

Decay of Heavy Mesons Into Possible Spin 3/2 Leptons

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Abstract The recent interest in the study of possible spin 3/2 leptons motivated our calculation of the decay rate of heavy mesons into these leptons. The mesons may be u particles ($b\bar{b}$ systems) or any other system of the $q\bar{q}$ (quark-antiquark) kind with mass larger than those of the u particles, for example, $t\bar{t}$ bound states (not yet detected). We calculate the decay rate of these mesons as a function of the mass of the spin 3/2 leptons. The results obtained are compared with those of the spin 1/2 leptons.

INTRODUCTION

Until now there is no experimental observation of the heavy spin 3/2 leptons. The single possible candidate to this category would be the lepton τ ¹. Nevertheless it is well established that the lepton τ has spin 1/2, and together with the neutrino ν_τ constitute the third generation of leptons. In spite of the non-existence of experimental evidence for the spin 3/2 leptons, there are sufficient motivations to study spin 3/2 fields. An example is the theory of supergravitation, where massless particles of spin 3/2 play a fundamental role. It is known that at presently available energies the leptons behave like structureless particles. Notwithstanding, we speculate that at higher energies the leptons would reveal indications of a possible structure², it would be natural to seek for spin 3/2 excitations. Weinberg and Witten argue for the absence of higher spin light particles³. Some theoretical difficulties arise when we try to describe the electromagnetic interaction with an elementary particle of spin 3/2. There is an inconsistency in the theory of local interaction of a charged spin 3/2 field with an external electromagnetic field⁴, and the theory is not renormalizable. The electromagnetic interaction of spin 3/2 elementary particles violates unitarity, and the cross sections grows wi-

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thout limit with increasing energy⁵. We overcome this problem by the introduction of electromagnetic form factors for the spin 3/2 leptons. The same trick is used in the calculations of the decay rate of heavy mesons like particles ψ , ψ' , T , T' , etc. into possible spin 3/2 leptons. Our results are compared with similar ones for the decay rate of these particles into spin 1/2 leptons⁶.

SPIN 3/2 FIELDS

The spin 3/2 fields satisfy the Rarita-Schwinger equation⁷ and

$$(\gamma \cdot p - m)u_\alpha(p) = 0 \quad (1)$$

and the constraints

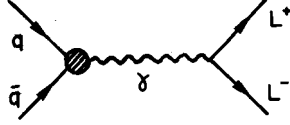
$$\gamma^a u_\alpha(p) = 0 \quad \text{and} \quad p^a u_\alpha(p) = 0 \quad (2)$$

The projectors on the positive and negative energy states are:

$$\begin{aligned} P_\pm^{\mu\nu} &= \pm \frac{1}{2m} (\gamma \cdot p \pm m) \times \\ &\times \left\{ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3} \frac{p^\mu p^\nu}{m^2} \pm \right. \\ &\left. \pm \frac{1}{3m} (\gamma^\nu p^\mu - \gamma^\mu p^\nu) \right\} \quad (3) \end{aligned}$$

DECAY CALCULATIONS

The heavy mesons may be considered as heavy quark- antiquark bound states, e.g. the ψ particles are $c\bar{c}$ (charm-anticharm) bound states and the T particles are $b\bar{b}$ (bottom-antibottom) bound states. The bound states are calculated⁶ using a phenomenological potential that describe confinement at large distances as well as asymptotic freedom at small distances. The decay of these heavy mesons into spin 3/2 leptons may then be described in a first approximation by a one photon transition as shown in the diagram



The most general form of the conserved electromagnetic current at the leptons vertex is

$$\begin{aligned}
 j^\mu(p', q') = & \bar{u}_\alpha(p') \left[f_1(k^2) \gamma^\mu g^{\alpha\beta} + \frac{i}{2m} f_2(k^2) (g^{\alpha\mu} k^\beta - g^{\mu\beta} k^\alpha) + \right. \\
 & \left. + \frac{i}{2m} f_3(k^2) (p^\mu - m\gamma^\mu) g^{\alpha\beta} + \frac{i}{2m^3} f_4(k^2) k^\alpha (p^\mu - m\gamma^\mu) k^\beta \right] v_\beta(q')
 \end{aligned}
 \tag{4}$$

where f_1, f_2, f_3 and f_4 are the form factors of the spin 3/2 leptons, $u_\alpha(p')$ and $v_\beta(q')$ are positive and negative energy Rarita-Schwinger spinors, $k = p' + q'$ and $P = p' - q'$. As a first approximation we assume a pointlike minimal electromagnetic coupling in the leptonic vertex which corresponds the choice $f_2 = f_3 = f_4 = 0$ and $f_1 = 1$. In this approximation the total cross section for the process

$$q\bar{q} \rightarrow \gamma \rightarrow L^+L^-$$

is

$$\begin{aligned}
 \sigma_T = & \frac{2e_q^2 e^2 |\vec{q}'|}{27 \pi s |\vec{p}|} \left\{ \frac{36m^2 M^2}{s^2} + 2(9m^2 - M^2) \frac{1}{s} - \left(1 + \frac{2M^2}{m^2} - \right. \right. \\
 & \left. \left. - \frac{1}{m^2} \left(1 - \frac{M^2}{m^2} \right) s + \frac{s^2}{2m^4} \right\}
 \end{aligned}
 \tag{5}$$

where e_q is the quark charge, M and m are the quark and lepton masses respectively, and

$$s = (p+q)^2 = (p' + q')^2 \tag{6}$$

In the center of masses system, the non-relativistic limit

($p \ll M$) gives

$$s = 4M^2, \quad |\vec{p}| = |\vec{q}| = \frac{M}{2} v_{\text{rel}}. \quad (7)$$

The transition probability of a weakly bound $q\bar{q}$ system, in the triplet spin state, angular momentum $\ell = 0$ and $J^{PC} = 1^{--}$, into spin 3/2 leptons L^+L^- , is given by $\sigma_T \times v_{\text{rel}}$. with the following modifications:

1. multiply by $|\psi(0)|^2$ (ψ is the bound state wave function)
2. multiply by 16/3 (from leptonic spin states)
3. multiply by $\sqrt{3}$ for each colour (three colours) and $1/\sqrt{3}$ from colour wave function .

With this recipe, the decay rate of the above mentioned $q\bar{q}$ bound system is

$$\Gamma(1^{--} \rightarrow L^+L^-) = \frac{64}{27\pi} \frac{e^2 e_q^2}{(4M^2)} \sqrt{1-\gamma^2} |\psi(0)|^2 \times \left\{ \frac{27}{4} \gamma^2 - \frac{3}{2} - 6 \gamma^{-2} + 12 \gamma^{-4} \right\} \quad (8)$$

where $\gamma = \frac{m}{M}$.

The decay rate of the same heavy meson into spin 1/2 leptons (e^+e^- or $\mu^+\mu^-$) is

$$\Gamma(1^{--} \rightarrow e^+e^-) \approx \frac{e^2 e_q^2}{\pi(4M^2)} |\psi(0)|^2 \quad (9)$$

The fraction of the decays defined by

$$R = \frac{\Gamma(1^{--} \rightarrow L^+L^-)}{\Gamma(1^{--} \rightarrow e^+e^-)} \quad (10)$$

in the same approximation of the above calculations is given by

$$R = 16 \sqrt{1-\gamma^2} \left\{ \gamma^2 - \frac{2}{9} - \frac{8}{9} \gamma^{-2} + \frac{16}{9} \gamma^{-4} \right\} \quad (11)$$

Clearly the R value increases without limit with increasing $\gamma^{-1} = \frac{M}{m}$, which is a consequence of the unitarity violation. This problem may be circumvented by introducing a neutral vector meson of mass m_x , which corresponds to multiplying the scattering amplitude of $q\bar{q} \rightarrow \gamma \rightarrow L^+L^-$ by a form factor for the leptonic vertex⁵

$$f(k^2) = \frac{m_x^2}{(-k^2 + m_x^2)} \quad (12)$$

with this correction, in the nonrelativistic limit, the fraction of the decays R becomes:

$$R \approx \frac{16m_x^4}{(4M^2 - m_x^2)^2} \sqrt{1 - \gamma^2} \left\{ \gamma^2 - \frac{2}{9} - \frac{8}{9} \gamma^{-2} + \frac{16}{9} \gamma^{-4} \right\} \quad (13)$$

with $m_x < 2M$.

Analyzing the behaviour of R at the limiting values of γ we have:

$$R \rightarrow 0 \quad (\gamma \rightarrow 1)$$

and

$$R \approx \frac{16}{9} \cdot \frac{1}{\left(1 - \frac{m_x^2}{4M^2}\right)^2} \left(\frac{m_x}{m}\right)^4 \quad (\gamma \ll 1)$$

But, to be coherent with the experimental facts, $R < 1$, which is satisfied if $m_x \ll M$. In that case

$$R \approx \frac{16}{9} \left(\frac{m_x}{m}\right)^4,$$

from which we get

$$m_x < \frac{\sqrt{3}}{2} m.$$

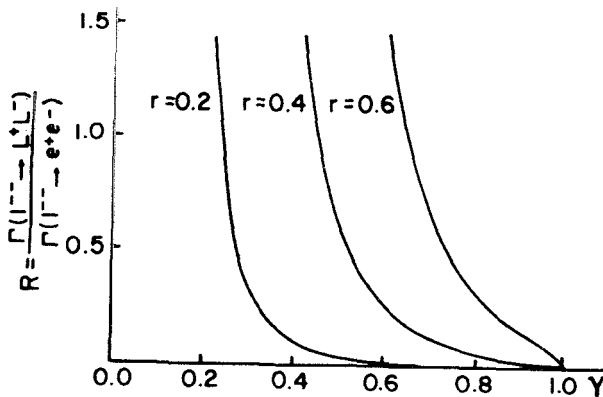
CONCLUSIONS

The fraction of the decays of a mesonic resonance into spin 3/2 or into spin 1/2 leptons (electrons or muons) is essentially a relation between the masses of the resonance (or of the quarks), of the spin 3/2 leptons and of the vectorial meson.

The minimal coupling hypothesis with pointlike interaction gives rise to a fast increase of the above mentioned fraction of the decays, for small values of the ratio of spin 3/2 lepton mass and the resonance mass. This is a consequence of the unitarity violation introduced by this type of coupling.

The introduction of a vector meson which dominates the electromagnetic leptonic form factor attenuates that inconvenient behaviour of the decay rate into spin 3/2 leptons, and the behaviour for small values of the masses of these leptons depends only on the fraction of the masses of the vectorial meson and of the lepton.

Looking at the Figure we verify that for values of the decay rate into 3/2 leptons, compatible with the experimental fact that these leptons have not yet been observed, the vectorial meson mass must be smaller than that of the lepton. We observe also that for a given value of the lepton mass the best value of R (the fraction of the decays into spin 3/2 and spin 1/2 leptons) corresponds to the smallest mass of the vectorial meson.



The fraction R of the decays of a mesonic resonance into spin 3/2 and into spin 1/2 leptons is plotted against $\gamma = m/M$, for several values of $r = m_x/M$, the fraction of masses of the vector meson and the quark.

We conclude that the small probability of decay into spin $3/2$ leptons may be a consequence of the large mass of the spin $3/2$ lepton and of the small mass of the vectorial meson.

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RESUMO

Devido ao recente interesse no estudo de possíveis leptons de spin $3/2$, calculamos a taxa de decaimento, nestes leptons, de mesons pesados tais como: partícula T (sistema $b\bar{b}$) ou qualquer outro sistema do tipo $q\bar{q}$ (quark-antiquark) de massa maior do que as das partículas ϵ e τ , como por exemplo, estados ligados $\tau\bar{\tau}$ (ainda não detectados). Fazemos uma estimativa da taxa de decaimento desses mesons em função das massas dos leptons de spin $3/2$. Confrontamos os nossos resultados com os obtidos para os leptons de spin $1/2$.