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Large Missing-Mass Clusters in Hydrodynamical Approach

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The previously proposed hydrodynamical description of the large missing-mass systems in hadron-hadron collisions is here applied to analysing recent large-mass data obtained at ISR. The results are still consistent with the data, indicating the usefulness of the large-cluster concept.

Aplica-se, para a análise de dados recentes obtidos no ISR, a descrição hidrodinâmica proposta anteriormente dos sistemas de grandes massas marcantes produzidos em colisões hadrônicas. Os resultados são ainda consistentes com os dados, indicando a utilidade do conceito de "clusters" grandes.

1. INTRODUCTION

Although the multiple production of soft particles constitutes the bulk of all high-energy hadronic collisions, our understanding of this phenomenon is still far from satisfactory. Among the several features of these events, one of the most well established is the appearance of the so called leading-particle effect, by which we understand a tendency shown by the incident particles to survive during the collision, emerging afterward conserving not only their quantum numbers, but also a large fraction of their energy and momentum.

"Which is its origin?" is a question which has yet no conclusive answer. Diffractive dissociation[1] is a possible mechanism. In this case, the other particles which appear in a collision would form a small cluster going to the oposite direction. Another possibility, of which the former is probably a particular case, is to invoke the multiparticle unitarity constraint[2]. In the present paper, we leave this fundamental question aside for a moment and go on just accepting the phenomenon as established.

The next question would be to know how many leading particles there are on average, whether one or two. In other words, once a proton (for definiteness let us assume pp collision) of $x \sim 1$ heaving been detected, we would like to know which is the probability of finding a second proton with $x \approx -1$? In annalysing data, often data leading particles are admitted, which would appear almost symmetrically with respect to the c.m. system. We think this is just a working assumption and by no means an established fact. It is clear that to justify the term leading particle, proton must appear really leading whole the system of secondary particles, possibly isolated from these, that is with its c.m. rapidity larger than the corresponding one of every other particle belonging to the same event, showing perhaps a large gap. Taking such a restriction into account, we find that the existent data[3,4] indicate the predominance of one-leading-particle events rather than two-leading particle ones. For instance, in Ref.3, the inclusive charged particle distribution has been measured in correlation with a leading proton, for several missing-mass intervals. The results show a smooth distribution in pseudo-rapidity η, even for the largest missing mass considered ($M^2 \simeq 260$ GeV^2 for $s = 934$ GeV^2), with a strong concentration in the backward hemisphere or, more precisely, almost symmetrical around $\eta = -\frac{1}{2}\ \ln \frac{s}{M^2}$. Although there exists some indication of two leading-particle events in the same data (which appear as a small backward peak), the above features strongly suggest that the bulk of the detected events are such that the backward proton, if any, is among the other particles and not leading them. In Ref. 4, a more specific measurement has been carried out, in which two leading protons are effectively detected. In terms of correlation factor $R \equiv \sigma \frac{d\sigma}{d\vec{p}_1 d\vec{p}_2} \Big/ \left[\frac{d\sigma}{d\vec{p}_1}\frac{d\sigma}{d\vec{p}_2}\right]$ the result is $R = 1.5 \pm 0.2$ in the range $|x_1|, |x_2| > 0.8$, $|t_1 + t_2| \lesssim 1.3$ GeV^2 and $M \equiv \sqrt{s(1-|x_1|)(1-|x_2|)} > 2$ GeV. It means that if one proton is detected as the forward leading particle, the probability of findinf a backward proton in the specified range is once and half larger than the corresponding one for umbiased events. In terms of cross section, however, it corresponds to a small fraction ($\lesssim$ 1 mb) of all the events, according to the same work. Thus, one arrives at the conclusion that any two-leading-proton events constitute a negligible fraction of the total number of events. We prefer, thus, not to separate the second proton from the remaining particles, and treat whole the system but the forward leading proton as forming a large cluster. By the way, this picture is also favored by cosmic-ray observation of energy spectrum and average lateral spread of gamma-ray families[5].

Once accepted this point of view, one would like to know what this large cluster looks like, or in other words how to describe it. In an earlier work[5] (which will be called I throughout this paper), we have proposed to treat such a system within the framework of Landau's hydrodynamical model[7], which would somehow take the complex interaction among the secondaries into account. The main difference of our treatrnent from the usual version of hydrodynamical approach is the explicit separation of one leading particle, the emergence of which probably depends on the quantum nature of the process and we consider it outside the applicability of a statistical description only. By assuming the conventional equation of state $p = \varepsilon/3$, we have compared in I the resulting prediction of the average multiplicity, the multiplicity distribution and the inclusive pseudo-rapidity distribution for several values of mass M, with the existing data and shown that all of them could consistently be interpreted in this way.

The main purpose of the present paper is to verify whether this model is still compatible with the more recent momentum-distribution data[8] obtained at ISR. We have, however, improved the previous treatment in two respects. First, instead of Milekhin's formula[9] for the invariant -momentum distribution of secondaries, we adopted the one proposed by Cooper and Frye[10], which correctly accounts for the energy-momentum conservation. In the second place, we partially took the transverse expansion into account, which has been completely neglected in I. As will become clear later, this second correction comes to be necessary when one applies Cooper and Frye's formula.

The plan of presentation is the following. In the next section, we will define the model and explain how the parameters are chosen (except for the one accounting for the transverse expansion mentioned in the last paragraph, which will be fixed in Sec.3. In Sec. 3, we will explain how to obtain the longitudinal momentum distributions and study both the effect of using the energy-momentum-conserving distribution as well as the one due to the transverse expansion. Sec. 4 will be devoted to comparisons with the existing data. Finally, the conclusions are drawn in Sec. 5, together with some further remarks.

2. THE MODEL AND THE CHOICE OF PARAMETERS

1) It is assumed that during the collision between two hadrons, a fireball is formed around one of the incident particles, emerging the other as a leading particle. The idea is very old[11] and it was in fashion in the early seventies[12], but it has not been exploited adequately, especially with regard to the dynamics of the fireball. There has been a proposal of employing statistical methods[13,14] for treating the missing--mass clusters, but the validity in directly applying it for large- mass fireballs as we intend to is doubtful.

2) Due to the short range natura of the strong interaction and the finiteness of the incident hadrons, which moreover appear Lorentz contracted at high energy, the fireball would probably be flattened at the moment of its creation, occupying a volume which is much smaller than the natural dimension of hadrons. Then, it should suffer an expansion before the final particles might appear. We apply Landau's hydrodynamical model for describing this process, which would take all the complex interaction of this system into account, generating the final particles. The natural frame of reference would be the center-of-mass system of the fireball, where we assume it is at rest initially.

3) In the lack of a better knowledge of the constituents of the fireball (perhaps a quark-gluon gas) and their dynamics, the thermodynamical properties of the gas are assumed to obey the equation of state

$$p = c_0 \, \varepsilon \quad , \tag{1}$$

with $c_0 = 1/\sqrt{3}$ as in the original version. Although we do not havea more fundamental justification for such a choice, it is worth while observing that this choice univocally fix the mass dependence of the average multiplicity[6]

$$\bar{n}(M) \propto M^{\frac{1-c_0^2}{1+c_0^2}} , \tag{2}$$

and the existing data are consistent with the above chosen sound velocity $c_0 = 1/\sqrt{3}$, in all the interval $10 \leq M^2 \leq 2 \times 10^3$ GeV^2 (see Fig. 1). We prefer to accept this fact as an evidence in favour of our choice of c_0 and not, for instance, take it as an adjustable parameter. The proportionality constant in *(2)* has been determined, by fixing, as in I, the radius of the initial disc to be $R = 1.64 m_\pi^{-1}$.

4) As the fluid expands, the temperature in each point decreases down to a certain critica1 value T_d, when the secondary particles emerge as free. Although the estimated value[7] is $T_d \simeq m_\pi$, the final results for the several physical quantities are known to be little sensitive to its exact value. In I, we have fixed $T_d = m_\pi$ and could verify that the multiplicity distribution is correctly reproduced as function of the mass M. Though a slightly larger value of T_d seems to be more satisfactory, we prefer to fix it at this value, considering the shortness of data.

5) The expansion of the fireball as mentioned in 2 is governed by the relativistic Euler equation

$$\partial^\mu T_{\nu\mu} = 0 , \tag{3}$$

where $T_{\nu\mu}$ is the energy-momentum tensor. Usually, the exact one- dimensional solution[15] of this equation is applied, which alreadygive, as shown in I, rather satisfactory agreements. The inclusion of the transverse expansion is quite complicated and we still have no satisfactory solution. However, as far as the bulk of the secondary soft particles are concerned, an important effect is expected, which is an additional colling, implying a smaller longitudinal expansion before the dissociation temperature T_d is reached. The same effect follows if one accepts the idea of early evaporation which has been introduced by Feinberg et

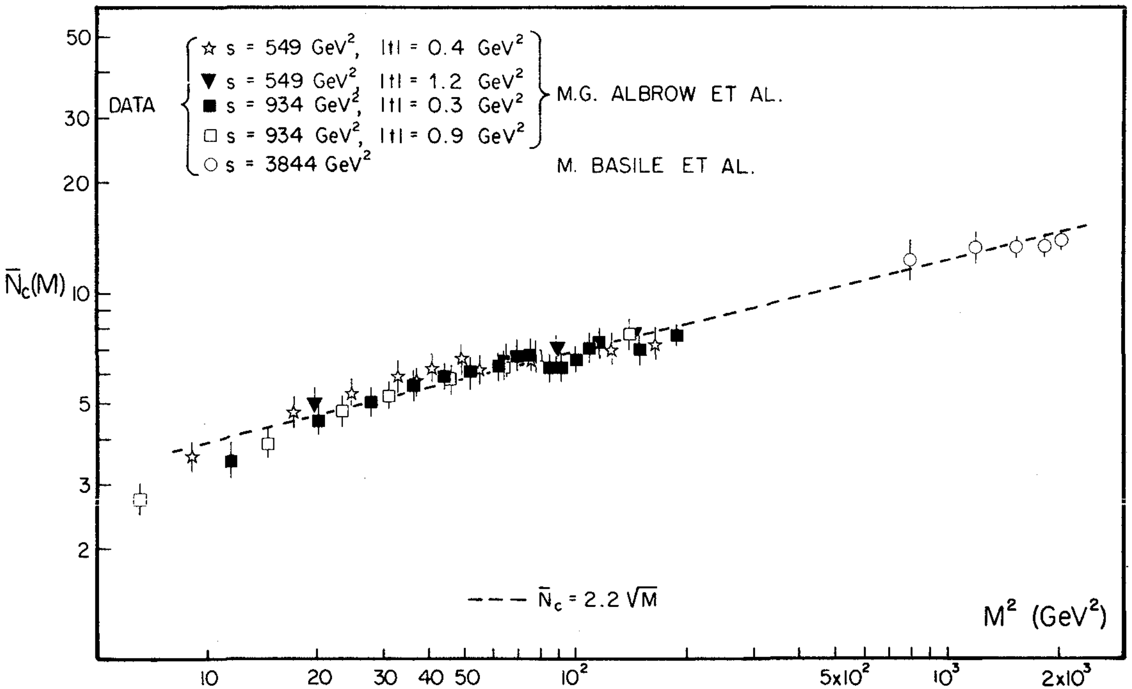

Fig.1 - Average multiplicity given by eq. (2) as a function of M^2. The data atributed to Ref. 8 have been estimated from their $\frac{1}{N_{ev}} \frac{dN_{tracks}}{dx_R^*}$, although an independent multiplicity determination has been published elsewhere[20]. We found a serious discrepancy between the two results, even if we abstracted the forward-backward asymmetry as implied by our model (fig. 4).

al.[16] in connection to the energy dependence of the inclusive high-transverse-momentum distribution. In the present work, we simulate this effect by modifying the fluid rapidity distribution as

$$\frac{dN}{d\alpha} = A\,\frac{dN}{d\alpha'} \quad , \qquad \alpha' = b\alpha \qquad (b \geq 1) \quad , \tag{4}$$

where A is a normalization constant which is fixed by the average multiplicity, b is a phenomenological parameter which shorten Khalatnikov's exact one-dimensional rapidity distribution $dN/d\alpha'$ (which corresponds to $b = 1$).

6) Besides the fluid rapidity distribution discussed above, another ingredient which determines the inclusive particle distribution is the thermal motion, given by Bose (or Fermi) distribution, which in the local rest frame reads

$$G(E_0,T) = \frac{g}{(2\pi)^3}\,\frac{1}{\exp(E_0/T) \mp 1} \tag{5}$$

Here, E_0 is the particle energy in the local frame and g is the statistical factor. Cooper and Frye have shown[10] that the usually accepted Mikelhin's formula[9]

$$E\,\frac{dN}{d\vec{p}} = \int \frac{dN}{d\alpha}\,\frac{g(E_0,T_d)}{\bar{n}_0}\,E_0\,d\alpha$$

$$= \int_{\sigma(T_d)} g(E_0,T_d)\,E_0\,u^{\mu}\,d\sigma_{\mu} \quad , \tag{6}$$

in which the distribution (5) is simply boosted to c.m. system and integrated over the collective motions is not consistent with the energy-momentum conservation. The correct formula, obtained starting from Boltzmann transport equation would be

$$E\,\frac{dN}{dp} = \int_{\sigma} g(E_0,T_d)\;p^{\mu}\,d\sigma_{\mu} \quad , \tag{7}$$

which shall be used in the present work. Although small, this amounts to a non-negligible effect as will be shown in Sec. 3.

It is clear that the modifications explained in 5 and 6 only affect the momentum distribution, leaving the other results given in I unchanged. Thus, is the following sections we shall just discuss the momentum distribution of the charged secondaries.

3. CALCULATION OF $\frac{1}{\sigma}\frac{d\sigma}{dx}$ and $\frac{1}{\sigma}\frac{d\sigma}{d\eta}$ [17]

Following the scheme outlined in Sec. 2, we start fromtheexact Khalatnikov's one-dimensional solution, which is modified according to eq. (4) and put into eq. (6). Next, by using the recipe proposed in Ref. 9, we substitute $E_0 u^{\mu} \to p^{\mu}$, arriving at

$$E\,\frac{dN}{d\vec{p}} = A \int g(E_0,T)\,\frac{E'_{,}(\alpha/b)}{E_0(\alpha)}\,p^{\mu}\,d\sigma_{\mu}(\alpha)\ . \qquad (8)$$

If $b=1$, this will reduce to eq. (7).

The computation of $\frac{1}{\sigma}\frac{d\sigma}{dx}$ and $\frac{1}{\sigma}\frac{d\sigma}{d\eta}$ is achieved by starting from eq. (8), passing to the center-of-mass system of the collision and carrying out the integration over appropriate transverse variables. In doing so, we have included, as in I, the part of the solution which corresponds to the progressive waves.

The effect of using eq. (7) instead of eq. (6) is illustrated on Fig. 2, where $\frac{1}{\sigma}\frac{da}{dx}$ has been computed starting from these two equations, for a particular choice of s and M^2. The use of ℓn scale in the vertical axis apparently reduces the discrepancy between the two curves, but one can clearly see that the proposed correction implies a'broader distribution.

As explained in Sec. 2, we have simulated theadditional cooling of the fluid by introducing a parameter $b \gtrsim 1$, the influence of which is shown by Fig. 3, where $\frac{1}{\sigma}\frac{d\sigma}{dx}$ predicted by eq. (8) has been computed at $s = 3844$ GeV^2 and $M^2 = 2050$ GeV^2, and compared with the recent data[8]. As expected, a small change in b from the exact one-dimensional expansion solution (b=1) improves considerably theagreement with thedata. With the parameter b fixed in this way, we are going to compute both $\frac{1}{\sigma}\frac{d\sigma}{dx}$ and

$\frac{1}{a}\frac{d\sigma}{d\eta}$ for several values of M^2 and to compare with other data in next section.

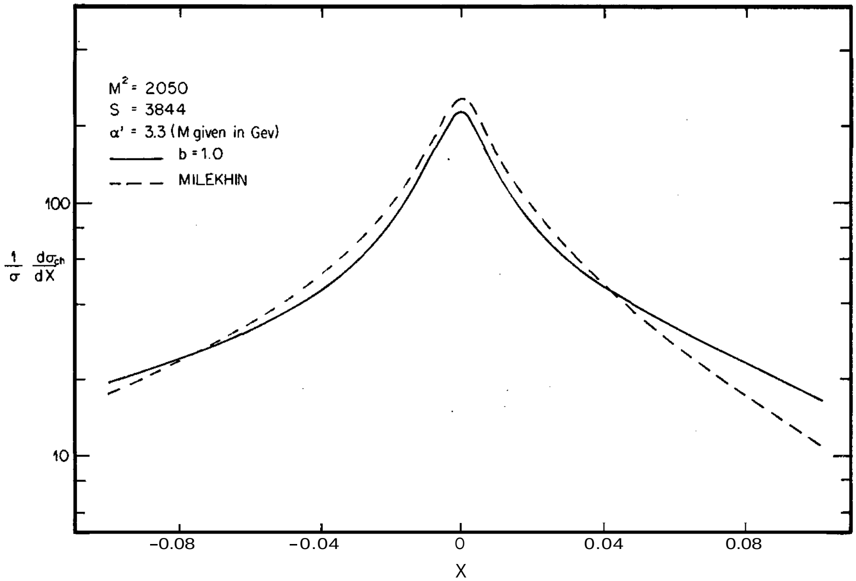

Fig.2 - Comparison of Milekhin's[9] distribution with the corrected version of Cooper and Frye[10].

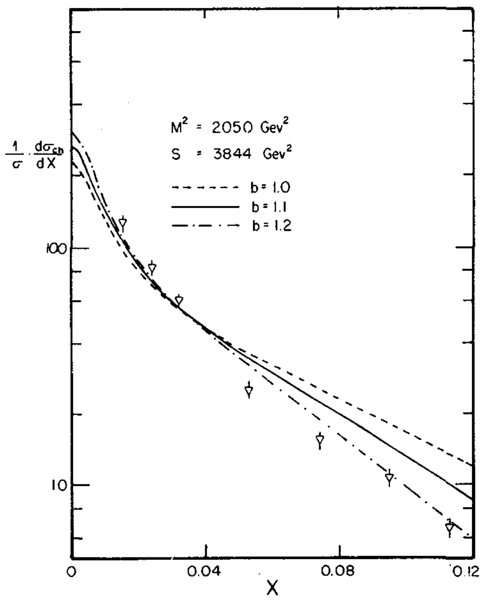

Fig.3 - The dependence of $\frac{1}{\sigma}\frac{d\sigma ch}{dx}$ on the parameter b defined in eq. *(4)*.

4. COMPARISON WITH THE DATA"

First of all, we have verified whether the changes introduced in the preceding sections noticeably modified the previous fit given in I. The answer is that no sensible change resulted, so that all the previous conclusions remain valid. In short, the change from eq. (6) to eq. (7) and taking b > 1 (instead of b = 1) act in opposite direction, compensating one another.

Next, the data at $\sqrt{s}$ = 62 GeV and different M^2 intervals have been compared with our prediction (Fig. 4). Notice that, whereas a detailed agreement has not been reached in our simplified model where just pions have been considered in the final states, even though the overall agreement is not bad . In particular, the data do show the same M^2 dependency predicted by the model. Observe that the data are incomplete in the sense that only the forward distributions exist, whereas the forward

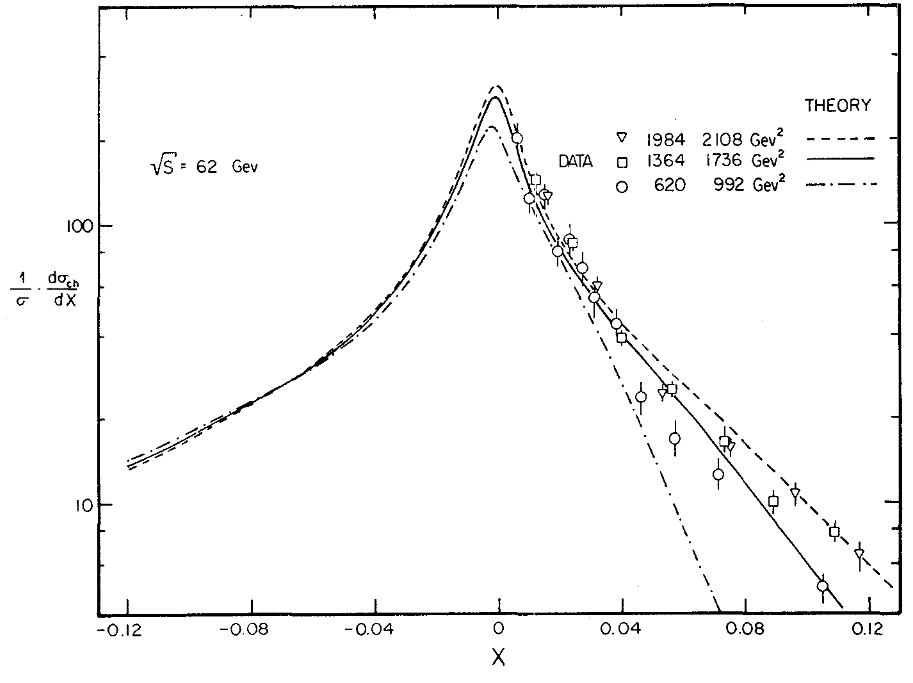

Fig. 4 - Comparison of $\frac{1}{\sigma}\frac{d\sigma ch}{dx}$ calculated at three different M^2 values with data[e].

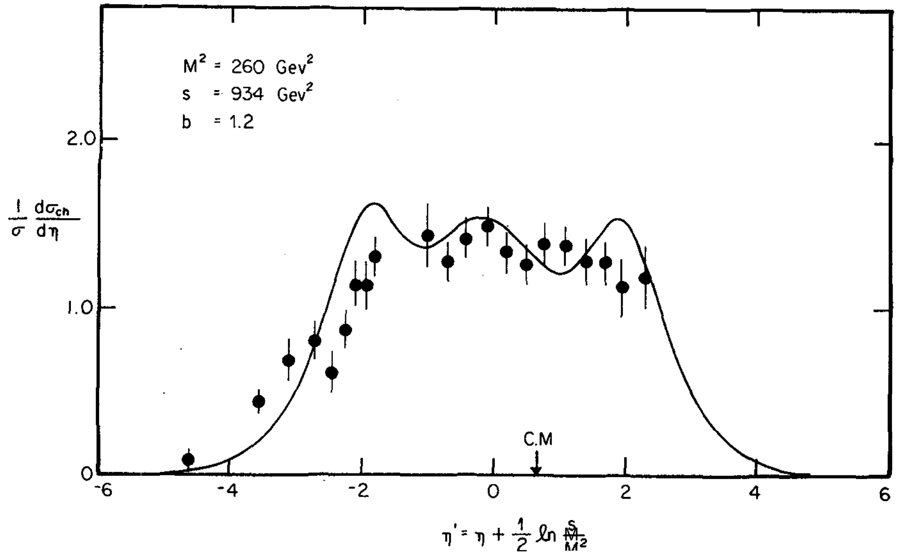

Fig. 5 - $\frac{1}{\sigma}\frac{d\sigma_{ch}}{d\eta}$ predicted by the model compared with data[3]. Similar comparisons at lower M^2 values have already been done in I[6].

-backward asymmetry as well as a near M^2-independence of the backward distributions are some of the properties of the present model.

Besides those one already analysed in I, Ref. 3 containsdata at higher M^2 values (260 GeV^2), although not covering the complete η interval. However, these are complementary to those given by Ref. 8 and discussed above, because the former cover all the backward hemisphere. They are shown on Fig. 5 in nice agreement with our curve.

At first sight, these results, with a main characteristic of being symmetrical[19] in rapidity with respect to $-\frac{1}{2}\ell n\frac{s}{M^2}$, seems largely to be a consequence of the energy-momentum conservation. However, a careful analysis shows that this is not so obvious. For instance, if one assumes an uncorrelated-jet-like model[2], with the energy-momentum conservation exactly taken into account, one will find serious difficulty in getting the correct correlation, as shown by Fig. 5, between one leading particle and another secondary. In our opinion, good agreements we have found give a strong support to the reality of large-mass clusters such as the ones we describe.

5. CONCLUSIONS

In the present paper, we have first compared Milekhin's formula[9] for particle momentum distribution with a more correct one due to Cooper and Frye[10]. Next, we have examined the effect of the transverse expansion (or evaporation) on the longitudinal momentum distribution. Both of them give relatively small changes in the final distributions, but if a quantitative comparison is to be achieved they must be taken into account. As given by eq. (4), the effect of the transverse expansion has been included in the present work through a phenomenological parameter $b > 1$. It is clear that a more precise prediction of the model has to be looked for by solving exactly the hydrodynamical equation, which is now being done following Milekhin's method[9].

In Sec. 4, we have extended the previous analysis reported in I, where we have examined the idea of large-mass cluster formation, accompanying a leading particle in hadronic collisions with multiple production. The conclusion is that this picture is consistent with all the existing data, related to the missing-mass associated to a leading particle, such as the average charged multiplicity as function of the missing-mass, the multiplicity distribution and its mass dependence, and the momentum distribution of the secondaries inside a missing mass.

Although other events such as two-leading-particle ones and those with large-$p_\perp$ jets are also possible, these are however rare events. Why just one cluster? This is in part due to our particular choice of events where one leading particle (in the interval $0.5 \lesssim x \lesssim 0.9$) is always present. It is perfectly natural that two-cluster events exist, which have been developped around each of the two incident particles. On the other hand, we do not see the need of including a larger number of clusters, in which case we would also have additional arbitrariness, since the properties of these clusters are not uniquely determined as in our model.

One weakness of the usual hydrodynamical model is the non-inclusion of quantum fluctuation effect during the first stage of the collision when highly compressed high-temperature fluid is formed, and which is also related to the lack of leading particles in that version. We think we are considering such a fluctuation in our model. It is true that there still remains the question of explaining how such a fluctuation operates and which is the resultant mass spectrum implied. These are questions which are intimately related to the one we have raised in Introduction, with regard to the origin of leading particles. However, as has been argued in I, if one imposes further constraints of reproducing correctly the overall average multiplicity and KNO scaling (without including the exact functional form) for the multiplicity distribution, one arrives to the missing-mass distribution of the form $\frac{d\sigma}{dM} \sim M^{\alpha}$, with $\alpha > -1$ (more likely to be $\alpha \simeq 0$), which is consistent with the data, perhaps excluding the extreme small values of M, which however give relatively small contribution.

The present work has been accomplished while one of us (F. W. Pottag) was a post-graduate fellow of Fundação de Amparo à Pesquisa do Estado de São Paulo.

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