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Completely Cornputable p-n Mass Difference

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Abstract Weinberg-'t Hooft mechanism for rendering mass differences finite, in conjuction with a new representation for these mass differences make it possíble to compute $A = Mp-Mn$. The value obtained is somewhat low but with the right sign.

Mass differences within isotopic multiplets, like $A = Mp - Mn$ for nucleons, are usually of the order of a times the mass. It is natural then to think that these mass differences should be calculable as an electromagnetic effect. But nowadays, we also believe that particles are systems composed of more elementary things (quarks) and in such a case, mass differences among the components could account for the whole or part of A. Is the order of magnitude of the mass differences then, just a mere accident?

In the framework of renormalizable unified gauge theories, 't Hooft[1] and Weinberg[2] proposed a mechanism whereby mass differences within a multiplet are calculable radiative effects. Weinberg showed that even semirealistic models for A could be constructed and went on to explain in detail how the mechanism works. Models of this type provide a justification for the idea that A should be computable.

For a detailed description of Weinberg-'t Hooft mechanism I refer the reader to the original papers as well as further elaborations[3,4]. Let me just mention that in the type of models that were studied by these authors, the gauge symmetry is broken sponeaneously in such a way as to leave all masses within a multiplet, equal in zero order. Mass differences are then calculable as radiative effects in second order in the gauge coupling. Unfortunately, elaborations of these ideas in the framework of a pure weak-electromagnetic theory (no strong interactions) have shown that, more likely than not, A would come out with the wrong sign[3,4].

The next thing to try would be an hybrid model in which Weinberg-'t Hooft mechanism is incorporated into the usual formalism for A (Cottingham formula[5,6], etc.). The problem with such a programme is the unknown substraction function that appears in Cottingham's expression when dispersion relations are written for the Compton amplitudes[6]. In order to overcome this difficulty I will derive for the mass shift, a new representation to which the substraction function does not contribute.

After a Cottingham rotation, the electromagnetic mass shift of a fermion can be written

$$\delta M = \frac{\alpha}{(2\pi)^3} \int \frac{d^4k_E}{Q^2}\, T(-Q^2, i\nu) \quad , \tag{1}$$

where $T(-Q^2, i\nu)$ results from a Cottingham rotation of the contracted Compton amplitude[7]

$$T(\vec{k}, k^0) = g^{\mu\nu}\, i \int d^4x\, e^{ikx} \langle p|T\{j_\mu(x), j_\nu(0)\}\,|p\rangle$$

$$= \frac{\pi}{\alpha} \{3k^2 t_1(|\vec{k}|, k^0) - (2\nu^2 + k^2) t_2(|\vec{k}|, k^0)\} \quad , \tag{2}$$

$Q = \sqrt{-k^2}$, $\nu = pk/M$ and dk_E is the four dimensional volume element in Euclidean k space.

Writing dispersion relations, once substracted for t_1 and unsubstracted for t_2 [8], we have

$$T(-Q^2, i\nu) = -\frac{3\pi Q^2}{\alpha}\, t_1(-Q^2, 0) + 2\left(1 + \frac{2\nu^2}{Q^2}\right) \int \frac{d\nu'\, \nu' W_2}{\nu'^2 + \nu^2}$$

$$+ 6\nu^2 \int \frac{d\nu'}{\nu'^2 + \nu^2} \left[\frac{W_1}{\nu'} - \frac{\nu' W}{Q^2}\right] \quad , \tag{3}$$

in terms of the usual electron-nucleon scattering structure functions W_1 and W_2. The stumbling blocks in the use of Cottingham's formalism for the calculation of Δ, are the logarithmic divergence of the integral over the deep inelastic region and the unknown function $t_1(-Q^2,0)$. The logarithmic divergence would disappear if the Weinberg-'t Hooft mechanism were operative. In the simplest version of the type of model where this mechanism is implemented, that of SU(2) x U(1) for example, the heavy neutral gauge boson (Z) contribution to δM combines with the photon contribution for a net change[3]

$$\frac{1}{Q^2} \to \frac{1}{Q^2} - \frac{1}{Q^2+m_Z^2} \tag{4}$$

Whenever that happens we will have, instead of Eq. (1),

$$\delta M = \frac{\alpha}{(2\pi)^3} \int \frac{d^4k_E}{Q^2(1+Q^2/m_Z^2)} \; T(-Q^2,i\nu) \tag{5}$$

In order to derive the new representation for δM let me start by defining

$$U(Q^2,\nu) \equiv \int_{Q^2}^{\infty} dQ'^2 \, \frac{T(-Q'^2,i\nu)}{1+Q'^2/m_Z^2} \quad , \tag{6}$$

the mass shift being a function of

$$\frac{T(-Q^2,i\nu)}{1+Q^2/m_Z^2} = - \frac{\partial U(Q^2,\nu)}{\partial Q^2} \tag{7}$$

The integral

$$I = \int \frac{d^4k_E}{Q^4} \, U(Q^2,\nu) \tag{8}$$

is invariant under a rescaling $k_i \to \lambda k_i$ of the momentum, implying that

$$0 = \lambda(\partial I/\partial\lambda) = \int \frac{d^4k_E}{Q^4} \left[2Q^2 \partial U/\partial Q^2 + \nu \, \partial U/\partial\nu \right] \tag{9}$$

From Eqs. (6)-(9) we see that the mass shift (5) can also be written as

$$\delta M = \frac{\alpha}{(2\pi)^3} \int \frac{d^4k_E}{Q^4} \nu^2 \int_{Q^2}^{m} \frac{dQ'^2}{(1+Q'^2/m_z^2)} \frac{\partial T(-Q'^2, i\nu)}{\partial(\nu^2)} , \quad (10)$$

to which the unknown substraction function $t_1(-Q^2,0)$ in Eq. (3) would not contribute It is simpleto seethat, afterthe angular integration, Eq. (10) can be rewritten in the form

$$\delta M = \frac{\alpha}{2\pi^2} \int_0^\infty \frac{dQ^2}{Q^4} \int_0^Q \sqrt{Q^2-\nu^2}\, \nu^2\, d\nu \int_{Q^2}^\infty \frac{dQ'^2}{(1+Q'^2/m_z^2)} \frac{\partial T(-Q'^2, i\nu)}{\partial(\nu^2)} , \quad (11)$$

formula on which the calculation of the proton-neutron mass difference will be based.

The off-shell Compton scattering can be divided intotwo parts: coherent and incoherent scattering of the system (quarks, etc.) that composes the nucleon. The coherent scattering can be well approximated by the Born term since the other resonant contributions are negligible in comparison[6]. The incoherent amplitude can be expressed in terms of the structure functions that are measured in deep-inelastic scattering experiments.

With the elastic structure functions[g]

$$W_1^{e\ell}(-Q^2,\nu) = (Q^2/4M^2) G_M^2(Q^2)\,\delta(\nu - Q^2/2M) , \quad (12)$$

$$W_2^{e\ell} = \frac{G_E^2(Q^2) + (Q^2/4M^2) G_M^2(Q^2)}{1 + Q^2/4M^2} \delta(\nu - Q^2/2M) , \quad (13)$$

it is easy to see that the elastic contribution to B is

$$T^{e\ell}(-Q^2, i\nu) = -\frac{3\pi Q^2}{\alpha} t_1^{e\ell}(-Q^2,0) + \frac{(Q^2+2\nu^2)(Q^2G_M^2+4M^2G_E^2) - 12\, M^2\nu^2(G_E^2-G_M^2)}{M\left[(Q^2/2M)^2 + \nu^2\right](Q^2+4M^2)} \quad (14)$$

This expression, replaced into Eq. (11), yields the Born contribution to δM given by

$$\delta M^{(1)} = - \frac{4\alpha M}{\pi^2} \int \frac{dQ^2}{Q^4} \int \sqrt{Q^2-\nu^2}\ \nu^2\ d\nu \int_{Q^2}^{\infty} \frac{dQ'^2 Q'^2 \left[2M^2 G_E^2(Q'^2) - Q'^2 G_M^2(Q'^2)\right]}{(Q'^4+4M^2\nu^2)^2\ (1+Q'^2/m_z^2)} \tag{15}$$

The form factors can be taken as

$$G_M(Q^2)/\mu = G_E/q = (1+Q^2/m_0^2)^{-2} \quad , \tag{16}$$

where q is the charge, μ the magnetic moment of the nucleon and $m^2 \approx 0.72$ GeV . Since G_E^2 and G_M^2 damp the integrand quite fast with Q'^2, we can supress the factor $(1+Q'^2/m_z^2)$ in Eq. (15) (Z has a large mass , $m_z = 90$ GeV, say).

With an integration by parts and appropriate changes of variables the integrations in Eq. (15) can be performed yielding[10]

$$\delta M^{(1)} = - \frac{\alpha M}{\pi} \left\{ q^2 \left[\left[\frac{1}{R} + 2\frac{\partial}{\partial R} + \frac{R}{\alpha}\frac{\partial^2}{\partial R^2} \right] \left(1 + \frac{R \tan^{-1}\sqrt{R-1}}{\sqrt{R-1}} \right) - \frac{1}{3R} \right. \right.$$

$$\left. \left. + \frac{\mu^2}{3R} \left[\frac{1}{R} - \left(\frac{\partial}{\partial R} + R\frac{\partial}{\partial R^2} - \frac{1}{R} \right) \left\{ 1 + \frac{R \tan^{-1} \sqrt{1-R}}{\sqrt{R-1}} \right] \right] \right\}, \tag{17}$$

in terms of $R = 4M^2/m_0^2 = 4.90$. After all that, the Born term contribution to the proton-neutron mass difference was computed as

$$\Delta^{(1)} = \delta M_p^{(1)} - \delta M_n^{(1)} = - 0.58 \text{ MeV} . \tag{18}$$

The contribution from the incoherent scattering to the mass difference is

$$\Delta^{(2)} = \frac{\alpha}{2\pi^2} \int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^4} \int_0^{Q} \sqrt{Q^2-\nu^2}\ \nu^2\ d\nu \int_0^{\infty} \frac{dQ'^2}{(1+Q'^2/m_z^2)} \frac{\partial}{\partial(\nu^2)} \left[T_p^{In} - T_n^{In} \right], \tag{19}$$

where the T^{In} are as in Eq. (3) with the structure functions restricted to the deep-inelastic region. The dominant part of $\Delta^{(2)}$, which will be seen to be small, is [10]

$$\Delta^{(2)} = \frac{2\alpha M}{\pi^2} \int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^4} \int_0^{Q} \sqrt{Q^2-\nu^2}\, \nu^2\, d\nu \int_{Q^2}^{\infty} \frac{dQ'^2 \left[5M_2(2,Q'^2)-6M_1(3,Q'^2)\right]}{Q'^4(1+Q'^2/m_Z^2)} , \tag{20}$$

where the $M_i(j,Q^2)$ are the Cornwall-Norton j moments of the non-singlet combinations $F_i^{e-p} - F_i^{e-n}$. Whenever the Callan-Gross[11] relation $2x\ F_1 = F_2$ [12] holds (as expected), Eq. (20) reduces, after the trivial ν integration, to

$$\Delta^{(2)} = \frac{\alpha M}{4\pi} \int_{Q_0^2}^{\infty} dQ^2 \int_{Q^2}^{\infty} \frac{dQ'^2 M_2(2,Q'^2)}{Q'^4(1+Q'^2\ /m_Z^2)} , \tag{21}$$

in terms of[13]

$$M_2(2,Q^2) \approx \frac{M_2(2,Q_0^2)\left[\ln(Q_0^2/\Lambda^2)\right]^{d_2}}{\left[\ln(Q^2/\Lambda^2)\right]^{d_2}} \tag{22}$$

To simplify things I took Q_0 = 2 GeV, A = 0.2 GeV and, for six flavours, $d_2 \approx 1/2$. From an integration of $F_2^{ep} - F_2^{en}$ given in Ref. (14) I obtained $M_2(2,4$ GeV) = 0.018. Then, a numerical integration of Eq. (21) gave $\Delta^{(2)}$ = 0.04 MeV. The complete proton-neutron mass difference is thus,

$$A = \Delta^{(1)} + \Delta^{(2)} = -\ 0.54\ \text{MeV} , \tag{23}$$

almost within a factor two of the experimental A_{exp} = - 1.29 MeV.

We have here a framework in which A can be completely calculated. $\Delta^{(1)}$ was obtained from a substracted Born term but, as it can be shown[15], an unsubstracted Born term[16] leads exactly to the same result. So, some of the ambiguities that plague the usual formalism based on Cottingham formula do not appear here.

The Weinberg-'t Hooft mechanism takes care of the high momentum divergence while the use of Eq. (11) makes unnecessary the knowledge of the substraction function $t_1(-Q^2,0)$ (and contributions from fixed poles if they exist). Within this framework we can now search for ways of improving the calculation and obtaining a better value than (23).

NOTES AND REFERENCES

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RESUMO

A conjunção do mecanismo de Weinberg-'t Hooft para tornar finitas diferenças de massa e de uma nova representação para essas diferenças de massa, torna possível o cálculo de $A = M_p - M_n$. O valor obtido é um pouco pequeno, mas o sinal é o correto.