldots,10,11$. The phases are drawn so that they converge to zero in the high energy limit. Their threshold value is[15]

$$\delta_\ell(\varepsilon=0) = (n_\ell + m_\ell)\,\pi \qquad (40)$$

where m_ℓ is the number of bound states in the partial wave R and n_ℓ is the number of redundant states given by $n_\ell = 4-\ell/2$ for even waves and

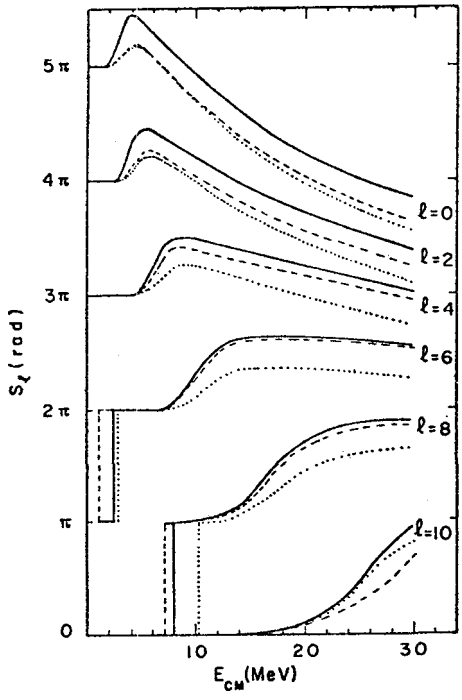

Fig.4 - Phase-shifts for even-ℓ. The solid lines are the results of a full RGM calculation; the dashed and dotted lines are obtained by solving the OCM with the potentials V_{eff} and V_D, respectively.

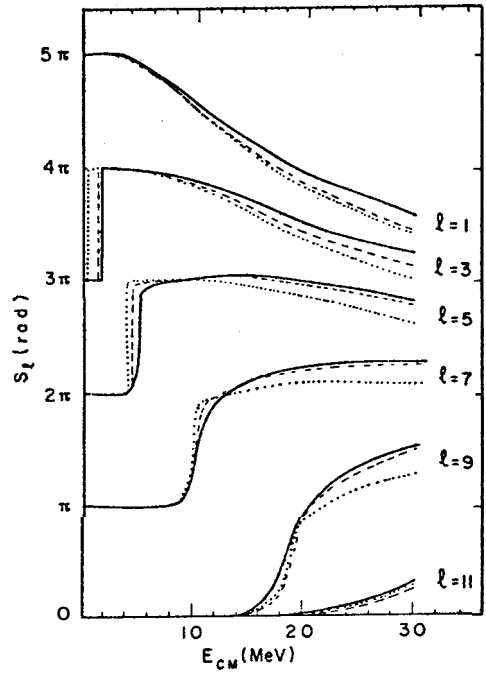

Fig.5 - Phase-shifts for odd-ℓ. The notations are the same as in fig.4.

$n_\ell = 4-(R-1)/2$ for odd waves.

The agreement between "exact" phase-shifts and phase-shifts obtained by the OCM with V_{eff} is variable. It is good for $\ell=4$, verygood for $\ell=6$ and 8 and fair for $\ell=0,2$ and 10. For odd waves, where the OCM calculation seems to be more accurate, the agreement is good for $\ell=1$ and 3 and very good for the remaining waves.

On the other hand, the phase-shifts obtained by the OCM with the potential V_D are consistently satisfactory for all waves (slightly better for odd waves). The overall agreement is, however, definitely poorer than in the previous case.

The phase-shifts obtained in both OCM calculations (with V_{eff} and with V_D) as well as the "exact" phase-shifts show bands of resonances; two bands for even waves and one band for odd waves. The resonance energies obtained from each calculation are listed in table 2, for even-R bands, and table 3, for the odd-R band. With the exception of a few resonances the energies predicted by both versions of the OCM are consistent with those of the RGM. One can notice also that the predictions with the potential V_{eff} are slightly better.

Table 2 - Even-waves resonance energies. $E^1_{exact}(E^2_{exact})$, $E^1_{eff}(E^2_{eff})$ and $E^1_{V_D}(E^2_{V_D})$ are energies (MeV) for the first (second) band, obtained with the RGM and the OCM with V_{eff} and V_D, respectively.

ℓ	0	2	4	6	8	10
E^1_{exact}	bound	bound	bound	2.9	8.1	-
$E^1_{V_{eff}}$	"	"	"	1.2	7.2	-
$E^1_{V_D}$	"	"	"	3.0	10.4	-
E^2_{exact}	2.8	3.6	6.0	9.8	17.4	25.4
$E^2_{V_{eff}}$	3.2	4.0	6.4	9.9	18.0	27.8
$E^2_{V_D}$	3.2	4.3	7.0	11.0	21.6	25.8

Table 3 - Odd-waves resonance energies. The notation is the same as in table 2.

ℓ	1	3	5	7	9	11
E^1_{exact}	bound	1.6	5.2	10.4	18.4	>30.0
$E^1_{V_{eff}}$	"	1.2	4.5	10.2	18.8	>30.0
$E^1_{V_D}$	"	0.2	4.1	9.9	18.4	>30.0

3.3. Ambiguities in the effective potential

In this subsection we study the dependence of effective potentials and corresponding GCM phase-shifts on details of the parametrization of eq. (30). For this purpose we compare the potentials $V^{(a)}_{eff}$ and $V^{(b)}_{eff}$ and the phase-shifts $\delta^{(a)}_R$ and $\delta^{(b)}_\ell$, derived from two sets of parameters $\{V_1^{(a)},\ldots V_4^{(a)};\ a_1^{(a)}\ldots\ a_4^{(a)}\}$ and $\{V_1^{(b)}\ldots,V_4^{(b)};\ a_1^{(b)},\ldots a_4^{(b)}\}$. With both sets the GCM energy surface is fitted with very small rms deviation ($\Delta\sim 0.05$ MeV). The potentials $V^{(a)}_{eff}$ and $V^{(b)}_{eff}$ are shown in fig. 6, which also includes V_D. Although $V^{(a)}_{eff}$ and $V^{(b)}_{eff}$ have the same tail these potentials diffar considerably a $r \leq 2$fm. In figs. 7 and 8 the phase-shifts $\delta^{(a)}_\ell$ and $\delta^{(b)}_\ell$ are pictured for ℓ=5 and ℓ=6, respectively. "Exact" RGM phase-shifts are also shown. Again, the sets of parameters a and b lead to significantly different results. In both cases the phase-shifts $\delta^{(a)}_\ell$ and $\delta^{(b)}_\ell$ differ by ~20 degrees. For the sharp resonance the situation is still worse. While the resonance energies predicted with $V^{(a)}_{eff}$ are not far from the "exact" values, the results obtained with thepotential $V^{(b)}_{eff}$ are much poorer. The resonance at R=5 is missed by -2 MeV and that at R=6 appears as a bound state.

The existence of the above ambiguities is an unpleasant feature of the technique used to derive the effective potential. The source of this difficulty could be in the unfolding of the convoluted matrix-elements of eq.(33), to derive the effective potential. The deconvolution of such expressions is a difficult operation which may become worse if the potential is expressed in terms of a large number of porameters and

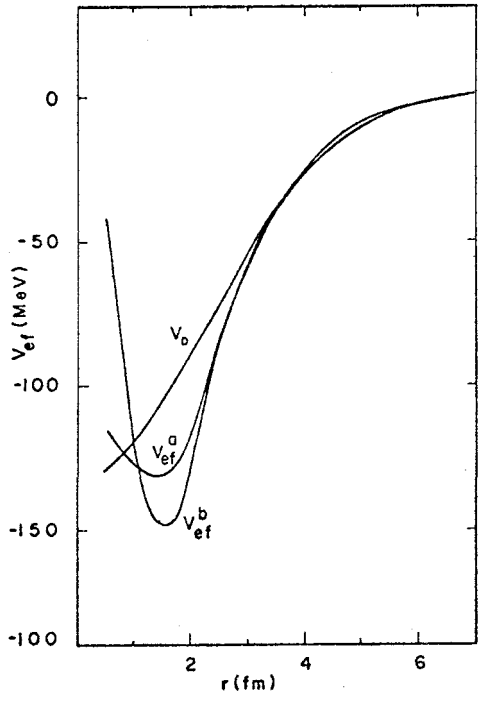

Fig.6 - Ambiguities in the effective potentials. V_D, $V^{(a)}_{eff}$ and $V^{(b)}_{eff}$ are defined in the text.

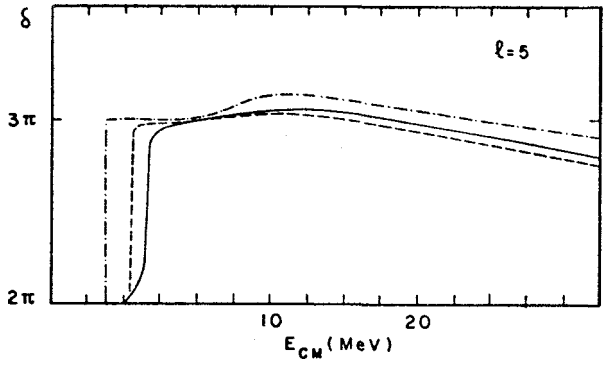

Fig. 7 - The α-^{16}O phase-shifts for R = 5. The solid line is the result of a full RGM calculation; the dashed and dot-dashed lines are the results of the OCM equation with $V^{(a)}_{eff}$ and $V^{(b)}_{eff}$, respectively.

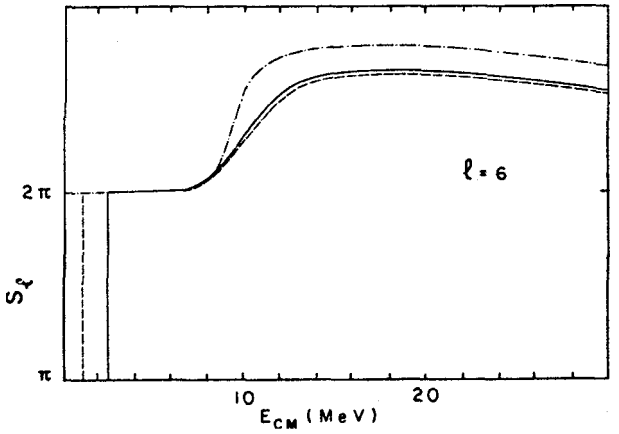

Fig.8 - The α-^{16}O phase-shifts for **R-6.** The notation are the same as in fig. 7. Notice that the sharp resonance predicted by the RGM calculation for the $V_{eff}^{(b)}$ case appears as a bound state.

these pararieters are chosen to **fit** the convoluted matrix elements with great accuracy. In some cases the unfolding can be improved by simply reducing the number of parameters and allowing for some inaccuracy in in the **fit**. Having this in mind we performed calculations with V_{eff} parametrized in terms of three and two gaussians with the result that the use of three gaussians led to ambiguities of the same kind andtoslightly poorer phase shifts and with two gaussians the phase shits were too poor and some ambiguity still remained. The ambiguities in V_{eff} cannot, therefore, be associated with the use of too many parameters in our calculation.

A different approach to handle this problem could be followed by introducing a few changes in the prescription[5] for deriving the effective potential. The major limitations of such a prescription, namely, restricting the **fit** to diagonal elements of the GCM kernels at a set of equally spaced generator coordinate values, can be partly dropped. The inclusion of extra pointsin the vicinity of the minimum of the energy surface, which plays an important role for sharp resonances, isstraightforward. Off-diagonal GCM kernels $\tilde{H}(\vec{\alpha}_i, \vec{\alpha}_j)$ with $\vec{\alpha}_i$ and $\vec{\alpha}_j$ along the same direction can also be included without appreciable cornplication. We be-

lieve that this approach can lead to improvements in the effective potential and in the phase shifts. Presently a study along these lines is in progress.

4. CONCLUSION

The collision α-^{16}O is studied with microscopic theories. The OCM, using both the local part of the RGM potential and an effectivepotential derived from diagonal elements of GCM kernels, is used to derive nuclear phase-chifts in the partial waves R = 0,1,...,11, at C.M. energies up to 30 MeV. The calculations are performed with the commonly used Volkov V2 interaction and the results are compared to "exact" RGM results. The OCM phase-shifts obtained with V_{eff} are in good agreement with those of the RGM, specially for the partial waves R = 5, 6, 7, 8,9. The overall agreement is better for odd partial waves. The OCM phase--shifts when calculated with V_D are also consistent with those of the RGM, although they are poorer than in the former case. The improvement resulting from the use of V_{eff} is appreciable. It is however less impressive than that for calculations[5] with the Brink-Boeker[16] B1 interaction, for which the OCM with V_D fails entirely.

We study the sensitivity of the results to details of the parametrization of V_{eff}. We find that this potential is ambiguous. We show that two equally good parametrizations (by the criteria of fitting our mesh of GCM kernels to the same r.m.s. deviation) can lead to rather different potentials and nuclear phase-shits. A detailed study of the origin of such ambiguities is presently in progress.

We are grateful to Dr. H. Friedrich for providing us with the OCM and the V_{eff} codes and for several helpful comments during the development of this work. We are also indebted to Dr. R. Donangelo for critically reading the manuscript.

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Resumo

O uso de teorias miscroscópicas aproximadas na colisão α-^{16}O é investigado. O modelo da condição de ortogonalidade (OCM) com o potencial direto do Método do Grupo Ressonante (RGM) e também com um potencial efetivo, V_{eff}, derivado a partir de "Kernels" do Método de Coordenada Geradora (GCM) é empregado para estudar colisões em energias (C.M.) até 30 MeV, para todas as ondas parciais relevantes. Embora as previsões do OCM sejam consistentes com resultados "exatos" do RGM em ambos os casos, as defasagens obtidas com o potencial efetivo são melhores. São constatadas ambiguidades na determinação de V_{eff}. A natureza destas ambiguidades é discutida.