Revista Brasileira de Física, Volume 13, n? 1, 1983

# Theory of Fidelity Measure in Degenerate Four-Wave Mixing

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Recebido em 30 de dezembro de 1982

**Abstract** Fhase-conjugate beam fidelity is studied in degenerate four-wave mixing with spatially varying pump beams. The analysis includes the effects of probe depletion, diffracting non-linear phase variation, focussing, and finally that of losses. We find relatively simple algebraic expressions for the phase conjugate reflectivity for the cases of collinear and near-collinear beam geometries. It is found that by focussing ths probe beam into the mixing medium, the fraction of energy in the phase conjugate beam which was transferred to other modes, may typically be reduced by one order of magnitude.

## 1. INTRODUCTION

F'hase conjugation optics is a new field in optics which has created exciting prospects of a gamut of applications in areas ranging frorn optical computing to nuclear fusion by laser. The gist of the concept of non-linear phase conjugation is the almost instantaneous creatlon of an electromagnetic wave which is at every point of space equals the phase-inverted image of a given object wave. In mathematical terms, if the rnonochromatic object wave is given by the expression

$$\vec{E}_p(x,y,z,t) = \vec{\mathcal{E}}_p(x,y,z)\; e^{i\vec{k}.\vec{r}-i\omega t} + \text{c.c.}$$

where c.c. stands for complex-conjugate, then the phase-conjugate wave is given by

$$\vec{E}_s(x,y,z,t) = r\, \vec{\mathcal{E}}^{\,*}_p(x,y,z)\; e^{-i\vec{k}.\vec{r}-i\omega t} + \text{c.c.} \tag{2}$$

(the subscrlpts $p$ and $s$ stand for "probe" and "signal" respectively), where $r$ is a constant.

Several techniques have been developed to create phase-inverted wavefronts, the most irnportant being three-wave mixing, backward Brillouin scattering, and four-wave mixing. The reader is referred to articles by Yariv[1], and Pepper[2], for revlews on the already extensive literature. Degenerate four-wave mixing (DFWM) has turned out to be the most popular of these methods since its proposal in 1977 by Hellwarth[3], and first experimental demonstrations in the same year by Bloom and Bjorklund[4] and Jensen and Hellwarth[5]. Suggestive of the importance of his contributions is the fact that Bloom was made a Fellow of the Optical Society of America only a few years later - and a few years after completing his Ph.D!

## 2. DERIVATION OF THE COUPLED-WAVE EOUATIONS IN A TRANSPARENT KERR MEDIUM

In DFWM the phase conjugate wave is produced by mixing a probe (object) wave in a non-linear mediurn wlth two counter propagating pump waves of the same frequency as the probe (see fig. 1). The third-order polarization inan isotropic medium may be written[6]

$$\vec{P}^{(3)}(x,y,z) = A(\vec{\mathcal{E}}.\vec{\mathcal{E}}^*)\vec{\mathcal{E}} + \frac{1}{2}B(\vec{\mathcal{E}}.\vec{\mathcal{E}})\vec{\mathcal{E}}^* \qquad (3)$$

where $A$ and $B$ are constants characterizing the medium, and the electric field is

$$\vec{E} = \sum_{\mu=1}^{4} \vec{\mathcal{E}}_\mu(x,y,z)e^{i\vec{k}_\mu.\vec{r}} \, e^{-i\omega_0 t} + \text{c.c.}$$

$$= \vec{\mathcal{E}}(x,y,z)e^{-i\omega_0 t} + \text{c.c.} \qquad (4)$$

The label μ stands for forward pump wave 1, backard pump wave 2, probe wave 3, and signal wave 4. In order that the phase conjugation be near -perfect, it is important that the pump intensity be nearly uniformover the interaction volume, eventhough, due to the absence of phase -matching conditions in four-wave mixing (as opposed to three-wave mixing) excellent phase conjugation is possible even when the probe beam is incident at an angle as large as 90° to the pump beams. In order to obtain maximum reflectivity, the parallel or near-parallel beam arrangement is favorable. It is then useful to choose the polarization ofthe probe beam to be orthogonal to that of the two pump beams. Writing the non-linear polarization $\vec{P}$ in the form

$$\vec{P}(x,y,z) = \Sigma_n \left[\vec{P}(nk_0)e^{ink_0z} + \vec{P}(-nk_0)e^{-ink_0z}\right]e^{-i\omega_0 t} + \text{c.c.}, \quad (4')$$

then equations (2) and (3) yield to first order in the probe and signal fields:

$$\vec{P}(k_0) = A(|\mathcal{E}_1|^2 + 2|\mathcal{E}_2|^2)\vec{\mathcal{E}}_1 + A(|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2)\vec{\mathcal{E}}_3 + \frac{1}{2}B(\mathcal{E}_1\mathcal{E}_2)\vec{\mathcal{E}}_4^*$$

and

$$\vec{P}(-k_0) = A(2|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2)\vec{\mathcal{E}}_2 + A(|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2)\vec{\mathcal{E}}_4 + \frac{1}{2}B(\mathcal{E}_1\mathcal{E}_2)\vec{\mathcal{E}}_3^* \quad , \quad (5)$$

where the pump beams have been assumed linearly polarized. We shall assume that the probe and signal fields are both weak comparedtothe pump field, which justifies neglecting their second-order contributions. Marburger and Lam[7] have solved the fully non-linear problem containing all four waves interacting to second order, but without including transverse spatial variation.

Equation (5) is substituted into Maxwell's equation, which when the paraxial ray ($\vec{\nabla}(\vec{\nabla}.E) = 0$) and slowly varying amplitude ($\partial\mathcal{E}/\partial z \ll k_0\mathcal{E}$) approximations are applied, yield

$$\nabla_t^2\mathcal{E}_1 + 2ik_0\frac{\partial\mathcal{E}_1}{\partial z} = A(|\mathcal{E}_1|^2 + 2|\mathcal{E}_2|^2)\mathcal{E}_1$$

$$\nabla_t^2\mathcal{E}_2 - 2ik_0\frac{\partial\mathcal{E}_2}{\partial z} = A(|\mathcal{E}_2|^2 + 2|\mathcal{E}_1|^2)\mathcal{E}_2 \quad (6a)$$

$$\nabla_t^2\mathcal{E}_3 + 2ik_0\frac{\partial\mathcal{E}_3}{\partial z} = A(|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2)\mathcal{E}_3 + \frac{1}{2}B\mathcal{E}_1\mathcal{E}_2\mathcal{E}_4^*$$

$$\nabla_t^2 \mathcal{E}_4 + 2ik_0 \frac{\partial \mathcal{E}_4^*}{\partial z} = A(|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2)\mathcal{E}_4^*$$

$$+ \frac{1}{2} B \mathcal{E}_1^* \mathcal{E}_2^* \mathcal{E}_3 , \qquad (6b)$$

where

$$\nabla_t^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 .$$

The constants $A$ and $B$ are assumed real, i.e., the dissipative part of the non-linear susceptibility is neglected here, as the frequency $w_0$ is assumed to be far from any atomic resonances.

Let us neglect for the moment transverse spatial variation. Writing $\mathcal{E}_1 = |\mathcal{E}_1| e^{i\phi_1}$ and $\mathcal{E}_2 = |\mathcal{E}_2| e^{i\phi_2}$, we see from equations (6a ) that

$$\frac{\partial |\mathcal{E}_1|}{\partial z} = \frac{\partial |\mathcal{E}_2|}{\partial z} = 0 , \qquad (7)$$

while

$$\frac{\partial \phi_1}{\partial z} = - (2k_0)^{-1} A(|\mathcal{E}_1|^2 + 2|\mathcal{E}_2|^2)$$

$$\frac{\partial \phi_2}{\partial z} = (2k_0)^{-1} A(|\mathcal{E}_2|^2 + 2|\mathcal{E}_1|^2 ) \qquad (8)$$

Combining equations (7) and (8) we obtain that the sum $\phi = \phi_1 + \phi_2$ is given by

$$\phi(z) = \phi(0) + (2k_0)^{-1} A(|\mathcal{E}_1|^2 - |\mathcal{E}_2|^2)z \qquad (9)$$

This shows that if pump intensities $|\mathcal{E}_1|^2$ and $|\mathcal{E}_2|^2$ are choosen to be equal, the phase $\phi$ is a constant, as is therefore the product $\mathcal{E}_1 \mathcal{E}_2$.

The inclusion of transverse spatial dependence is more compii-cated to treat. As the ideal situation for phase conjugation is $\mathcal{E}_1 \mathcal{E}_2$ = constant, and this is not rigorously attainable in practice, let us try for the solutian

$$\arg\,(\mathcal{E}_1\mathcal{E}_2) = \text{constant} \tag{10}$$

Thus let

$$\mathcal{E}_1 = F(x,y,z)\ e^{i\phi(x,y,z)}\ ,$$

$$\mathcal{E}_2 = F(x,y,z)\ e^{-i\phi(x,y,z)}\ , \tag{11}$$

where F and $\phi$ are real. The phase in eq. (10) has been set equal to zero, without los of generality. Substituting eqs. (11) into (6a), yields, after equating real and imaginary parts, two equations:

$$\nabla_t^2 F - (\nabla_t^2\phi + 3\ AF^2)F = 2kF\ \frac{\partial\phi}{\partial z}$$

$$\vec{\nabla}_t\phi\ .\ \vec{\nabla}F + \frac{1}{2}\nabla^2\phi\ F = -\ k\,\frac{\partial F}{\partial z}\ . \tag{12}$$

The first of these is an eikonal (or Hamilton-Jacobi) equation, the second expresses energy conservation when integrated over the x-y plane.

We note that equations (11) may be satisfied at the surface of the mirror in fig. 1, provided that its curvature matches that of incident wavefront. Likewise, physical solutions of equations (12) normally exist, leading us to the conclusion that equation (10) may indeed by satisfied.

The condition (10) is not always attainable, for example, when the medium is not perfectly loss-less, or if it is strongly scattering.

The remaining equations (6b) are now written in the form

$$L\mathcal{E}_3 = \kappa(x,y,z)\mathcal{E}_4^*\ ,$$

$$L\mathcal{E}_4^* = \kappa^*(x,y,z)\mathcal{E}_3\ , \tag{13}$$

where $L$ is an operator given by

$$L = i\partial/\partial z + \frac{1}{2k_0}\nabla_t^2 - (2k_0)^{-1}A(|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2)\ , \tag{14}$$

and

$$\kappa(x,y,z) = (4k_0)^{-1} B\,\mathcal{E}_1(x,y,z)\,\mathcal{E}_2(x,y,z) \quad . \tag{15}$$

$$= \kappa(x,y,z)^*$$

Equations (13) couple the probe field $\mathcal{E}_3$ to the phase-conjugate signal field $\mathcal{E}_4$. We now investigate the conditions under which $\mathcal{E}_4 = \mathcal{E}_3^*$ *outside* the non-linear medium.

### 3. CASE OF INFINITE PUMP BEAMS

For *infinite pump* beams $A' = (2k_0)^{-1} A(|\mathcal{E}_1|^2 + |\mathcal{E}_2|^2)$ and $\kappa$ are constants. The conditions $\kappa = \kappa^*$ need not be applied for the moment. Fourier transform equations (13)

$$\left[- k_z - (2k_0)^{-1} k_x^2 - A'\right] \mathcal{E}_3(k) = \kappa\,\mathcal{E}_4^*(k)$$
$$\left[- k_z - (2k_0)^{-1} k_x^2 - A'\right] \mathcal{E}_4^*(k) = \kappa^*\mathcal{E}_3(k) \tag{16}$$

Equations (16) may be solved for $k_z$ assuming non-zero values for the fields

$$k_z = \frac{1}{2k_0} k_x^2 - A' \pm |\kappa| \tag{17}$$

$$\mathcal{E}_3(\vec{k}) = \mp \frac{\kappa}{|\kappa|} \mathcal{E}_4^*(\vec{k}) \tag{18}$$

The *solution* of the *fields* $\mathcal{E}_3(\vec{r}) = \mathcal{E}_3(\vec{k}) r^{i\vec{k}.\vec{r}}$ *and* $\mathcal{E}_4(\vec{r}) = \mathcal{E}_4(\vec{k})\ e^{i\vec{k}.\vec{r}}$ which satisfies $\mathcal{E}_4 = 0$ at $z = L$ is then found to be given by [8] (in the entrance plane $z = 0$)

$$\mathcal{E}_4^*(\vec{k}) = \frac{i\kappa^*}{|\kappa|} \tan\ (|\kappa| L)\, \mathcal{E}_3(\vec{k}) \tag{19}$$

Observe the following properties of equation (19):

(a) as $|\kappa|L \to \pi/2, \mathcal{E}_4^* \to \infty$. The phase conjugate mirror may "oscillate" without introduction of an external feed-back,

(b) the reflectivity $i\kappa^*/|\kappa| \tan(|\kappa|L)$ is independent of $\vec{k}$, that is, it is the same for all Fourier-components of the field.

From property (b) we arrive at the property expressed by equation (2), *outside* the non-linear medium, provided the external medium is loss-less and at most weakly scattering in the Born approximation[9]. In addition, equation (19) shows that

(c) the phase-conjugate reflectivity has the phase of the product $(\mathcal{E}_1\mathcal{E}_2)^*$, and finally

(d) it is independent of the non-linear phase modulation term in A'.

All four of these properties are modified when the spatial variation of the pump fields is taken into account.

## 4. PHASE CONJUGATION WITH FINITE PUMP BEAMS

We now consider the more realistic situation in which the pump beams have a non-uniform spatial dependence, resulting in less than perfect fidelity of the phase conjugate signal beam. The condition (10) is assumed to apply, as this condition is at least in principle always achievable. Equations (16) are written in the form

$$\left[ i\frac{\partial}{\partial z} + (2k_0)^{-1}\nabla_t^2 - A'(x,y,z) \pm \kappa(x,y,z) \right] \mathcal{E}^{\pm}(x,y,z) = 0 \qquad (20)$$

where $\mathcal{E}^{\pm}(x,y,z) = \mathcal{E}_3(x,y,z) \pm \mathcal{E}_4^*(x,y,z)$.

The solution of equation (20) which satisfies the boundary condition $\mathcal{E}_4(x,y,L) = 0$ is

$$\mathcal{E}_4^*(x,y,0) = \hat{R}\,\mathcal{E}_3(x,y,0) \qquad (21)$$

where $\hat{R}$ is the anti-hermitian reflection operator, which is defined in terms of a unitary operator $U(x,y,L)$ by a Cayley transform[10]

$$\hat{R} = \frac{1 - \hat{U}(x,y,L)}{1 + \hat{U}(x,y,L)} \tag{22}$$

Finally, the operator $\hat{U}(x,y,z)$ is defined as the product

$$\hat{U}(x,y,z) = \hat{U}^{-}(x,y,z)^{\dagger} \quad \hat{U}^{+}(x,y,z) \tag{23}$$

where $\hat{U}^{-}$ and $\hat{U}^{+}$ satisfy equations (20), and $\hat{U}^{-}(x,y,0) = \hat{U}^{+}(x,y,0) = 1$.

From the unitary property of $\hat{U}$ a useful distortion criterion can be found. Let the functions $f^{-}(x,y,z)$ and $f^{+}(x,y,z)$ be solutions of equations (20) which satisfy

$$f^{-}(x,y,0) = f^{+}(x,y,0) \equiv \mathcal{E}_{i}(x,y) \ . \tag{24}$$

Now define a parameter $\varepsilon$ according to

$$\varepsilon \equiv 1 - \frac{|\int f^{-}(x,y,L)^{*} \ f^{+}(x,y,L) \ dxdy|^{2}}{\int |\mathcal{E}_{i}(x,y)|^{2} \ dxdy} \tag{25}$$

then, in the case that $\varepsilon \ll 1$ (which applies to collinear or near collinear beam geometries), the quantity

$$\delta = \frac{1}{2}(\tan \kappa_0 L + \cot \kappa_0 L) \ \varepsilon = \ \varepsilon \ \csc 2\kappa_0 L \tag{26}$$

represents the fraction of energy in the signal field which was coupled to other modes than the probe field distribution. The constant $\kappa_0$ was defined by writing $\kappa(x,y,z) = \kappa_0 + \Delta\kappa(x,y,z)$. The consequence is that the parameter $\varepsilon$ can be regarded as a convenient definition of distortion. We have compared numerical results based on the complete field, given in equation (21) with the values of $\varepsilon$ and $\delta$, showing excellent correspondence.

An explicit expression for the signal field is obtained from equations (20) by treating $\nabla_t^2$ as a perturbation. For simplicity we assume $\kappa(x,y)$ to be independent of $z$, then to first order[10]

$$\mathcal{E}_s(x,y,0)^* = i \tan \kappa(x,y)L \ \mathcal{E}_p(x,y,0) +$$

$$\frac{[L^2\nabla_t^2 \kappa(x,y) - 2iL^3|\vec{\nabla}_t \kappa(x,y)|^2]\mathcal{E}_p(x,y,0) + 2L^2\vec{\nabla}_t\kappa.\vec{\nabla}_t\mathcal{E}_p(x,y,0)}{4k \cos^2\kappa(x,y)L} \qquad (27)$$

Observe that the right- hand side of (27) contains a non-local term proportional to $\vec{\nabla}_t\kappa.\ \vec{\nabla}_t\mathcal{E}_p$. This term possesses two properties which we exploit to obtain this paper's principal result. First, it is very sensitive to the phase of $\mathcal{E}_p$. The other property is that the leading part of this expression is in phase with the principal (first) term in equation (27). It should be possible, therefore, to shape the phase of the probe field in such a way, by appropriately filtering of the beam before entering the PCM, that the distortion caused by the spatial dependente of $\kappa(x,y)$ is cancelled by the non local cross-gradient term[12]. A simple calculation shows that if $\kappa(x,y)$ falls-off quadratically near the axis, a negative radius of curvature of the incident wave front, satisfying

$$R = - L \qquad (28)$$

performs the desired function. This focussing of the probe beam is induced by placing a converging lens in front of the PCM. Because of the phase conjugation property of the PCM, the signal wave is automatically corrected for the added wavefront curvature when it passes through the lens.

Obviously the adaptative technique we have described can not work for an arbitrary incident wavefront. It is in fact only useful if it works reasonably well for a certain class of wavefronts which vary from one another within certain predescribed limits that are determined by the nature of the fields which the PCM is likely to receive, and the function it is designed to perforrn. It is therefore necessary to know the efficiency of the adapting lens when certain characteristics of the probe field are varied, the properties of the PCM and the adapting lens being maintained constant.

Because equation (27) does not accurately describe focussed fields, another method must be applied. If the pump beams overlap strongly with the probe, we are allowed to approximate

$$\kappa(x,y,z) = \kappa_0\{1 - r_p^{-2}(x^2 \cos^2\Theta + z^2 \sin^2\Theta - xz \sin 2\Theta + y^2) \tag{29}$$

Equations (20) have solutions in terms of Herrnite Gauss functions, and an analytic expression for $\varepsilon$ is available. These solutions have the form

$$f^{\pm}(x,y,z) = f_n^{\pm}(x,z)\ f_m^{\pm}(y,z)$$

where

$$f_n^{\pm}(x,z) = (2^n n!\ \Pi\ \omega^{\pm}(z))^{-1/2}\ e^{-i(n+\frac{1}{2})\phi^{\pm}(z)}$$

$$\times \exp\{ik[F^{\pm}(z) + \frac{1}{2}\mu^{\pm}(z)(x-x_0^{\pm}(z))^2 + \sigma^{\pm}(z)x]\}\ H_n\left(\frac{x-x_0(z)}{\omega^{\pm}(z)}\right) \tag{30}$$

and a similar definition of $f_m^{\pm}(y,z)$. The parameters appearing in (30) are: complex wavefront curvature

$$\mu^{\pm}(z) = \frac{i}{k\omega^{\pm}(z)} + \frac{1}{R^{\pm}(z)} = \frac{\mu_0\ U^{\pm}(z) \pm \alpha'^2}{\mu_0 + U^{\pm}(z)}, \tag{31}$$

the geometric optics ray position $x_0^{\pm}(z) = b^{\pm}(z)/\ V(z)$, the geometric optics ray direction $dx_0^{\pm}(z)/dz = \sigma^{\pm}(z)$. The quantities in the phase we. termed the homogeneous axial phase

$$F^{\pm}(z) = \int_0^z \pm\ (\kappa_0 + \frac{1}{2}\alpha^2 \sin^2\Theta\ z^2 - \frac{1}{2}\alpha^{\pm}(z)^2)dz - \frac{1}{2}kU^{\pm}(z)\ x_0^{\pm}(z)^2 \tag{32}$$

and the diçtributed axial phase

$$\phi^{\pm}(z) = \tan^{-1} \beta_0^2/(U^{\pm}(z) + 1/R(0)) \qquad (33)$$

where $\beta_0 = 1/\omega(0)$ and $\mu_0 = \mu(0)$. Finally, the auxiliary parameters are defined

$$U^{-}(z) = a' \cot \alpha' z, \quad U^{+}(z) = a' \coth \alpha' z$$

$$V^{-}(z) = \alpha'/\sin \alpha' z, \quad V^{+}(z) = \alpha'/\sinh \alpha' z$$

$$a^{\pm}(z) = \tan\Theta \; (1 - zU^{\pm}(z)), \quad b^{\pm}(z) = \tan\Theta \; (zV^{\pm}(z) - 1) \qquad (34)$$

where $\alpha' = a \cos\Theta = (2\kappa_0/kr_p^2)^{1/2} \cos \Theta$.

This solution was applied to equation (25), and alsoa slightly qeneralized version which included lateral offset. The value of ε was then calculated for a variety of different conditions which are too many to describe completely. Only the effect of focussing will be mentioned here. If is found that distortion, as measured by ε, can be reduced typically by as much as 90% under likely experimental conditions, for a considerable range of values of probe beam diameter, lateral and angular off-set, etc. The compensation by focussing also turned out to be remarkably insensitive to the exact location of the focal plane. This is explained by the fact that a delicate balance exists between spot size matching of the $f^{+}$ and $f^{-}$ functions, and wavefront curvature matching. Near the focal planes $R^{+}(z) \cong R^{-}(z)$ the wavefront curvatures are well rnatched, but the spot match is poor due to the small spot sizes. Farther away from the focal planes the reverse situations exists. The result is the existence ofawide range of values of the ratio $R/L$, as well as of other parameters, where considerate distortion compensation occur.

The effect of beam offset is as follows. When probe and pump beams are well centered, the reduction of ε is most effective with decreasing probe beam radius. In the presence of lateral or angular off set a minimum probe beam radius exists below which focussing yields diminishing returns. It is concluded therefore that this technique does not have much value for complex probe-phase structures, and may even be counter productive in those cases. On the other hand, for simple beams it appears to be very valuable.

Foreseeable applications are likely to be in tht areas of atmospheric communication by laser, the transmission of energy to satellites and the nuclear fusion with lasers - all of which are likely to involve electro-magnetic fields with a small number of degrees of freedom in transverse spatial directions.

## 5. THE EFFECTS OF LOSSES ON PHASE CONJUGATE REFLECTIVITY

Most high conversion PCM's are build from resonant absorbing materials. For such media, therefore, the theory so far presented isnot very satisfactory. Unfortunately, the inclusion of loss effects presents a most troublesome problem, in particular when non-linear phase variations of the pump fields are present. A theory which includes these effects will be published elsewhere[13], for the moment only a simple result applicable when only linear losses are present will be given. In the case of collinear or near-collinear beam geometries, with normally incident pump beams, and under the conditions that the structure of the probe is relatlvely simple, the reflectivity can be written[10]

$$R(x,y) = \frac{i\kappa(x,y)\,\sin\frac{1}{2}\,S(x,y)}{\omega(x,y)\,\cos\frac{1}{2}\,S(x,y) + \alpha\sin\frac{1}{2}\,S(x,y)} \tag{35}$$

Here a is the linear loss constant, $\omega(x,y)^2 = \kappa(x,y)^2 - \alpha^2$ and $S(x,y)$ is the total phase mis-match at the cell-back of the functions $f^+$ and $f^-$, defined in section 4 (with $\kappa(x,y)$ substltuted by $\omega(x,y)$).

## 6. SUMMARY

This paper reviews some of the principal results we obtained in our work on phase conjugation optics. Addltional results, including numerical, may be found in our more complete paper ref. 10. In particular, the topic of four wave mixing at large angles, not discussed here, is treated there.

These calculations have applications to communications, high powered lasers and laser amplifiers, interferometry and optical computing. We are investigating laser cavities in which one mirror is a phase-conjugate mirror[13].

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## RESUMO


É estudada a fidelidade da onda conjugada em fase em mistura de quatro ondas degeneradas, com ondas de bomba cujas intensidades variam em posição dentro da região de interação. A análise inclue os efeitos de esgotamento da onda "sonda", difração, variação não linear da fase, focalização e finalmente os das perdas. Derivamos expressões para a refletividade do espelho conjugador de fase as quais são relativamente simples nos casos das geometrias dos feixes colinear e quasi colinear. Achamos que por meio de focalizar a onda sonda dentro do meio não linear, a fração de energia no feixe da onda conjugada em fase, que foi transferida para outros modos, pode ser reduzida tipicamente por uma ordem de magnitude.