Theoretical and Phenomenological Problema Concerning $J^P = 1^+$ Mesons

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Both from the experimental and theoretical points of view, the researches on $J^P = 1^+$ mesons have been very important for hadronic physics. The complex problems involving these states (particularly in what regards the determination of resonances and other non-resonant effects) have occupied a great part of the hadron-hadron phenomenology literature in the last decade. We give a description of the principal views of the subject with a particular emphasis on the case of the $A_1(\rho\pi)$ meson because it is the most important source of papers on $J^P = 1^+$ objects and presents still now, open problems. An almost exhaustive set of references on the subject and correlated topics is given.

Tanto do ponto de vista teórico quanto experimental as pesquisas sobre mesons de $J^P = 1^+$ têm sido muito importantes para a física hadrônica. Os complexos problemas envolvendo esses estados (particularmente no que diz respeito à determinação de ressonâncias e de outros efeitos não-ressonantes) ocuparam grande parte da literatura sobre fenomenologia hadron-hadron na última década. Apresentamos uma descrição dos principais aspectos do assunto com particular ênfase no caso do meson $A_1(\rho\pi)$, já que é a principal fonte de artigos sobre objetos com $J^P = 1^+$ e apresenta, ainda, problemas abertos. Uma bi-

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1. INTRODUCTION

Our principal motivation here was to describe the complexity of problems involving spin-parity $J^P = 1^+$ states and, at the same time, to make a critical review of the models proposed up to now. To promote a status of bona fide resonances for these states (predicted by SU(3)), has required to overcome enormous experimental difficulties on the one hand and has suggested a great quantity of theoretical models on the other hand. As an example of the problems connected with the dilemma "resonance" versus "Deck effect" (false proposition) have triggered a large production of papers as we will see in the following. In spite of many problems being still completely open, some of them have become obsolete or have been forgotten due to the coming of a new phenomenology - using the language of Quarks and Partons -. But what are those problems and what suggestions could we give to solve them? We intend here to answer at least partially these questions. There are many theoretical models to describe particular views of the subject but we feel that one lacks a global view to take into account all particular facts in a self-contained form.

First of all, we describe the experimental results - (section 2) - and next we discuss the theoretical and phenomenological points of view - (section 3). These two parts of the paper can be read independently. The reader can go directly at any one of them. We finish with a conclusion (sec.4) - that summarizes all points discussed in the text. As a general remark closing this Introduction, its is gratifying that the SU(3) prediction for these $J^P = 1^+$ objects have been now almost fulfilled by the new experimental results, although some difficulties still exist like the observation of the $A_1^0$ that continues to be a problem, (specially in charge exchange reactions) as we will see in the following and the detection of $H$ and $H'$.
2. EXPERIMENTAL RESULTS

2.1. Generelities

We begin with a general view of the experimental results obtained in the last years. The aim of some of these experiments was the identification of a number of resonances $B, \,(H,H'),\,(D,D' \equiv E)\, (Q_A, Q_B)$ and $A$, as $J^P = 1^+$ meson states', belonging to two SU(3) monents given in Table (1)$^2$. We have choosen a certain number of distributions, (invariant mass, transfer momentum, angular) partial wave analysis (PWA) and relative or phase-shift analysis, to give an idea about the experimental situation. In fact we display only a little number of results, the most significative and recent ones. Reactions of the diffractive dissociation type (see Fig. (1)) have been preferentially chosen due to the great number of diffractive productions. As it is evident from Fig. (1), by diffractive production for $2 \times 3$ (particles) $(a+b+c+d)$ we mean those reactions for which the squared center of mass energy $S = (p_1 + p_2)^2$ and the sub-energy $S_1 = (p_1 + p_2)^2 = M_{1.2}^2$ (squa-

Table I - All states $J^P = 1^+$ are classified according to SU(3). (see ref.2). $J^P$ is the spin-parity and $I$ is the isospin quantum numbers.

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>$1^+$ &quot;$B$-Nonet&quot;</th>
<th>$1^+$ &quot;$A_1$-Nonet&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$B(1235)$</td>
<td>$A_1(1.07)$</td>
</tr>
<tr>
<td>Strange</td>
<td>$Q_B(1.3-1.4)$</td>
<td>$Q_A(\equiv C)$</td>
</tr>
<tr>
<td>1/2</td>
<td></td>
<td>$(1.24-1.29)$</td>
</tr>
<tr>
<td>0</td>
<td>$H$</td>
<td>$D(1285)$</td>
</tr>
<tr>
<td>(Singlet/octet)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mixing?</td>
<td>$H'$</td>
<td>$D'(\equiv E)(1422)$</td>
</tr>
</tbody>
</table>
red invariant mass of $1+2$) is small. In these reactions we also observe a strong concentration of events at small $t_2$ (momentum transfer between b and 3). The high value of $S_2$ and small values of $t_2$ justify the assumption of Pomeron (P) exchange for the (b3) vertex. We report also other experimental results, as non-diffractive reactions, forward and backward productions, charge exchange reactions. In these cases for reasons related to searches for $A_1^0$ and H, we favour the charge exchange reactions. By contrast, we go hastily through the experimentally well established results (e.g., B and D). When it will be necessary we make some theoretical comments in this section.

2.2. Budha

The constituent of the SU(3) nonet, $j^{PC} = 1^{+-}$ called Budha (B) (Table I) is now well established as a good resonance. We give below, a number of distributions with the purpose of pointing out the existence of this object among others with the same spin-parity. Its main properties are:

a) $j^{G,PC} = 1^{+1-} (B^+, B^- \text{ and } B^0)^4$, where $j^{G}$ refers to the isospin and G-parity.
b) Mass and Width, $M_B = 1231 \pm 10$ (MeV), $\Gamma_B = 129 \pm 10$ (MeV).

c) Decay mode, $B \to \pi \pi \pi$ (only seen).

d) Studied reactions', $\pi N$, $K N$ and $p \bar{p}$ at different energies.

e) Cross-sections:

\begin{enumerate}
\item $\pi^- p \to \pi^- \omega, 6$ at $6.7$ (GeV/c) $160 \pm 22$ (pb)
\item $\pi^- p \to p \omega^0, 7$ at $9.1$ (GeV/c) $123 \pm 22$ (ub)
\item $\pi^- p \to p \pi^+ \pi^0, 8$ at $3.2$ (GeV/c) $108 \pm 30$ (ub)
\item $K^- p \to \Sigma^- B^+$ (Backward Production) at $4.2$ (GeV/c) $3.2 \pm 0.5$ (ub)
\end{enumerate}

We show in Fig. (2) the invariant mass distributions for two different reactions and energies, the $\pi N$ and $K N$ backward production of $B$. In both reactions the $B$ production is clear. The mass and width values obtained for the $B$ from the reaction $\pi N$ (Fig. (2a)), are: $M_B = 1242 \pm 10$ (MeV) and $\Gamma_B = 140 \pm 40$ (MeV).

From Fig. (2b), the backward production of $B$ in $K N$ reaction is estimated at $M_B = 1208 \pm 18$ (MeV) and $\Gamma_B = 163 \pm 50$ (MeV).

Both experiments fit the data with a Breit-Wigner formula and backgrounds of different types. In Fig. (3) we show a transfer momentum distribution for $\pi N$ interaction ($p_t = t_2$, $p_f$ = proton target and $p_f$ = final proton).

A strong concentration of events occurs at small values of $t_2$. This situation is not exceptional and characterizes the contribution of peripheral mechanisms present in $B$ production. In spite of the fact that the $B$ is a well-stablished resonance, other peripheral mechanism (non-resonant background) can be very important in an exact determination of production cross-sections in each specific reaction. All results from the published literature confirm the resonance hypothesis for $B$ in a clearer way than for most of other hand $J^P = 1^+$ objects as we will see in the following.
Fig. 2 - a) Mass distribution for $\pi\pi$ from ref. 7. b) $\pi\pi$ mass distribution for $\Xi^-$ backward reaction at 4.2 Gev/c from ref. 9.
2.3. $H$ and $H'$

These two states seem to be particularly difficult to detect experimentally. Only now, after almost ten years, does a reasonable evidence about the $H$ existence emerge and even now its parameters are not yet very conclusively established: a) $J^{PC} = 1^{+-}$, $M_H \approx 1.0$ (GeV); b) possible decays: $K\bar{K}\pi$ and $\rho\pi$. One of the difficulties is that the region of mass of this resonance overlaps with that of the $A_1$. Although the isospin values are different for $A_1 (I=1)$ and $H (I=0)$, it is very difficult to make a clear separation in the global mass spectrum of $\pi^+\pi^-\pi^0$. Some authors argue that the $\rho^-\pi^+$ charge states are more suitable to search for the $H$ and $A_1^0$. This is due to the different $\rho^-\pi^+$ mass spectrum obtained in comparison with $\rho^+\pi^-$ and $\rho^0\pi^0$ in the small masses region. Together with other, however, we think that the $\rho^0\pi^0$ is the best case to search for the $H$, since in this case, the $A_1^0$ contribution is obviously absent. In comparison with the $K\bar{K}\pi$ decay, we have in this case only one neutral particle ($\pi^0$) since the $\rho^0$ is identified by $\pi^+\pi^-$ in the final state. We call attention to the fact that the difficulty in
experimental identification of the $H$ and $A_1^0$ are very similar. We believe also that if the resonance $H$ exists, it is enhanced (similarly $A_1^0$) by strong contributions coming from kinematical threshold Deck like effects. From a theoretical point of view the situation is not less confused. There are contradictory predictions coming from the naive Quark Model and duality schemes respectively. In section (2.6), particularly in the $A_1^0$ subsection we return to many of these comments.

2.4. $D$ and $D' (=E)$

While the $D(1285)$ meson is well established as a good resonance the $E(1420)$ meson is not yet definitely identified as a resonant state although recent results obtained from $\pi^- p$ at 3.95 (GeV/c) confirm the previous quantum numbers assignment for the $E$ meson. The main characteristics for these $D$ and $E$ states are:

a) Both have been seen initially in $p\bar{p}$ annihilations and have afterwards been produced in other reaction (e.g. $\pi N$ and $KN$).

b) The mass spectra are compatible with a Breit-Wigner formula.

c) $J^{PC} = 0^+, 1^{++}$

d) $M_D = 1284 \pm 10$ (MeV), $\Gamma_D = 27 \pm 10$ (MeV)

$M_E = 1418 \pm 10$ (MeV), $\Gamma_E = 50 \pm 10$ (MeV)

e) Decays:

$$D \rightarrow 4\pi, \ K\bar{K}\pi, \ \eta\pi\pi, \ \delta\pi$$

$$E \rightarrow K\bar{K}\pi, \ (K^*\bar{K}+K\bar{K}^*), \ \eta\pi\pi, \ \delta\pi$$

We show in Fig. (4a) the mass spectrum of the $(\eta\pi\pi)$ final state obtained from the reaction $\pi^- p \rightarrow \eta^+ \pi^- \pi^0$ at 8.45 (GeV/c) where the $D$ is well seen. The plot of the relative phase versus effective mass $m_{\eta\pi\pi}$ is shown in Fig. (4b) and we note that $(\delta \pi)$ decay is preferred. In Fig. (5) we show the effective mass of $K^+ K^0 \pi^0$ final states for $E(1420)$ production. Finally to complete these information about $D$ and $E$ we pre-
Fig. 4 - a) Histogram of $\eta^+\pi^-$ mass from ref. 16b. b) Results of the phase-shift analysis from ref. 16b.
sent in Table (II) some branching-ratios, cross-sections and decay modes. From a theoretical point of view the \( D \) and \( E \) mesons are predicted by the Quark model\(^2\),\(^6\) although we should mention a recent controversial interpretation of the \( E'(1420) \) as an object related to the existence of glueball\(^2\).\(^7\)\(^d\).
Table II - Decay, cross-sections and Branching-Ratios for some reactions initiated by $\pi$, K and N, for the $B(1285)$ and $E(1420)$ production.

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Decay Modes</th>
<th>Cross-Section $[\mu B]$</th>
<th>Branching-Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ref. 18)</td>
<td>$\eta\pi\pi$</td>
<td>$3.7 \pm 1.0$</td>
<td>$D \rightarrow \frac{K\bar{K}\pi}{\eta\pi\pi} = 0.42 \pm 0.15$</td>
</tr>
<tr>
<td>$K^- p \rightarrow \Lambda\bar{D}$</td>
<td>$\eta\pi\pi$</td>
<td>$5.5 \pm 1.5$</td>
<td>$D \rightarrow 4\pi$ $\rightarrow \eta\pi\pi = 0.70 \pm 0.50$</td>
</tr>
<tr>
<td>4.2 $[\text{Gev/c}]$</td>
<td>$2\pi^+\pi^-$</td>
<td>$2.3 \pm 0.5$</td>
<td>$D \rightarrow \frac{\delta^+\pi^+ \rightarrow \eta\pi^+\pi^-}{\eta\pi^+\pi^-} = 0.72 \pm 0.15$</td>
</tr>
<tr>
<td>$1.265 \lesssim M_{\eta\pi\pi} \lesssim 1.32$</td>
<td>$4\pi$</td>
<td>$1.2 \pm 0.7$</td>
<td></td>
</tr>
<tr>
<td>(Ref. 19)</td>
<td>$K\bar{K}\pi$</td>
<td>$3.6 \pm 2.1$</td>
<td>$D \rightarrow \frac{\delta^+\pi^+ \rightarrow \eta\pi^+\pi^-}{\eta\pi^+\pi^-} = 0.6 \pm 0.3$</td>
</tr>
<tr>
<td>$\pi^- p \rightarrow \left{ \eta\pi^+\pi^- \pi \right}$</td>
<td>$\eta\pi\pi$</td>
<td></td>
<td>$D \rightarrow \frac{K\bar{K}\pi}{\eta\pi\pi} = 0.5 \pm 0.2$</td>
</tr>
<tr>
<td>12 and 15 $[\text{Gev/c}]$</td>
<td>$1.2 \lesssim M_{\eta\pi\pi} \lesssim 1.36$</td>
<td></td>
<td>$D \rightarrow \frac{\delta^+\pi^+ \rightarrow \eta\pi^+\pi^-}{K\bar{K}\pi} = 1.0 \pm 0.3$</td>
</tr>
<tr>
<td>For other results</td>
<td>$\eta\pi\pi$</td>
<td></td>
<td>$E \rightarrow \frac{\eta\pi\pi}{K\bar{K}\pi} \lesssim 0.5$</td>
</tr>
<tr>
<td>from a $\bar{p}$ see ref. 20</td>
<td></td>
<td></td>
<td>$E \rightarrow \frac{\delta^+\pi^+ \rightarrow \eta\pi^+\pi^-}{K\bar{K}\pi} \lesssim 0.3$</td>
</tr>
<tr>
<td>(Ref. 21)</td>
<td></td>
<td></td>
<td>$\gamma \approx 100 \pm 12$</td>
</tr>
<tr>
<td>$\bar{p}p \rightarrow K^0 K^\pm \pi^\mp + \pi$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7 $[\text{Gev/c}]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For ten other Final states see ref 21.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.5. $Q_A$ and $Q_B$

Each meson $Q(Q_A, Q_B)$ is a constituent of one different nonet (see Table I). They are produced mainly from reactions initiated by $K$. These two mesons have motivated a great amount of theoretical and experimental work due to the difficulty in determining these states as two resonances. Their main properties are:

a) $I = 1/2, J^{PC}(Q_A) = 1^+ -, J^{PC}(Q_B) = 1^+$

b) $M_{Q_A} = 1280$ (MeV); $\Gamma_{Q_A} = 120$ (MeV)
$M_{Q_B} = 1400$ (MeV); $\Gamma_{Q_B} = 150$ (MeV)

c) Decay:

\[
\begin{align*}
Q_A & \{ \begin{array}{l}
K\pi\pi \text{ dominant} \\
K\omega \text{ recently seen}^{23,1a} \\
K\rho \text{ favoured} \\
K^{*}\pi
\end{array} \\
Q_B & \{ \begin{array}{l}
K^{*}\pi \text{ favoured} \\
K\rho
\end{array}
\end{align*}
\]

d) Cross-sections$^{24c}$

i) $\sigma(Q_A \rightarrow K\rho) = 6.2 \pm 0.6$ (\(\mu b\))

ii) $\sigma(Q_A \rightarrow K^{*}\pi) = 1.7 \pm 0.5$ (\(\mu b\))

iii) $\sigma(Q_B \rightarrow K\rho) \geq 0.2$ (\(\mu b\))

iv) $\sigma(Q_B \rightarrow K^{*}\pi) < 0.5$ (\(\mu b\))

e) Helicity Conservation:

s - channel - (mode $K\rho$)

t - channel - (mode $K^{*}\pi$)
We choose a certain number of distributions to characterize them, in forward and backward production and for different reactions and energies. The effective mass of the $(K^\mp\pi^\pm)$ system is shown in figure (6).\textsuperscript{24} For $K^- p \to K^\mp \pi^\pm p$ at 10, 14 and 16 (GeV/c) it is shown in figure (6a) and in figure (6b) for $K^- p \to K^0\pi^-\pi^0 p$. From the latter figure, we identify two peaks corresponding to the $Q_A(1,27)$ and to the $Q_B(1,37)$ respectively. Other data are shown in Fig. (6, c) for the reactions $K^+ d \to K^+\pi^+\pi^- d$ at 12 (GeV/c) and in Fig. (6d) for backward productions. The (PWA) and relative phases are given in Fig. (7) (for $K^- p \to K^-\pi^+\pi^- p$ and $K^- p \to K^0\pi^-\pi^0 p$ reactions) and in Fig. (8) (for $K^+ p \to K^+\pi^+\pi^- p$ at 13 (GeV/c). Two peaks are seen in the $s$-wave ($\lambda=0$) associated to the $Q_A$ and $Q_B$ states. These data support the interpretation of the meson $Q_A \to K\rho$ as a good resonance but the evidence seems less conclusive in favour of $Q_B \to K^*\pi$. Similar results coming from other experiments and favouring the existence of two resonant states are given in ref.\textsuperscript{25}. Other more recent analysis\textsuperscript{26} (PWA)-obtained from the reactions $K^- p \to K^-\pi^+\pi^- p$ and $K^- p \to K^0\pi^-\pi^0 p$ at 4.2 (GeV/c) are consistent with two $s$-wave resonances. For the study of possible mechanism at work, one may turn now to angular and momentum transfer ($t_2$) distributions. In Fig. (9)\textsuperscript{27} we show the $t_2$-distribution and we note that the $K^-\pi^-\pi^+$ system has a slope greater than the $K^0\pi^-\pi^+$ system. This exponential behaviour is typical of diffractive productions. These $(d\sigma/dt_2)$ distributions present a well known cross-over\textsuperscript{27} for $K^-$, $K^+$ that was a motivation for the phenomenological models presented in the section 3. The mass-slope correlation parameters are given in Fig. (10) and Table III. In conclusion we think that the situation regarding these two $J^P=1^+$ objects is not yet completely well established. By analogy with all states of this set we believe that there are two resonances, but that other mechanisms (such as e.g., the Deck effect) are also contributing.
Fig. 6 a, b) Kπ mass distribution for the combined data at 10, 14 and 16 (GeV/c) for the reactions $K^-p \rightarrow K^-\pi^+\pi^-p$ (a) and $K^-p \rightarrow K^0\pi^-\pi^0p$ (b) from ref. [24a]
Fig. 6 - (c) $M(K^{+}\pi^{+}\pi^{-})$ (GeV) for all $K^{0}d \rightarrow K^{+}\pi^{+}\pi^{-}d$ events from ref. 24b.
Fig. 6 - d) $(K\pi\pi)^+_{\text{effective mass spectra}}$ for the sum of $K^+p + E^n\pi^0n^0$, $K^+p + E^n\pi^0n^0$ and $K^+p + E^n\pi^0$ + neutrals, from ref. 24c. For other results on backward reactions see also ref. 24d.
Fig. 7 — Cross-Sections of \( \gamma \rightarrow K^- p \rightarrow K^- \pi^+ \pi^- p \) as function of the \((K \pi \pi)\) mass. See ref 24a for notations and other information.
Fig. 8 - a) Comparison of the $1^+0^+$ and $1^+1^+$ $pK$ cross sections and relative phases. See ref. 25. b) Comparison of the $1^+0^+$, $1^+1^+$ and $2^+1^+K\pi\pi$ cross sections and relative phases.
Fig. 9 - The $dN/dt'$ distributions for $\bar{K}^+ p \rightarrow \bar{K}^+ \pi^+ \pi^-$, $\bar{K}^+ p \rightarrow \bar{K}^0 \pi^+ \pi^0$, and $K^+ p \rightarrow K^0 \pi^+ \pi^-$, and for $1.0 \leq M(K\pi\pi) \leq 1.5$ GeV.
Fig. 10 - Slope-mass correlations for the indicated reactions, from ref. 27.

Table III - Slopes for several mass ($K\pi\pi$) intervals at different energies.

<table>
<thead>
<tr>
<th>$M(K\pi\pi)$ (GeV)</th>
<th>$B(\bar{K}^0\pi^0)$ (GeV$^{-2}$)</th>
<th>$B(K^-\pi^+\pi^-)$ (GeV$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. 28 10,16 (GeV/c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.05 - 1.20</td>
<td>10.5 ± 1.0</td>
<td>12.8 ± 0.7</td>
</tr>
<tr>
<td>1.20 - 1.35</td>
<td>9.0 ± 0.8</td>
<td>9.0 ± 0.6</td>
</tr>
<tr>
<td>1.35 - 1.50</td>
<td>6.9 ± 0.6</td>
<td>7.6 ± 0.5</td>
</tr>
<tr>
<td>1.50 - 2.0</td>
<td>5.7 ± 0.6</td>
<td>6.4 ± 0.5</td>
</tr>
<tr>
<td>Ref. 29 4-12 (GeV/c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 - 1.2</td>
<td>8.7 ± 1.1</td>
<td>13.8 ± 1.4</td>
</tr>
<tr>
<td>1.2 - 1.3</td>
<td>6.6 ± 1.1</td>
<td>11.6 ± 1.4</td>
</tr>
<tr>
<td>1.3 - 1.4</td>
<td>5.5 ± 1.1</td>
<td>8.9 ± 1.1</td>
</tr>
<tr>
<td>1.4 - 1.5</td>
<td>3.3 ± 1.0</td>
<td>6.9 ± 1.1</td>
</tr>
<tr>
<td>1.5 - 1.75</td>
<td>2.9 ± 1.0</td>
<td>5.4 ± 1.0</td>
</tr>
</tbody>
</table>
2.6 \( -A_1 \)

The axial vector meson \( A_1^{30} \) was surely the subject of the greatest number of theoretical experimental papers among those of \( J^P=1^+ \) family. For example it is at the origin of Deck Model (see section 3), and still nowadays we have a number of interesting problems not completely solved associated with this object. Its principal characteristics are:

a) \( I^G = 1^-; \quad \nu^{PC} = 1^{++} \)

b) \( M_{A_1} = 1.1 \text{ (GeV); } \Gamma_{A_1} = 300 \text{ (MeV)} \)

c) principal decay mode: \( \rho \pi \)

d) different reactions studied:

i) \( \pi p \rightarrow (3\pi) \pm p \) (forward and backward production). Favourable reaction for observing the \( A_1^0 \) - in charge and hypercharge exchange reactions -.

ii) \( \pi^+ n \rightarrow (3\pi)^0 p \)

iii) \( \pi^+ p \rightarrow (3\pi)^0 \Delta^{++} \)

iv) \( K^- p \rightarrow (3\pi)^0 \Lambda \)

v) \( \pi^- p \rightarrow \pi^+ \pi^- \pi^0 n \) (this is the only charge exchange reaction where \( A_1^0 \) was observed)\(^{10b}\)

e) No s-channel helicity conservation\(^{31}\).

f) Untii very recently, all searches for \( A_1^0 \) production in charge exchange reactions gave basically negative results. Recently however strong evidences has been given for an \( A_1^0 \) resonant state in reaction (v) above 8.45 (GeV/c).

We try now to illustrate the various aspects of the problem for and against a resonant interpretation and due to the important literature associated with the subject we will try to be fairly complete. The reactions chosen are backward and forward productions, diffractive and non diffractive interactions and others. We recall first of all that while the diffractive reactions favoured the non-resonant interpretation of the \( (\rho \pi) \) enhancements, the others reactions favoured a resonant
interpretation via a Breit-Wigner formula. In Fig. (11) we show the total mass spectrum of $(3\pi)$ from $\pi^-p \rightarrow (\pi^+\pi^-)p$ at 11 and 25 (GeV/c) where we see some evidence for two peaks associated with the $A_1$ and $A_2$ states where the latter is the well known $J^P=2^+$ resonance at 1310 MeV. The solid curve is the results of a fit made by the authors of ref. 32 with the Deck-Model. An example of background production in $\pi N$ reactions at 9 (GeV/c) is shown in Fig. (12) where the solid line represents the result obtained from a fit with two Breit-Wigner formulas for the $A_1$ with $m_{A_1} = 1050 \pm 11$ (MeV) and $\Gamma_{A_1} = 195 \pm 32$ (MeV) and for the $A_2$ respectively. To compare with the experimental data we calculate the $(pn)$ mass distribution, using a double-Regge Model without the optimization of the Regge parameters (see ref.35). The result, shown by a dashed line in Fig. (12), is very large and centered at $M_{A_1} = 1.18$ (GeV).

![Graph](image)

**Fig.11** - $3\pi$ mass distribution of the reaction $\pi^-p \rightarrow (\pi^+\pi^-)p$ at 11, and 25 GeV/c. $^{32}$

This result can be improved by small variations of the parameters used. In Fig. (13) we show a backward production from the reaction $K^-p \rightarrow \pi^- + \pi^+ + \pi^+$ at 4.15 (GeV/c) $^{35,36}$. The (PWA) results are shown in Fig. (14) $^{36}$ and support the evidence of the $A_1$ as a $1^+ S(^1 S^0\pi^+)$ wave. While the combined
\[ \pi^-, 9 \text{ GeV/c} + 12 \text{ GeV/c} \]
\[ U', \pi^-, p_f < 0.5 \text{ (GeV)}^2 \]
\[ m \left( p_f \pi^- \right) < 1.8 \text{ GeV rejected} \]
\[ r - p \rightarrow p_f \pi^+ \pi^- \pi^- \]

**Fig. 12** - \( (\rho^0 \pi^-) \) mass spectrum for \( (9+12 \text{ GeV/c}) \) data \(^{34}\). Events with \( m(\rho^0 \pi^-) < 1.8 \text{ GeV} \) rejected; only events with \( \rho_f p < 0.5 \text{ GeV}^2 \) have been retained. The solid curve results from fits explained in ref. 34 where the mass and width of \( A_2 \) are \( m_{A_2} = 1050 \pm 11 \text{ MeV} \) and \( \Gamma_{A_2} = 195 \pm 32 \text{ MeV} \). The dashed curve is obtained from a Double-Regge model \(^{346}\).

Results obtained from \( \pi^- p \rightarrow \pi^- \pi^+ p \) at 25 (GeV/c) and 40 (GeV/c) do not show any significant variations of the relative phases (see Fig. 15). Other more recent results \(^{39} \) from \( \pi^- p \rightarrow 3 \pi^- + p \) at 63 and 94 (GeV/c) provide the strongest piece of evidence in favour of the resonance interpretation of the \( A_1 \) (see Fig. 16). The solid lines (Fig. 16a-d) are the result of the analysis after the \( A_2 \) contribution has been subtracted out in the form of a Breit-Wigner and take into account also a Deck contribution. The mass and width found for the \( A_4 \) are \( m_{A_4} = 1280 \text{ (MeV)} \) and \( \Gamma_{A_4} = 300 \text{ (MeV)} \), values that do not agree with others of the current literature \(^{1a} \) (\( m_{A_4} \approx 1.1 \text{ (GeV)} \)). It is quite possible that threshold phenomena \(^{40} \) may be responsible for the different values found. The authors of ref. 39 claim that only a resonance or a Deck amplitude separately could not account for the effects observed in these global spectra \(^{1a} \). However, we call attention to the fact that the Deck contri-
Fig. 13 - (3-) mass spectrum for $K^- p \rightarrow \Sigma^- \pi^+ \pi^+ \pi^-$ at 4.15 Gev/c$^2$. 

Mass ($\pi^+ \pi^+ \pi^-$) GeV
Fig. 14 - (3π) mass spectrum for $K^- p \rightarrow \Sigma^- \pi^+ \pi^-$ at 4.15 (GeV/c) from partial wave analysis of ref. 36.
Fig. 15 - (31) mass spectrum of different partial waves and interference phases in A, region for reaction $\pi^- p \rightarrow \pi^- \pi^- p$ at 25 (Gev/c) and 40 (Gev/c) combined.
Fig.16 - Results obtained for a "Resonant $A_2$ plus rescattered Deck" fits in $1^S$ intensity and phase with respect to $2^D1^S$, $A_2$ phase subtracted. (a), (b), (c), and (d) indicate different $t'$ intervals; from ref. 39.
bution used in these fits takes into account only the \(\pi\)-exchange term. We return to this point in Section 3.

\(A_{1}^{0}\) Observation

As we have already mentioned, the observation of the \(A_{1}^{0}\) has been particularly difficult in charge exchange reactions\(^{4,3}\) and only recently\(^{10b}\) a pronounced relative phase variation for the \(A_{1}^{0}\) state has been observed confirming the resonant interpretation of this object. Also from the theoretical point of view the situation is quite confused as we can see in Table (IV) where several predictions of the cross-sections for \(A_{1}^{0}\) production are reported. These predictions turn out to be very dependent on the mass and the approach employed.

There are other experiments\(^{4,2}\) \(- K^{-}p \rightarrow \pi^{+}\pi^{-}n\) at 4-5 (GeV/c) and \(K^{+}\)p at 12.7 (GeV/c) that identify the \(A_{1}^{0}\) in \((\pi^{+}\pi^{-}\pi^{0})\) mass spectrum. For both reactions the effective mass distribution (Fig. (17)) shows a peak around 1.05 (GeV) associated with the \(A_{1}^{0}\). The reactions where the resonant interpretation is favoured\(^{4,3}\) \((\pi^{+}n \rightarrow \pi^{+}\pi^{-}\pi^{0}p)\) at 4. (GeV/c)\(^{4,3}\) \(\pi^{-}p \rightarrow \pi^{+}\pi^{-}\pi^{0}n\) at 12 and 15 (GeV/c)\(^{4,3b}\) \(\pi^{+}p \rightarrow \pi^{+}\pi^{-}\pi^{0}\Lambda^{0}\) at 7 and 15 (GeV/c)\(^{4,3c}\), \(K^{-}p \rightarrow \pi^{+}\pi^{-}\pi^{0}n\) at 4.2 (GeV/c)\(^{4,3d}\) exhibit a strong cancellation responsible for not observing the \(A_{1}^{0}\) in these reactions. An example is given in Fig. (18) where we show data from the same experiment at 15 (GeV/c)\(^{4,3c}\); the \(A_{1}^{+}\) signal is absolutely in the channel \(\pi^{+}p \rightarrow \pi^{+}\pi^{+}\pi^{-}\) in \(1^{+}(\rho\pi)\) S wave, whereas no structure is seen in the \(\pi^{+}p \rightarrow A_{1}^{++}(\pi^{+}\pi^{-}\pi^{0})\) results. As we have already pointed out in Section 2.3 in the cases of \(H\) and \(H'\)\(^{11}\) also in the mass region of the \(A_{1}^{0}\) there are other competing resonant states, and this makes very difficult the analysis and it is only in one charge exchange reaction\(^{10b}\), that a clear signal has been observed recently. Fig. (18c-e) show these results for \(A_{1}^{0}\) as well as for the \(H\) mesons. The analysis for these states is made simultaneously since they are very close in mass \((m=1.13\) (GeV) in this experiment) exhibiting analogous difficulties. More data and analysis are necessary to make consistent the finding of the various charge exchange experiments\(^{4,3,10b}\).
Table IV - Several approaches for the \( A_1^0 \) cross-sections and their different results (see ref. 64a).

<table>
<thead>
<tr>
<th>Approach</th>
<th>Plab. (Gev/c)</th>
<th>Reaction</th>
<th>( \sigma_{\text{Tot}} ) (Theo.) [( \mu^2 )]</th>
<th>( M_{A_1} = 1.1 \text{(Gev)} )</th>
<th>( M_{A_1} = 1.3 \text{(Gev)} )</th>
<th>( M_{n/A_1} = 1.5 \text{(Gev)} )</th>
<th>( \gamma_{\text{exp.}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No D wave in ( A_1 \rightarrow \rho \pi )</td>
<td>7.</td>
<td>( \pi^+ p \rightarrow A_1^0 \Delta^++ )</td>
<td>1.7</td>
<td>0.8</td>
<td>0.4</td>
<td>( &lt;2 ) ( \text{c} )</td>
<td>( &lt;0.5 ) ( \text{c} )</td>
</tr>
<tr>
<td>Broken ( SU(6)_w ) result</td>
<td>7.</td>
<td>( \pi^+ p \rightarrow A_1^0 \Delta^++ )</td>
<td>16.0</td>
<td>2.4</td>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>( \pi^+ p \rightarrow A_1^0 \Delta^++ )</td>
<td>6.0</td>
<td>1.1</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.4</td>
<td>( \pi^- p \rightarrow A_1^0 n )</td>
<td>2.0</td>
<td>0.9</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Algebra Result.</td>
<td>7.</td>
<td>( \pi^+ p \rightarrow A_1^0 \Delta^++ )</td>
<td>72.0</td>
<td>2.3</td>
<td>9.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15.</td>
<td>( \pi^+ p \rightarrow A_1^0 \Delta^++ )</td>
<td>28.0</td>
<td>10.0</td>
<td>4.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.4</td>
<td>( \pi^- p \rightarrow A_1^0 n )</td>
<td>69.0</td>
<td>26.0</td>
<td>12.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 17 - The three-pion ($\pi^+\pi^-\pi^0$) mass spectra for $\Delta^0$ production in $KN$ reactions. (a) Events in $K^-p \rightarrow K^-p\pi^+\pi^-\pi^0$ at 4.6 and 5.0 (GeV/c) from ref. 42a. (b) Events in $K^+p \rightarrow K^+p\pi^+\pi^-\pi^0$ at 12.7 (GeV/c) from ref. 42b.
Fig. 18 - (3π) mass spectra for $1^+(\rho\pi)^*$ partial wave at 15. (GeV/c). 

(a) Reaction $\pi^+p - p\pi^+\pi^+\pi^-$ where the $\Delta^+_1$ is seen and fitted (solid curve) by a Breit-Wigner formula with $m = 1.152 \pm 0.009$ (GeV), $\Gamma = 0.264 \pm 0.011$ (GeV) and $\sigma = 129.8 \pm 7.8$ (mb). (b) Reaction $\pi^+p - \Lambda^0\pi^+\pi^-\pi^0$ with a complete absence of a resonance structure. (c) Partial wave analysis results from ref. 10b for the H and (d) for the $\Delta$ in (1+0+1+1) spectrum and others for different sets of quantum numbers. (e) Relative phase between different sets of quantum numbers representing $A_1$ versus $\Delta$, $A_1$ versus exotic and $\Delta$ versus exotic respectively. The notation $I/JP$ (isobar) $\Delta H$ are given in references 10b.
New experimental results** coming also from lepton-hadrons interactions provide supplementary support in favour of the resonant interpretation of the $A_1$ meson. In spite of the small number of observed events, these experiments show that the heavy lepton $\tau$ ($J=1/2$, $m=1784\pm4$ (MeV) decays into $A_1(\rightarrow\rho\pi)\nu_\tau$. Fig. (19) shows the mass spectrum from
\(e^+e^-\) interactions with \(\Lambda_1\) identification. We return to this point in the next Section.

The interested reader may find many other results and angular distributions\(^{36,45}\) in the references listed here.

3. THEORETICAL APPROACHES, SCHEMES AND MODELS

3.1. Generelities

An evidence of the importance of the subject for particle phenomenology is the number of papers about \(J^P=1^+\) mesons including papers dealing with theoretical schemes and models. We give here a short description of each of the main approaches. If, on the one hand \(SU(3)\) predicts easily these resonances on the other hand, their experimental detection has been very difficult. Experimentally a great step in improving this analysis has been the Partial Wave Analysis\(^{37,43}\) of Ascoli and Collaborators. These analysis are now determinant in the identification of a resonance. For the sake of simplification, we can classify in three main categories the various theoretical schemes proposed so far, according to which mechanism they make responsible for the enhancements observed in the different reactions and which we call \(1^+\) mesons:

I - Fure resonant states described by Breit-Wigner formula;
II - pure non-resonant states interpreted kinematically via Deck-like models.
III) composite models where it is assumed that resonances exists but a Breit-Wigner formula is unable to account for all the spectrum, since these objects are produced the threshold of a new channel and other effects do also contribute. Thus the kinematical effects which give rise to Drell-Hiida-Deck-like models must also be taken into account in the complete amplitude.

Approaches (I) and (II) are too simple minded to provide a realistic description of the data. We believe like everybody else that approach (III) is the correct one. The difficulty is at a technical level.
in the sense of taking into account all contributions without incurring in the sin of double counting. The development of this subject occurred in parallel in diffractive dissociation reactions. We do not intend here to give an exhaustive description of each approach but to give a good idea of the main ones and some information about the others.

3.2. Resonant Approach

Usually, in the same reactions, $1^+$ mesons are produced together with some well identified resonances such as $A_2$, $K_{1*1520}$, etc., but the former are much more difficult to detect.

A well known approach consists in interpreting the enhancements observed in the invariant mass distribution of a reaction like $a+b ightarrow a^*b$ where $a^*+1+2$ (see Fig. (1)), as objects described by a Breit-Wigner (B.W.) formula which we write here for pedagogical purposes including threshold effects:

$$BW = \frac{PQ}{m_R^2 - S_1 - i\pi R \Gamma}$$

$$\Gamma = \Gamma_R \left(\frac{q}{q_R}\right)^{2S+1} \frac{M_R}{\sqrt{S_1}}, \quad S_1 = M_R^2$$

(see ref. 46h for the notation and definitions of variables). In a more complete analysis, we examine also the phase-shifts ($\delta_{\phi}(S_1)$) associated with each partial wave produced to verify which of them, if any, goes through $\pi/2$ around $S_1 = M_R^2$, ($M_R$ = mass of the resonance). These relative phase variations (PWA)$^{46}$ give an enhancement the status of a good resonance or not.

Many ambiguities are inherently present in the definition of a resonance like tail effects, background contamination, superposition of closed-by resonance which all make the BW formula somewhat unrealible beyond a certain level. We also mention that the symmetric curve produced by a (B.W.) - like formula is not always in agreement with experimental spectra. In general, near the threshold one finds an asymmetry that is well described by others mechanisms, and this is an indication
that a pure \((B,W.)\) formula does not describe completely the effect observed experimentally. In the particular case of the \(A_1^+\) (seen in \(p^0\pi^- + \pi^+\pi^-\)) other effects coming from Bose symmetry must be taken into account. It was shown that this symnetrization increases the enhancement due to the \(A_1\) resonance. (Note that this is not the case for \(A_1^0\)).

Some classes of models deserve a special consideration, the so-called dual models, whereby the full amplitude is constructed as a never ending superposition of resonances. A complete discussion of this class of models is, however, outside the scope of our present review and we refer the interested reader to the large literature existing on the subject.

Finally, the resonance approach with or without ambiguities is very simple, perhaps too simple and the \(J^P = 1^+\) mesons are the proof that it is not always possible to use it to fully describe the physical reality.

### 3.3. Kinematical Effects and Drell-Hiida-Deck (D.H.D.) Approaches

The main points about the (D.H.D.) model are given in the following with a somewhat more detailed description. In the original form, the (D.H.D.) model consists in considering the dissociation of the beam of particle into two virtual others that interact with the target (in general a nucleon or nucleus), diffractively (high energy and small transfer of momentum). For example, take the \(\pi N \rightarrow p \pi N\) reaction shown in Fig. (20). This is given by the product of a pion-exchange and an off-mass-shell elastic subreaction characterized by a Pomeron exchange in the Regge language, i.e., the diffractive part of the global process. The cross-section that this mechanism gives for the diagram of the Fig. (20) is:

\[
\frac{d\sigma}{dt} = G \frac{|M_{\pi N}|^2}{(t - \mu^2)^2} \delta^4(p_1 + p_2 + p_3 - p_4 - p_5) \frac{dp_1}{E_1} \frac{dp_2}{E_2} \frac{dp_3}{E_3}
\]

where \(G\) summarizes all numerical constants-flux factor and eventual off-on-mass-shell corrections for the elastic subreaction \(\pi N \rightarrow \pi N \rightarrow M_{\pi N}\) which is parametrized as
\[ |M_{\pi N}|^2 = (8\pi S_2)^2 \left( \frac{d\sigma}{dt_2} \right)_{t_2=0} e^{bt}, \]

where \( b \) is the slope of diffraction peak from \( d\sigma/dt_2 \) distribution and \( \left( \frac{d\sigma}{dt_2} \right)_{t_2=0} \) is the differential forward cross-section.

The principal points associated with the development of the (D.H.D.) model are:

i) Reggeization of the (D.H.D.) amplitude\(^9\),

ii) Dualization\(^5\),

iii) Considerations about others components\(^6\).

Since their first version\(^**\), the (D.H.C.) model had a great success for the mass spectrum description of \( A_1 \to \rho \pi \), and looked initially as a competitive possibility for the resonant approach. It is easily understood why a D.H.D. amplitude produces a non symmetric enhancement. We recall the well known kinematical relation \( \frac{S_1 S_2}{S} \approx \text{const.} \) and we look for the phase space in the invariants \( S \) and \( S_2 \). Since \( S \) and \( S_2 \) are great \( (M_{\pi N} \text{ is dominated by Pomeron exchange}) \) \( S_1 \) is necessarily small by energy-momentum conservation \( (S_1 + S_2 + S_3 = S + m_1^2 + m_2^2 + m_3^2) \). We see also that the amplitude is directly proportional to \( S_2 \) and \( e^{bt_2} (t_2 < 0) \). Let
us look to the Chew-Low plot ($S_1$ versus $S_2$) in the mass spectrum of Fig. (21); a small increase in $S_1$ corresponds to a rapid variation in $t_2$ arising from the exponential $e^{b t_2}$; consequently we have rapid decrease of the curve in $S_1 (= m^2_{\rho \pi})$ in spite of the peripheral $\pi -$ exchange term $(1/(t_1 - \mu^2))$.

![Fig.21 - Comparison between a typical mass-spectra and a Chew-Low plot for ($\rho\pi\rho$) like model.](image)

However, this naive Deck model does not describe other aspects of this reaction since so far we have taken into account only one of the three possible contributions: $\pi -$ exchange, $\rho -$ exchange and direct $r-$pole-exchange (for $\pi N \rightarrow \rho\pi N$ e.g.). There are many reasons proving that besides considering these three components it is also necessary to take into account all phase space, mass-slope-$\cos \theta^GJ$, correlations, angular distributions, $S$ and $t$ channels helicity conservation etc. Finally, this model can be considered as a particular case of the Double Regge Model, and we return to this point in section 3.5.

### 3.4. Contributions from Rescattering Components

A model with final particles rescattering corrections is presented by some authors\(^2\) (see Fig. (22)) to take into account $A$, resonance effects as well as Deck like contributions. This model adds coherently the three following terms:

1) one $\pi-$exchange Deck type,
ii) one term representing the rescattering correction from final $\rho\pi$ states, and,

iii) one term representing the direct resonance production (via a Breit-Wigner formula) of the $A_1$ decaying into $\rho\pi$.

The results $^{53b}$ of this model, are claimed to give support to the existence of the $A_1$ resonance in spite of the small phase variation found. The physical interpretation appears to be that the phase shifts due to the resonance term and those coming from the rescattering term would cancel each other leaving only those due to the Deck non-resonant term. But it is not obvious to us that the way in which the resonant and Deck components are added in this approach does not lead to double counting by duality arguments. If there is double counting we can ask what would it be the results within a more complete model where the three Deck terms, $\pi$ and $\rho$ exchange and $\tau$-direct-pole-exchange corresponding to the $\pi, \rho$ and $S$ channels of the subreaction $n + \rho\pi$ are all taken into account. An advantage of this approach with a rescattering term is that it can take into account directly the phase shifts of the $\rho\pi$ elastic system via the final state (Watson) theorem$^{54}$. Other authors$^{55}$ take into account all effects produced with resonant and Deck effects - with rescattering component - considering the corrections coming from unitarity in the Deck amplitude and keeping also the coupled channel contribution at the resonance (e.g. A, with $\rho\pi$ and $K^*K$; Q with $K\rho$ and $K^*\pi$) via the $\chi$ matrix formalism$^{56}$. In principle, double counting should be absent from these approaches, in practice, however, the parameters used for the $A_1$ resonance ($M_{A_1} \approx 1300$ (MeV)), $\Gamma_{A_1} \approx 400 \pm 100$ (MeV)$^{55}$, ($M_{A_1} \approx 1450$ (MeV), $\Gamma_{A_1} \approx 380$ (MeV))$^{57}$ are in contradiction
with the current experimental results. Another problem is the great
number of free parameters used to take into account rescattering cor-
rections, Deck effects, resonances, coupled channel in a unique am-
plitude; but it is well possible that this is the price to pay to take in-
to account all these components at the same time. On the other hand, the
parameters obtained for $Q_1$ (+$K\rho$) and $Q_2$ (+$K^*\pi$) are compatible with
the experimental masses and width. Concerning the coupled channel reso-
nances in the first case, $(A_1^+)$, the two threshold $(K^*\pi$ and $\rho\pi)$ are very
far away in comparison with the second case ($K\rho$ and $K^*\pi$). Perhaps this
is responsible for the mass shift obtained for the $A_1$ meson. These ap-
proaches retain only two components of the Born term for the Deck am-
plitude corresponding to the $t$ and $u$ channels of the subreaction $\pi p \rightarrow 1+2$. It is clear that if we take into account also the third Born
term, the direct pole diagram - see Fig. (23) - we must be careful with
the double counting Duality problem. In a way, the contribution coming
from rescattering (see Fig. (22b)) is justified in the context of the
Landau singularities (triangle singularities in the present case) since the authors of ref. 59 show how the $J^P=1^+$ mesons $A_1$ and $Q_2$ were
an evidence for these singularities (the peaks corresponding to the $A_1$
and $Q_4$ taking place for $m_{\pi\pi} \approx 1.1$ (GeV) and $m_{K^*\pi} \approx 1.2$ (GeV) respectively). This point may suggest that the shift from 1.2 to 1.3 (GeV) for
$M_{A_1}$ is not due to the rescattering term. More information about interesting
subject is in ref. 59 and therein.

![Fig. 23 - The a-direct pole graph for $\pi N + (\pi\pi) \pi N$ interactions from ref. 47a.](image)

![Fig. 24 - Rescattering diagram representing a triangle singularity for a $\pi N + (\pi\pi) \pi N$ interactions from ref. 47a.](image)
3.5. Double Regge Approach

As we said above in Section 3.3, the $2+3$ reactions and particularly the Diffractive Dissociation reactions, interpreted via (D.H.D.) model can be considered in general via a Double Regge (D.R.) exchange model that presents a great flexibility of applications. If we take the Regge trajectory $\alpha_2 = \frac{\alpha_D}{s}$ (see Fig. (25)), we obtain a simple (D.H.D.) model. A great advantage of this Double Regge exchange mechanism is that can be applied without difficulties to forward as well backward production with a small number of parameters. And as it was pointed out recently, a Dual amplitude like Double Regge, corresponding to the diagram shown in Fig. (25),

$$A_s(S_1, S_2, t_1, t_2) = \xi_1(t_1)\xi_2(t_1, t_2)\frac{\alpha_1(t_1) - \alpha_1(t_2)}{S_2} V_{12}(t_1, t_2) +$$

$$+ \xi_2(t_2)\xi_2(t_1, t_2)\frac{\alpha_2(t_2) - \alpha_2(t_1)}{S_1} V_{21}(t_1, t_2)$$

$$\alpha_i(t_1) = \alpha_i^{\pm}(s_i - m^2)$$

$$\xi_i(t_1) = \tau_i + \exp[-i\pi\alpha_i(t_1)]$$

$$\xi_{ij}(t_1, t_2) = \tau_i \tau_j + \exp[-i\pi(\alpha_i(t_1) - \alpha_j(t_2))]$$

$$V_{ij} = V_0/\alpha_i(t_1)[\alpha_j(t_2) - \alpha_i(t_1)]$$

$i = 1, 2$

Fig. 25: Double Regge Exchange diagram. $\alpha_1$ and $\alpha_2$ represent the trajectories exchanged. The kinematical variables $S, S_1, S_2, t_1$ and $t_2$ were defined before.
contains implicity resonances and background. Due to the general properties of this amplitude, we also have a good result in forward as well in backward production, in spite of the arbitrariness in the parametrization. We know that this was the case with many problems connected with the \( A_1 \), \( Q \) and other \( J^P = 1^+ \) mesons. This model however, is not able to account for the strong suppression of the \( A_1 \) in contribution charge exchange reactions. For the cross-sections of this special case \( A_1^0 \), we give in Table (IV) a list of theoretical predictions according to the model used. A strong variation is observed with the mass attributed to the \( A_1 \), and with the approach taken.

3.6. New Interactions and Results

Interactions like\(^6\) lepton-hadron, \( RN \rightarrow L + V(A) + N \) where \( R \) = lepton, \( N \) = nucleon and \( V(A) \) is a vector (axial-vector) particle, are used for the observation of vector and axial mesons. More specifically for the diffractive physical region these reactions have been studied in the sense of \( A_1 \) productions. The study of these lepton-hadron reactions is very important for the subsequent information which can be obtained for charged and neutral currents and is of particular relevance for our purpose of clarifying the problems connected with \( J^P = 1^+ \) mesons. These reactions have also a possible Deck like background see Fig. (26) - which contributes. Figure (26) shows the two possible components, resonance and Deck to interpret the results of these reactions. Here we think that it is particularly necessary work in the scheme \( \Pi \) mentioned at the

![Diagram](image)

Fig.26 - Possible resonant (a) and Deck (b) Diagrams for \( R\gamma \rightarrow L'(\pi N)N' \) reactions.
beginning of section 3.1. To investigate the system $A_1(\pm pn)$ in the semi-leptonic decay mode $\tau^+A_1\nu_\tau\to(\rho n)v_\tau$ different techniques have been employed (some of which well known $^{66}$), in an attempt to clarify the problems of purely hadronic reactions. The matrix element$^{66b}$ for this decay can be written (Fig. 27),

$$M = \mu_A\mu_\nu$$

where $\mu_A$ and $\mu_\nu$ represent the semi-leptonic and hadronic vertex shown in Fig. (27) and have the following form,

$$\mu_A = \bar{u}_\tau \gamma_\mu(1 - \gamma_5)u_\tau$$

and

$$\mu_\nu = \bar{v}_\nu$$

\[ \text{Fig. 27 - } \tau \text{ decay into } A_1\nu_\tau \to (\rho n)v_\tau \]

\[ H_\mu = \begin{bmatrix} e_\mu(\rho) - Q_\mu \frac{e_\mu(\rho) - p(\pi)}{M^2} \\ -Q_\mu \frac{e_\mu(\rho) - p(\pi)}{M^2} \end{bmatrix} F_1(M^2) + \]

where $M^2 = Q^2$. $F_1$ and $F_2$ are the form factors used in many theoretical speculations$^{66b,d}$. We can see in Fig. (28,29) that the approach of ref. 66b applied to these reactions is compatible with those of Diffractive Dissociation. Some solutions$^{66b,c}$ are compared with the data.

Reactions of another type that begin now to produce interesting results in the context of the problems we are discussing are those with
Fig. 28 - The \((3\pi)_{\nu\nu}\) spectrum. (a) The continuum and dashed curves are theoretically obtained from ref. 67c in comparison with the data from ref. 67a and (b) with the data from ref. 67b.
Fig. 29 - The $(\rho \pi^\pm)$ mass spectrum. (a) The data from ref. 67a and (b) from ref. 67d. ihe theoretical curves are obtained from ref. 65c in the Current Algebra context.
polarized target. Recently measurements of a $3\pi$ system diffractively produced in the reaction $\pi^- p \rightarrow \pi^- + \pi^+ \pi^- p$ have been made at $17 \text{ (GeV/c)}$ confirming the results obtained previously in other experiments. The axial vector $A_1$ meson is observed in this reaction with a mass $m_{A_1} = 1.2 - 1.3 \text{ (GeV)}$ and a width $\Gamma_{A_1} \approx 300 \text{ (MeV)}$. The interested reader will find more information in ref.68. Here we just call attention to these "new" reactions and hope that new results will emerge from $pp$ annihilation at intermediary energy.

4. CONCLUSIONS

We summarize now the main points discussed above. We have shown that the study of $J^P = 1^+$ objects is interesting both from the experimental as well as from the theoretical point of view since they have been a good "theoretical laboratory" for the development of many issues of hadron spectroscopy. Many experiments that were realized to search for these mesons have yielded valuable information about hadron interactions in general. Many problems were solved in the last ten years and the improvement of the experiments with the increase of statistics (number of events) and accuracy of the techniques used is quite evident. From a strictly phenomenological point of view, some of these $J^P = 1^+$ states are now well established as good resonances ($B(1235), D(1285)$); others can be considered as almost definitively established ($A_1(1.1), G_A(1.24-1.29), G_B(1.3-1.4), D^*(1420)$; while $H(1.1)$ and the $H'$ rest today without a clear determination. From the experimental point of view it still remains necessary to make compatible the several existing results. For example for a set of experiments the $A_1$ mass is $\approx 1.1 \text{ (GeV)}$ and for another $1.2-1.3 \text{ (GeV)}$. The results obtained from charge exchange reactions concerning $A_1^0$ production gave good determination in ref.10b whereas no evidence was found in ref.43. Thus, we would need a good compatible set of experimental results which could be the principal tool to exclude some theoretical models. In this sense, the recent results about $A_1^0$ production in charge exchange reaction$^{10b}$ is to be considered an important experimental step. Next, we would need a deeper theoretical study of the several existing models to state all the problems, and compare the virtues and failures of the different approaches. A work that
would explain clearly all these experimental and theoretical problems
would surely be well come to give a final answer to all the contradictions pointed out in this paper.

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