Revista Brasileira de Física, Volume 13, nº 1, 1983

Statistical Features of Pre-Compound Processes in Nuclear Reactions

M. S. HUSSEIN

Instituto de Física, Universidade de São Paulo, Caixa Postal 20.516. São Paulo, 01000, SP, Brasil

and

R. A. REGO

Centro Técnico Aeroespacial, Instituto de Atividades Espaciais, São José dos Campos, 12.200, SP, Brasil

Recebido em 24 de janeiro de 1983

Abstract Several statistical aspects of multistep compound processes are discussed. The connection between the cross-section auto-correlation function and the average number of maxima is emphasized. The restrictions imposed by the non-zero value of the energy step usedin measuring the excitatlon function and the experimental error are discussed. Applications are made to the system $^{25}Mg(^{3}He,p)^{27}Al$.

1. INTRODUCTION

In recent years the basic mechanism underlying nuclear reactions have been profoundly reexamined in view of the increasing experimental evidence in support of new types of processes that lie, in complexity, between the usual direct and compound ones.

The theoretical description of these processes is rendered complicated as a result of their being more complicated than the simple direct processes, usually describable within DWBA or coupled channels theory, and yet less complicated than the usual compound processes normally accounted for by the statistical Hauser-Feshbach theory. This necessarily implies that the description of these preequilibrium processes rnust, somehow, contain both the statistical features, dominant in conpound reactions, and some coherent effects (e.g. peaking in the forward-angle region) that characterize direct processes.

One possible way of simplifying the theoretical descriptionof preequilibrium reactions is to separate the average cross-section into two well-defined and different pieces; one describing that part of the processes which is forward peaked (called multistep direct part by the MIT group[1]) and the other, symmetrical about 90^{O}, considered as a ge-

neralized Hauser-Feshbach cross-section that describes what is called precompound or multistep compound part. In particular, this last part has been the subject of extensive theoretical discussion in the last several years. Principally, three theories have been advanced[1,2,3], the common feature of which is their final result summarized as a generalized Hauser-Feshbach expression for the average cross-section. This expression is given as a sum of N distinct terms related to the contributions from the N different classes of compound doorways resonances assumed populated in the process of the formation of the compound nucleus. The system is allowed to decay to the open channels from any of these stages. From the characteristics of these decay processes one should be able to learn something about the nature of the compound nucleus configurations through which the trapped incident flux percolates.

In particular, in the Nested-Doorway Model (NDM) developed recently in Ref. 3, a major role is given to the statistical fluctuations around the average cross section (Ericson's fluctuations) in providing the above mentioned information about the different CN stages. Through a careful study of the cross-section auto-correlation function, it is suggested that one may be able to extract at least several distinct correlation widths attached to the different lifetimes of these stages.

Clearly, for these statistical analyses to be viable, one is forced to restrict oneself to transitions leading to well-separated discrete states of the residual nucleus. The excitation functions of these transitions are expected to exhibit clear statistical fluctuations, for not too high incident energies. As the incident energy is increased, the discrete parts of the spectra of emitted particles become mostly direct (forward-peaked) in nature. The multistep compound component would, in this case, contribute mostly to the continuum region, accounting partly for the emission of "fast" (non-evaporation) particles exhibiting 90^{o}-symmetrical angular distributions.

Recently, several excitation functions for discrete transitions of both light[4]- and heavy[5]- ion induced reactions, have been statistically analysed, and consequently a very clear evidence has been

established in support of the exlstence of more than one correlation width. In these analyses, the generalized cross-sectlon autocorrelation function of Ref. 3 has been used. Subsequent to these studies, it was suggested in[6] that the number-of-maxima method (NMM) of Brink ans Stephen[7] should be applied to the analysis of these excitation functions, in conjunction with the auto-correlation method in order to check the conslstency of results. Owing to the fact that the pre-compound excitation functions analysed with NMM in Ref. 6 exhibit several correlation widths, the treatment was necessarily crude. The importance of the above double-checking of the results with NMM certainly calls for a more precise theoretical discussion.

In the present paper we present the above discussion in the form of a generalizatlon of the method of Bizzeti and Maurenzing[8] which better serves the purpose of relating the average number of maxima in the excitation function to the cross-section auto-correlation function.

For completeness, we present in Section 2, a brief summary of the Nested-Doorway Model of precompound reactions. In Section 3, we present a short account of the NMM of Brink and Stephen and the related discussion of Ref. 8. The generallzation of the BM method to the multistep case and the numerical consequences are given in Section 4. Finally, In Section 5 we present several concluding remarks.

2. A RESUME OF THE NESTED-DOORWAY MODEL OF PREGOMPOUND PROCESSES

For a better understanding of the Nested-Doorway Model itwould be worthwhile emphasizing again some of the points mentioned already in the previous section. We show in Fig. 1 a typical spectrum of emitted particles 'in a light ion induced reaction. This figure constitutes the prototype one usually used[1] to describe a nuclear reactions at not too small energies. The preequilibrium portlon of the spectrum is seen to be in the continuum region.

At somewhat smaller energies, the form of the spectrum, changes, as even the discrete part of the spectrum becomes compound-nucleus dominated (90^{O}-symmetrical angular distribution). A possible picture of the spectrum at these lower energies is shown in Fig. 2. As is clearly

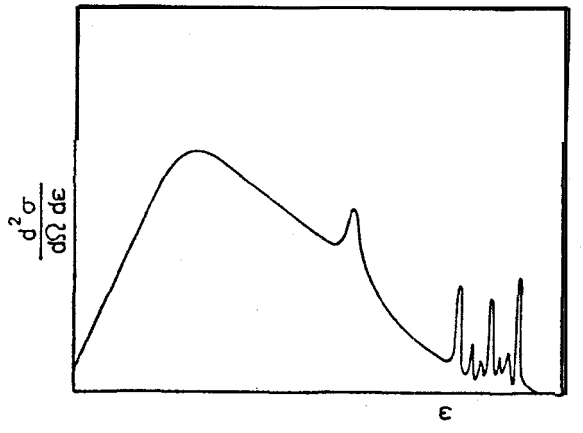

Fig. 1 - A schematic plot showing a typical nuclear reaction spectrum. The individual peaks at the end of the spectrum represents direct transitions to discrete states In the residual nucleus. The broad bump indicates the evaporation component.

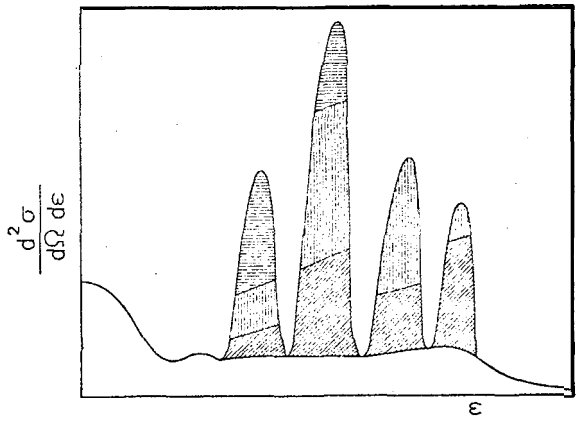

Fig. 2 - A schematic plot showing the spectrum of emitted particles at lower energies. The individual peaks represent compound transitions to discrete states in the residual nucleus. The different components in each peak represent the contributions from the different stages.

Implied, individual peaks in the spectrum receive contributioris from the decay of the compound system at the different stages through which it passes on its way to equllibrium.

One would therefore expect that the average cross section for a given transition from channel c to channel c' is (all formulae below refer to the contribution of a given partial wave)

$$\langle \sigma^{J\ell}_{cc'} \rangle = \sum_n \sigma^{J\ell}_{n,cc'} \tag{1}$$

where n refers to three, five, etc. - excitons configurations. Such a simple form for $\langle \sigma^{J\ell}_{cc'} \rangle$ emerges fram all three theories of MSCP. In the NDM, the individual terms in the sum of Eq. (1) come out all in the Hauser-Feshbach form

$$\sigma^{J\ell}_{n,cc'} = \frac{(P_n - P_{n+1})_{cc} \, (P_n - P_{n+1})_{c'c'}}{\mathrm{Tr}(P_n - P_{n+1})} \tag{2}$$

where P_n refers to the transmission matrix defined with respect to an averaged S-matrix, $\langle S \rangle_{I_n}$, constructed in such a way as to represent absorption in the systern due to the coupling to doorway classes, $n+1$, $n+2,\ldots,$ etc.. We are, here, adopting the following convention for labeling the doorway classes: the complexity increases according to the order: $1,2,3,\ldots,N$. Class N, therefore represents the equilibrated compound nucleus configuration and thus is the longest-lived. The energy interval I_n is chosen in such a way as to encompass all classes with average lifetlmes larger than $\hbar/I_n$ (or, equlvalently, with average widths, Γ_n, smaller than I_n). Explicitly, P_n is given by

$$P_n = 1 - \langle S \rangle^{\dagger}_{I_n} \langle S \rangle_{I_n} \tag{3}$$

The total S-matt-ix is accordingly given by

$$S \equiv \langle S \rangle_{I_n} + \sum_{m=n}^{N} S^{J\ell}_m \tag{4}$$

Since the final result for $\sigma^{J\ell}$ is obtained using an averaging interval I larger than the widthsof all classes, we may write

$$S = \langle S \rangle_{I_1} + \sum_{n=1}^{N} S_n^{J\ell} \tag{5}$$

where $\langle S \rangle_{I_1}$ is the optical S-matrix, obtained from an optical model description of elastic scaterring. It was shown in Ref. 3b that the multiple averaging $\langle\langle \ldots \langle \sigma^{J\ell}_{n,cc'} \rangle_{I_n} \ldots \rangle_{I_2} \rangle_{I_1}$, needed to obtain the final averaged fluctuation cross-section, does not change the structure of $\sigma^{J\ell}_{n,cc'}$, Eq.(2), ar long as the individual $S_n^{J\ell}$ in Eq. (4) (or Eq.(5)) are chosen in such a way as to average to zero, i.e.

$$\langle S_n^{J\ell} \rangle_{I_n} = 0 \tag{6}$$

Eq.(6) clearly shows, also, that $\langle S_n^{J\ell} \rangle_{I_{n+1}} = 0$, etc.. The important first step used in ref. (3) to obtain the above result was a generalization of the optical background representation method of Kawai-McVoy and Kerman[9], which gives

$$S^{J\ell}_{n,cc'} = -i \sum_{i \in n} \frac{g_{ni,c} g_{ni,c'}}{E - \varepsilon_{ni}} \tag{7}$$

where $g_{ni,c}$ are the usual form factors specifying the residues at, and ε_{ni} are the positions of, the poles in S due to the doorway resonances of class n. The $g_{ni,c}$, by construction, are random variables with zero mean, $\langle g_{ni,c} \rangle_{I_n} = 0$. Eq. (6) is then satisfied automatically.

With the help of Eqs. (6) and (7), the S-matrix autocorrelatim function.

$$C^{(s)}_{cc'}(\varepsilon) = \langle S^{J\ell *}_{cc'}(E + \varepsilon)\, S^{J\ell}_{cc'}(E) \rangle_{I_1} \tag{8}$$

may be easily evaluated to give

$$C^{(s)}_{cc'}(\varepsilon) = \sum_{n=1}^{N} \frac{\Gamma_n}{\Gamma_n{}^{E}}\, \sigma^{J\ell}_{n,cc'} \tag{9}$$

where $\sigma^{J\ell}_{n,cc'}$, are given by Eq. (2). The cross-section auto- correlation function, $C_{cc'}(\varepsilon)$, is then easily obtained from Eq. (9), neglecting contributions from direct reactions (i.e., $\langle S \rangle_{I_1}$ is considered diagonal in channel space)

$$C_{cc'}(\varepsilon) = \left| C^{(s)}_{cc'}(\varepsilon) \right|^2 = \left| \sum_{n=1}^{N} \sigma^{fl}_{n,cc'} \frac{\Gamma_n}{\Gamma_n + i\varepsilon} \right|^2 \qquad (10)$$

In Eqs. (9) and (10), Γ_n, denotes the correlation width associated with the n-th class of overlapping doorway resonances. The above result for $C_{cc'}(\varepsilon)$ is a straightforward generalization of Ericson's result for the one-class case. For a more detailed discussion concerning the derivation of Eq. (10) see refs. 3b and 3c.

Recently, several analyses[4,5] of excitation functions, using Eq. (10), have been reported. These analyses indicated clearly the presente of, at least, two distinct correlation widths. It should be stressed that a clear verification of the multistep nature of the discrete compound transitions of figure 2, referred to above, does not necessarily require the presence of more than one correlation width in the $C(\varepsilon)$ of each transition. It is sufficient to find distinctly different correlation widths in $C_{cc'}(\varepsilon)$ for different exit channels. This is so since the nature of the final channel might be such as to indicate strong coupling to a given class of doorways.

Ihe above mentioned analyses are quite important for the understanding of the reaction mechanism involved. Recently, these studies were extended to heavy-ion induced cornpound reactions[5] and fusion[10]. The only feature of the statistical theory that is being tested in these analyses, has been the existence of several distinct "life-times". To develop a more stringent test of the theory, however, other consistency checks, of the results obtalned in the above studies, are required. One possible constraint would be a sum-rule involving the correlation widths, suggested recently by one of us". This sum rule states

$$2\pi \sum_{n} \frac{\Gamma_n}{D_n} = \mathrm{Tr}\, P_1 \qquad (11)$$

where D_n is the average level spacing in class n, and P_1 is the optical transmission matrix. So far it has been difficult to put Eq. (11) into use, due to the fact that not all the Γ_n,s are extracted experimentally.

Another, potentially important, test is the number maxima method, developed originally for the one-class case, by Brink and Stephen[7]. This theory supplies the following relation between the average number, $\bar{n}$, of maxima in the excitation function per unit energy, and the correlation width, Γ,

$$\bar{n} = \frac{0.55}{\Gamma} \tag{12}$$

Eq. (12) has been extensively used in the past, in conjunction with the auto-correlation method. Recently[6] a generalization of Eq. (12) to the multi-class case has been made. In this case, the NMM does not supply another mean of obtaining the Γ_n,s, but, rather, it may be used to check the values of the extracted Γ_n,s and $\sigma^{f\ell}_n$,s. However, as already recognized in the past by several authors, several important corrections have to be made, before a confrontation of Eq. (12) with the data is attempted. These corrections are primarily related to the finite size of the energy step and the nonzero value of the error bar.

In Ref. (6) some of the above mentioned corrections were introduced into the generalized BS relation. However, it was realized[6] that the correction related to the error bar could, almost, be accounted for only approximately within the BS method. Although the theoretical values of $\bar{n}$ obtained in Ref. (6) for the reaction $^{25}Mg(^3He,p)^{27}A\ell$ studied recently by Bonetti *et al.*, came out close to the experimental $\bar{n}$, a better account for the above mentioned corrections is called for in view of their important effect in reducing the number of maxima *to be counted as real maxima.* In the following section, we present a more refined treatment of these corrections.

3. AVERAGE NUMBER OF MAXIMA IN THE EXCITATION FUNCTIONS FOR PRE-COMPOUD PROCESSES

In this section we generalize the method developed by Bizzetti and Maurezing[8] for the calculation of $\bar{n}$. This method is better suited for discussing the correction referred to in the previous section. In the BS method the real and imaginary parts of the fluctuation S-matrix are taken to be random variables. Here we take the fluctua-

tion cross section, $\sigma^{f\ell}$ calculated at three energies, E, $E+\varepsilon_0$ and E-E,, to be the three basic random variables. The joint probability distribution of these cross section is taken to be Gaussian. Such a procedure is valid when many channel are opened.

The condition that at energy E the cross section attains a maximum value is

$$\sigma^{f\ell}(E) > \sigma^{f\ell}(E \pm \varepsilon_0) \tag{13}$$

where for the moment we have ignored the effect of the non-zero value of the error bar. The average number of maxima per unit energy interval, $\bar{n}$, is then given by[12]

$$\bar{n} = \frac{2}{\varepsilon} \int_{-\infty}^{\infty} d\sigma_2 \int_{-\infty}^{\sigma_2} d\sigma_1 \int_{-\infty}^{\sigma_1} d\sigma_3 \; F(\sigma_1,\sigma_2,\sigma_3) \tag{14}$$

where o, $= \sigma^{f\ell}(E)$, $\sigma_2 = \sigma^{f\ell}(E+\varepsilon_0)$, $\sigma_3 = \sigma^{f\ell}(E-\varepsilon_0)$, with $\sigma_1>\sigma_2>\sigma_3$ (for which reason the appearance of the factor 2) and $F(\sigma_1,\sigma_2,\sigma_3)$ is their joint probability distribution given by

$$F(\sigma_1,\sigma_2,\sigma_3) = (2\pi)^{-3/2} \; D^{-1/2} \exp\left[-\frac{1}{2} x^T A x \right] \tag{15}$$

where $x = \begin{vmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{vmatrix}$ and D = det C, C being the correlation matrix, whose ij matrix element is given by (see Eq. (10))

$$C_{kj} = \left| \sum_n \frac{\sigma^{f\ell}}{1 + i(k-j)\dfrac{\varepsilon_0}{\Gamma_n}} \right|^2 \tag{16}$$

The matrix A is just C^{-1}.

The triple Gaussian integral, Eq. (14) is easily evaluated to give

$$\bar{n} = \frac{1}{\pi\varepsilon_0} \tan^{-1} \sqrt{4 \frac{C(0) - C(\varepsilon_0)}{C(0) - C(2\varepsilon_0)} - 1} \qquad (17)$$

The $C(\varepsilon_0)$, etc. is given by Eq. (10).

Equation (17), in the limit of zero energy step size and one class case, reduces to

$$\bar{n} = \frac{0.55}{\Gamma} \qquad (18)$$

which is BS relation valid for the large number of channels case. Eq. (17), though valid for many classes and contains the effect of finite energy step ($\varepsilon_0 \neq 0$), still suffers from the absence of any information concerning the nonzero value of the error bar. To remedy this we have to modify the basic condition for having a maximum in the cross-section, Eq. (3). Thus we require for the maximum the following

$$\sigma(E) > \sigma(E \pm \varepsilon_0) + \gamma \sqrt{V(\sigma)} \qquad (19)$$

where $V(\sigma)$ is the variance of the cross section

$$V(\sigma) = \langle\sigma^2\rangle - \langle\sigma\rangle^2 \qquad (20)$$

Equation (20) guarantees that only the "maxima" defined within the error bar (accounted for by the parameter $\gamma > 0$) are to be counted. Let us now introduce the following parameters

$$\begin{aligned} \sigma(E+\varepsilon_0) &= \langle\sigma\rangle + \sqrt{V(\sigma)}\, x_1 \\ \sigma(E) &= \langle\sigma\rangle + \sqrt{V(\sigma)}\, x_2 \\ \sigma(E-\varepsilon_0) &= \langle\sigma\rangle + \sqrt{V(\sigma)}\, x_3 \end{aligned} \qquad (21)$$

The average number of real maxima per unit energy is then given by

$$\bar{n} = \frac{2}{\varepsilon_0} \int_{-1/\sqrt{R(0)}}^{\infty} dx_2 \int_{-1/\sqrt{R(0)}}^{x_2-\gamma} dx_1 \int_{-1/\sqrt{R(0)}}^{x_1} dx_3 \; F(x_1,x_2,x_3) \tag{22}$$

where

$$R(0) \equiv V(\sigma) \; / \; \langle\sigma\rangle^2$$

For the Gaussian description of F to be valid, $R(0) << 1$. Thus we extend limits in the integrals above to $-\infty$. The joint distribution function $F(x_1,x_2,x_3)$ is just as in Eq. (15). Making the following change in variables

$$x_2 = y \quad , \quad x_1 = y - \rho \sin\theta \, , \; x_3 = y - \rho \cos\theta$$

and calling $\tan\theta \equiv t$, we obtain, after integrating over y, the following integral form for $\bar{n}$

$$\bar{n} = \frac{1}{\pi\varepsilon_0} \cdot \frac{1}{\sqrt{DM}} \cdot \int_0^1 \frac{dt}{C_1+2C_2t+C_1t^2} \exp \left[\frac{C_1+2C_2t+C_1t^2}{t^2} \right] \tag{23}$$

where D is the determinant of the correlation matrix $\boldsymbol{C}$ and is given by

$$\begin{aligned} D = {} & (1 - C^2(\varepsilon_0)) + C^2(\varepsilon_0)\left[C(2\varepsilon_0)-1\right] \\ & + C(2\varepsilon_0)\left[C^2(\varepsilon_0)-C(2\varepsilon_0)\right] \end{aligned} \tag{24}$$

and

$$DM = 2(1-C(2\varepsilon_0)) + (1-C^2(2\varepsilon_0)) + 4C(\varepsilon_0)\left[C(2\varepsilon_0)-1\right] \tag{25}$$

$$C_1 = D^{-1}\left\{(1-C^2(\varepsilon_0)) - \frac{(1-C(\varepsilon_0))^2.(1-C(2\varepsilon_0))^2}{DM}\right\} \tag{26}$$

$$C_2 = D^{-1}\left\{(C^2(\varepsilon_0)-C(2\varepsilon_0)) - \frac{(1-C(\varepsilon_0))^2.(1-C(2\varepsilon_0))^2}{DM}\right\} \tag{27}$$

In all of the above equations $C(x)$ is given by Eq. (10), namely

$$C(x) = \left| \sum_{n=1}^{N} \frac{\sigma_n^{f\ell}}{1 + i\,\frac{x}{\Gamma_n}} \right|^2$$

Clearly in the limit $\gamma=0$, we recover the error-bar uncorrected Eq. (17).

The integral in Eq. (23) cannot be evaluated in closed form. We have calculated $\bar{n}$ through numerical integration.

Equations (23)-(27) with $C(x)$ given by Eq. (10) constitute the principal results of this section. They show clearly the way the multistep aspect of the excitation function, as exemplified through the generalized cross-section auto-correlation function $C(x)$ of Eq. (10), enters in the determination of the average number of maxima, subject to the usual experimental restrictions of having to deal with a finite energy step size, ε_0, and a non-zero error bar, γ. The zero step-size limit ($\varepsilon_0=0$) and perfect measurement ($\gamma=0$) is easily obtained by taking the appropriate limit in Eq. (17). Considering a two-class case we obtain

$$\bar{n} = \frac{\sqrt{3}}{\pi}\, N(\sigma_1,\sigma_2,\Gamma_1,\Gamma_2) \tag{28}$$

where

$$N(\sigma_1,\sigma_2,\Gamma_1,\Gamma_2) = \left\{ \frac{\sigma_1^2/\Gamma_1^4 + \sigma_2^2/\Gamma_2^4 + 2\sigma_1\sigma_2\left[\Gamma_1^{-4} + \Gamma_2^{-4} + \Gamma_1^{-2}\Gamma_2^{-2} - \Gamma_1^{-2}\Gamma_1^{-1}\Gamma_2^{-1} - \Gamma_2^{-2}\Gamma_2^{-1}\Gamma_1^{-1}\right]}{\sigma_1^2/\Gamma_1^2 + \sigma_2^2/\Gamma_2^2 - 2\sigma_1\sigma_2\left[\Gamma_1^{-1}\Gamma_2^{-1} - \Gamma_1^{-2} - \Gamma_2^{-2}\right]} \right\}^{1/2} \tag{29}$$

with

$$\sigma_1 = \sigma_1^{f\ell}/(\sigma_1^{f\ell} + \sigma_2^{f\ell}) \;,\; \sigma_2 = \sigma_2^{f\ell}\;(\sigma_1^{f\ell} + \sigma_2^{f\ell})$$

Eqs. (28) and (29) are a simple generalization of the BS formula for the two-doorway-class case valid when the number of channels is large

(the numerical factor $\sqrt{3}/\pi$ is just 0.55, see Eq. (18)[6]. Actuallt by setting either σ_1 or σ_2 equal to zero, in Eq. (29), we recover Eq. (18) for the one-class case (i.e. $N \to \frac{1}{\Gamma_1}$ if $\sigma_2=0$ or $N \to \frac{1}{\Gamma_2}$ if $\sigma_1=0$).

Close inspection of the function $N(\sigma_1,\sigma_2,\Gamma_1,\Gamma_2)$ shows that its value for a given σ_1 and σ_2 lies between $1/\Gamma_1$ and $1/\Gamma_2$. Using our convention in ordering the doorway classes, we have

$$\Gamma_1^{-1} < N(\sigma_1,\sigma_2,\Gamma_1,\Gamma_2) < \Gamma_2^{-1} \qquad (30)$$

The above inequality clearly indicates that the average number of maxima to be expected in an excitation function, is generally smaller than the number predicted by BS for the equilibrated one-class situation (only Γ_2) and larger than the number associated with the simple doorway class alone (only Γ_1). This is quite reasonable since the presente of overlapping-doorway modulation on top of fine structure (FS) fluctuations in $\sigma(E)$ will make some of the FS maxima practically disappear.

Let us turn now to the effect of ε_0 and γ. Both the non-zero value of ε_0 and γ result in a reduction in the value of $\bar{n}$. As ε_0 increases in value ($\varepsilon_0 \gtrsim \Gamma_2$) some FS maxima start disappearing and eventually when ε_0 reaches the value Γ_1, one starts countingbasicallythe doorway-generated maxima only. In so far as the non-zero value of the error bar (γ) is concerned, the resulting reduction in $\bar{n}$ is quite easy to understand. The larger the error bar in the data, the larger would be the uncertainty in the nature of the peaks in the excitation function and accordingly the smaller the number of the *real* maxima to be expected. This feature is quite clearly seen in the formula for $\bar{n}$, Eq. (23).

To exhlbit the dependence of $\bar{n}$ on the several physical quantlties that specify the multistep nature of the reaction, i.e. a_1, a_2, $\sigma_3,\ldots,\Gamma_1,\Gamma_2,\Gamma_3,\ldots$, we consider below the specific case of the reaction $^{25}Mg(^3He,p)^{27}Al$, studied recently by Bonetti *et al.*[4]. Theseauthors analysed several excitation functlons for transition to discrete states in ^{27}Al, and found that in all cases two correlation widths seem to be present: a larger one, Γ_1 = 200 keV, attached to a simple, five-

exciton, configuration, and a smaller one, Γ_2 = 50 keV, associated with the life-time of the equilibrated compound nucleus ^{28}Si.

The average number of maxima according to Eq. (23) was calculated as a function of ε_0 and γ and for different combinations of a, and σ. Our results are summarized in Figures 3-6.

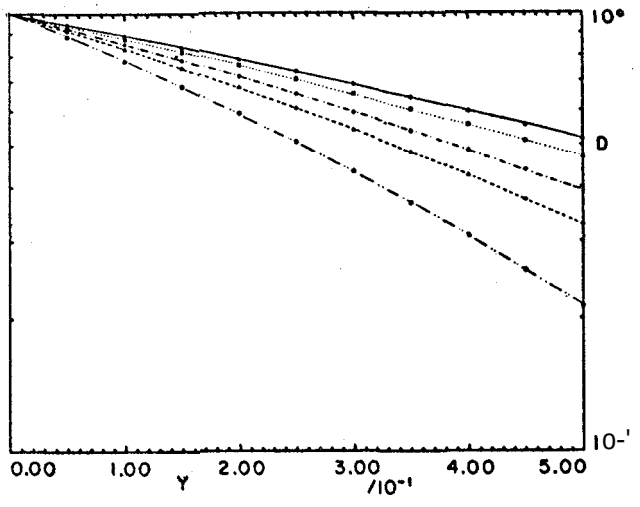

Fig. 3 - The correction factor associated with the finite size of the error bar, for the combination σ_1 = 0.75, σ_2 = 0.25, Γ_1 = 100 keV and Γ_2 = 50 keV. The full curve corresponds to ε_0 = 200 keV, dotted one, ε_0 = 150 keV, dashed-dotted, ε_0 = 100 keV, dashed, ε_0 = 75 keV and dashed-dotted-dotted, ε_0 = 50 keV. The corresponding values of $\bar{n}(\gamma=0,\varepsilon_0)\Delta E$, (Eq. 17)) are 41, 29, 23, 15 and 12 respectively. The error bar corrected $\bar{n}$ is obtained by multiplying $\bar{n}(\gamma=0,\varepsilon_0)\Delta E$ by the appropriate value of d, plotted in the figure as a function of γ (see text for more details).

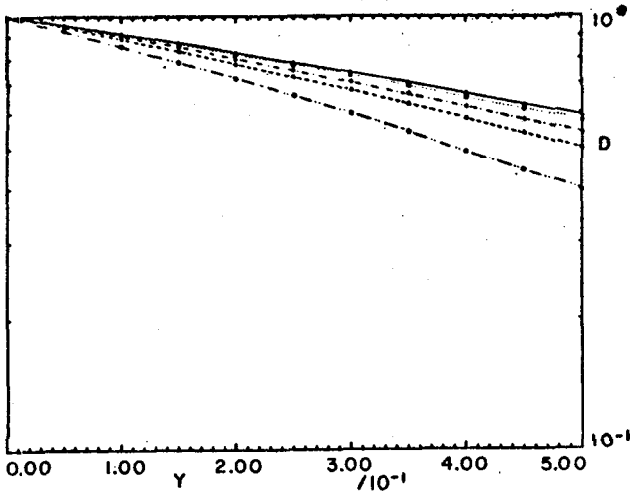

Fig. 4 - Same as the previous figure for the combination $a_1 = 0.25$, $\sigma_2 = 0.75$, $\Gamma_1 = 200$ keV and $\Gamma = 50$ keV. The value of $\bar{n}(\gamma=0,\varepsilon_0)\Delta E$, is 45, 32, 25, 17 and 13 for $\varepsilon_0 = 50$ keV, 75 keV, 100 keV, 150 keV and 200 keV, rsspectively.

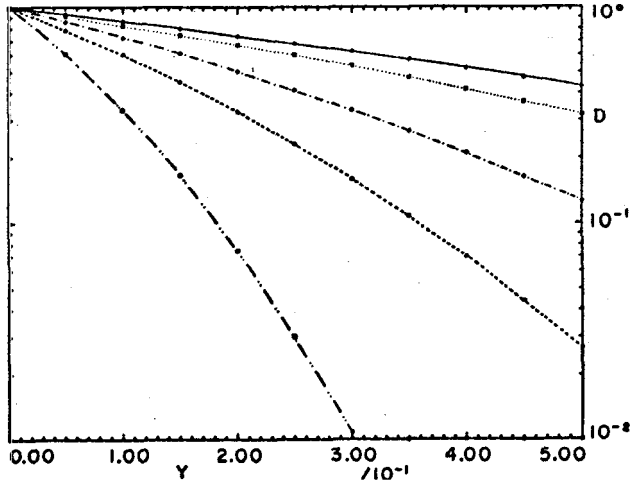

Fig. 5 - Same as the previous figure for the combination $a_1 = 1.0$, $a_2 = 0.0$,. $\Gamma_1 = 200$ keV, and $\Gamma_2 = 50$ keV. The value of $\bar{n}(\gamma=0,\varepsilon_0)\Delta E$, is 20, 18, 17, 14 and 11, for $E = 50$ keV, 75 keV, 100 keV, 150 keV and 200 keV, respectively.

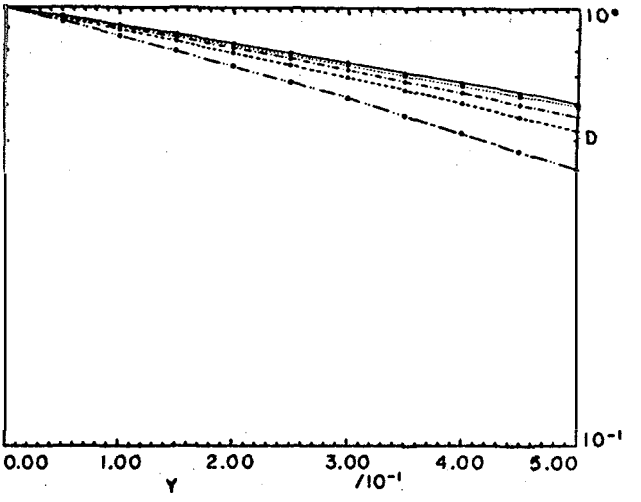

Fig. 6 - Same as the previous figure for the combination $a_1 = 0.0$, $a_2 = 1.0$, $\Gamma_1 = 200$ keV and $\Gamma_2 = 50$ keV. The value of $\bar{n}(\gamma=0,\varepsilon_0)\Delta E$, is 45, 33, 26, 17 and 13 for $\varepsilon_0 = 50$ keV, 75 keV, 100 keV, 150 keV and 200 keV, respectively.

To better appreciate the effect of includind the error bar in the calculation of $\bar{n}$, we present our result in the figures in the form of a factor defined as $d(\gamma,\varepsilon_0) = n(\gamma\neq0,\varepsilon_0)/n(\gamma=0,\varepsilon_0)$, which when multiplies $\bar{n}$ given by Eq. (17), namely $\bar{n}(\gamma=0,\varepsilon_0)$, supplies the desired result. As is clear from the results, $d(\gamma,\varepsilon_0)$ depends strongly on γ and mildly on E, for most of the different cases studied. The only case where the E - dependence of $d(\gamma,\varepsilon_0)$ is very strong is the one-class case associated with the doorway resonances ($\sigma_1=1.0$, $a_2=0.0$, Fig.5) when γ is taken to be large (0.5). Although $\bar{n}(\gamma=0,\varepsilon_0)$ always decreases with increasing E, the correctlon factor $d(\gamma,\varepsilon_0)$ tends to increase with E, the correctlon factor $d(\gamma,\varepsilon_0)$ tends to increase with ε_0, for a glven value of γ (see, e.g., Fig. 3, $d(\gamma=0.5, \varepsilon_0=50) = 0.2$, $d(\gamma=0.5, \varepsilon_0=200) = 0.47$). Clearly this trend in d, depends on the nature of the correlation function. As long as there are fine structure fluctuations ($\sigma_2\neq0$), the increase in d with ε_0 is very mild even for large values of γ. (Fig. 3,4 and 6). Once the fine structure fluctuations are removed, then the increase of d with ε_0, in the region $\varepsilon_0\lesssim\Gamma$, is seen to be quite drastic especially for large values of y. For the

case shown in Fig. 5 (σ_1=1.0, σ_2 =0.0), $d(\gamma$=0.5, ε_0=50) = 5x10^{-5} and $d(\gamma$=0.5, ε_0=200) = 0.43, i.e. a change of several orders of magnitude.

In cases where $d(\gamma,\varepsilon_0)$ changes slowly with E, it would be natural to seek a form for $\bar{n}$ that contains the ε_0- and y-corrections as multiplicative factors, in the sense

$$\bar{n} = \frac{\sqrt{3}}{\pi} N(\sigma_1,\sigma_2;\Gamma_1,\Gamma_2)\, d_1(\varepsilon_0) d_2(\gamma) \qquad (31)$$

where, though not indicated in Eq. (31), both d_1 and d_2 should depend on σ_1, σ_2, Γ_1 and Γ_2, and N is the function defined in Eq. (29). A possible way of contracting $d_2(\gamma)$ is to define an average of $d(\gamma,E_n)$ over several value of E, (i.e. the average curve in Figs. 1, 2 and 4). The functlon $d_1(\varepsilon_0)$ is just Eq. (17) divided by Eq. (28). Further work on the validity of the approximation implied by the form of $\bar{n}$ given in Eq, (31), is required.

4. DISCUSSION AND CONCLUSIONS

In this paper we have discussed several statistical aspects of multi-step compound processes. In particular we have considered the interconnection between the number of maximum method, appropriately generalized to the multistep case, and the cross-section auto-correlation functlon recently dlscussed in connection wlth pre-equilibrium reactions.

It is emphasized that any realistic application of the number-of-maxima methods must unvoidably consider the limitations imposed by the non-zero value of the energy step size and the error bar. Both of these effects result in a reduction in the average number of maxima.

Our results should be quite useful in supplyingadouble check, through the comparison of $\bar{n}$ wlth experiment, of the results of the generalized Erlcson analysis of multistep excitation functions.

REFERENCES

1. H.Feshbach, A.K.Kerman and S.E.Koonin, Ann. Phys. (NY), 125, 429 (1980).

2. D.Agassi, H.A. Weidenmüller and G. Mantizouranis, Phys. Rep. *22C* 145 (1975).

3. M.S. Hussein and K.W. McVoy, Phys. Rev. Lett. 43, 1645 (1979). W. A. Friedman, M.S. Hussein, K.W. McVoy and P.A. Mello, Phys. Rep. *77C*, 47 (1981). M.S. Hussein, Talk delivered at the Third International Conference on Nuclear Reaction Mechanism, Varenn, Italy, June, 1982. To appear in the proceedings, IFUSP/P-335.

4. L. Colli Milazzo, R. Bonetti and A. Garegnani, Lett. Al. Nuovo Cimento, 29, 496 (1980). R. Bonetti, L. Colli Milazzo, M. Melanotte, A. De Rosa, G. Inglima, E. Perillo, M. Sandoli, V. Russo, N. Saunier and F. Shahin, Phys. Rev. *C25*, 717 (1982).

5. R. Bonetti, L. Colli Milazzo, M. Melanotte and M. S. Hussein, Phys. Rev. *C25*, 1406 (1982).

6. R. Bonetti, M.S. Hussein and P.A. Mello, IFUSP/P-346, submitted for publication.

7. D.M. Brink and R.O. Stephen, Phys. Lett. 5, 77 (1963).

8. P.G. Blzzeti and P.R. Maurenzig, Il Nuovo Cim. 47, 29 (1967).

9. M. Kawai, A.K. Kerman and K.W. McVoy, Ann. Phys. (NY) 75, 156 (1973).

10. O. Civitarese, B.V. Carlson, M.S. Hussein and A. Szanto de Toledo, IFUSP/P-365, submitted for publication.

11. M.S. Hussein, Phys. Lett. *107B*, 307 (1981).

12. S.O. Rice, In *Selected papers* on *noise* and *stochastic processes*, ed. N. Wax (Dover, 1954) p. 217.

RESUMO

Vários aspectos estatísticos de processos compostos de múltiplas etapas são discutidos. A relação entre a função de auto-correlação da seção de choque e o número médio de máximos é ressaltada. As restrições impostas pelo valor não nulo no passo da energia, usado na medida da função de excitação e o erro experimental são discutidas. São teitas aplicações ao sistema $^{25}Mg(^{3}He,p)^{27}Al$.