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Proton-Neutron Mass Difference with Off-Shell Form Factors

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The proton-neutron mass difference is calculated in a Born -like approximation but taking into account off-mass-shell variations of the form factors. In this way, a mass difference with the right sign is obtained.

A diferença de massa entre o proton e o neutron é calculada na aproximação de Born, tomando em conta porém as variações dos fatores de forma fora da camada de massa. É obtida assim uma diferença de massa com o sinal correto.

"The proton-neutron mass difference, probably the oldest puzzle in hadron physics, has challenged and frustrated generations of theorists. The solution is nowhere in sight". These are the opening sentences of the review by A. Zee[1] on the proton-neutron mass difference ($\Delta = Mp - Mn$) problem. I have borrowed Zee's words to set the stage for the present work where a new attempt is made at solving the puzzle.

Since the time people started worrying about Δ (which is, experimentally, - 1.29 MeV) it was clear that the Coulomb interaction would make it positive. A possible way out was suggested by Feynman and Speisman (FS)[2] who pointed out that by taking the negative anomalous magnetic contribution into account, the right sign might emerge if the integrals are cut off at a suitable energy. Then, Cini, Ferrari and Gatto (CFG)[3] showed that the nucleon form factors made the artificial cut-offs of FS unnecessary, but, at the same time, lead to a Δ with the wrong sign.

All the above was done in a purely electromagnetic Born approximation. With the failures, however, other types of ideas started to come in: feedback mechanisrns, tadpoles, Regge poles, fixed poles, relation with electroproduction (via Cottingham[4] formula) and other approaches described in Zee's review. More recent attempts invoke the unified electroweak (Salam-Weinberg-Glashow) theory. In most of these approaches the tendency is to shift the responsability for producing a negative A from the low to the high energy region.

In the present work I go back to FS and CFG with two differences in relation to the latter paper. First, I use for the nucleon electromagnetic current a parametrization naturally suited for the Sachs form factors which are taken to be of the usual dipole type when both nucleon legs are on-mass-shell. Second, and more important, the forrn factors are allowed to vary when oneof the nucleons is off-mass--shell. With a plausible ansatz as to what the off-mass-shell behaviour of the form factors is, and after some tedious calculation, a A with the right sign and, possibly, with the right magnitude is obtained.

Let me start with the nucleon electromagnetic current which I choose to write as

$$(p_0 p_0'/M) \langle p'|J_\mu(0)|p\rangle = \bar{u}(p')\Gamma_\mu(k)u(p)$$

$$= e(1-k^2/4M^2)^{-1}\ \bar{u}(p')\,[M^{-1}G_E(k^2)P_\mu -$$

$$- (2M)^{-2}G_M(k^2)(\gamma_\mu \not{P}\not{k} - \not{k}\not{P}\gamma_\mu)]u(p)\ , \qquad (1)$$

where $P_\mu = \frac{1}{2}(p_\mu+p_\mu')$, $k_\mu = p_\mu - p'_\mu$ and G_E and G_M are the electric and magnetic Sachs form factors. For the metric etc., I use the notation of Bjorken and Drell[5].

The parametrization (1) for the current, which I will use in the following, is, of course, not new although it had not **been** much employed. One of its advantages is that it is specially suited for the Sachs form factors. Another, is that it provides convenient convergente factors for integrals that will appear later.

In lowest order, the electromagnetic self-mass of a fermion can be written in terms of the forward Compton scattering amplitude with an off-mass-shell photon $[T_{\mu\nu}(p,k)]$ as

$$\delta M = e^2 \int \frac{d^4k}{(2\pi)^4} \frac{(g^{\mu\nu} - k^\mu k^\nu/k^2)}{k^2 + i\varepsilon} T_{\mu\nu}(p,k) \quad , \tag{2}$$

with

$$\frac{M^2}{p_0} \bar{u}(p) T_{\mu\nu}(p,k) u(p) = \\ = \int \frac{d^4x}{(2\pi)^3} e^{-ikx} \langle p|T\{\dot{J}_\mu(x), \dot{J}_\nu(0)\}|p\rangle . \tag{3}$$

In a standard fashion, we can arrive at a Low equation for $T_{\mu\nu}$ which, in the Born approximation, is of the form

$$\bar{u}(p) T^B_{\mu\nu} u(p) = \\ = i \sum_{s'} \int \frac{d^3p' \, \delta^3(p' - p - k) \langle p|\dot{J}_\mu(0)|p', s'\rangle \langle p', s'|\dot{J}_\nu(0)|p\rangle \theta(k_0)}{p_0' - p_0 - k_0 \quad i\varepsilon} + \begin{bmatrix} \mu \leftrightarrow \nu \\ k \rightarrow -k \end{bmatrix} \tag{4}$$

where p' is the momentum and s' the spin of the intermediate nucleon.

Using parametrization (1) for the matrix elements of the currents appearing in Eq. (4) and putting the $T_{\mu\nu}$ thus obtained in Eq. (2) leads us, after some γ matrices gymnastics, to the formula

$$\delta M^N = \frac{-i8e^2}{M} \int \frac{d^4k}{(2\pi)^4} \frac{[2M^2 k^{-2}\, {}^N G_E^2(k^2) + {}^N G_M^2(k^2)] [M^2 k^2 - (pk)^2] (2M^2 - kp)}{(k^2 + i\varepsilon)(k^2 - 2pk + i\varepsilon)(k^2 - 4M^2)^2} \tag{5}$$

where N can be either p (proton) or n (neutron). We would, of course, have arrived at this same expression, had we written the formula for the lowest order Feynman diagram for the electromagnetic mass of an elementary fermion and then changed the electromagnetic vertex according to $\bar{u}\gamma_\mu u \rightarrow \bar{u}\Gamma_\mu u$, with Γ_μ as in Eq. (1).

One disturbing feature of Eq. (5) is the double pole at $k^2 = 4M^2$. At the pair production threshold that pole is killed by the factors $[M^2k^2-(pk)^2]$ and $(2M^2-pk)$ which vanish at'that point. In order to properly assess what happens at $k^2 = 4M^2$ in general, one should go beyond the Born approximation and take also into account intermediate states with one nucleon plus one pair. In this paper I will assume that the nucleon form factors provide sufficient damping to guarantee that whatever happens at $k^2 \sim 4M^2$ has a negligible effect on the final result.

The integrals in Eq. (5) can be performed once a suitable $i\varepsilon$ terrn is added to the (k^2-4M^2) factor. This prescription is in accord with the proper location of the poles as dictated by the causality principle. From now on it will be anderstood that in integralsover the Minkowski region all denominators contain an $i\varepsilon$ term even when it is not explicitly written. With all the poles properly located onecan perform a Cottingham rotation[4] on Eq. (5) and obtain

$$\delta M = \frac{4e^2M^2}{\pi^3}\int_0^\infty \frac{dQ^2}{Q^2}\int_0^{MQ} \frac{d\omega(Q^2 - \omega^2/M^2)^{5/2}\,[2M^2G_E^2 - Q^2G_M^2]}{Q^2(Q^2+4M^2)^2(Q^4+4\omega^2)}\,, \tag{6}$$

where $Q^2 = -k^2$ and $\omega = -i\nu$ with $\nu = -pk$.

G_E and G_M appeering in Eq. (5) will be taken to be of the standard dipole form

$$\frac{{}^nG_M(k^2)}{\mu_n} = \frac{{}^pG_E(k^2)}{\mu_p} = {}^pG_E(k^2) = G_D(k^2) = \left[(k^2/\beta M^2) - 1\right]^{-2}\,, \tag{7}$$

where μ_p and μ_n are the proton and neutron magnetic moments and the mass $m^2=\beta M^2$ has to be taken from experiment.

I calculated the proton and neutron mass shifts from Eq. (5) with the form factors (7) and obtained, as everybody else, the wrong answer: proton heavier than neutron. So, the question is: the simple Born approximation is not sufficient; how can we get something better?

In dispersion relation language, the higher intermediate states contribute to a sidewise dispersion relation of the form factors[7] (dispersion in the mass of one of the nucleon legs). In Feynman diagram language, the intermediate nucleon is off-mass-shell with an effective mass

$$p'^2 = W^2 = M^2 + k^2 - 2pk \ . \tag{8}$$

In either way of looking at the problem the conclusion is the same: the form factors should be taken with one nucleon leg off-mass-shell, i.e., $G_D(k^2;W^2)$.

It would be interesting to study the off-mass-shell form factors in a sidewise dispersion in order to see their connection with the electroproduction amplitudes. In the meantime I propose the following ansatz:

$$G_D(k^2,W^2) = \left[k^2/\beta W^2) - 1\right]^{-2} = \left[\frac{k^2}{\beta(M^2+k^2-2pk)} - 1\right]^{-2} \ . \tag{9}$$

Notice that in the Bjorken limit $(-k^2) \to \infty$, $(-pk) = \nu \to \infty$, with $x = (k^2/2pk)$, G_D scales as

$$G_D \to (1 - x)^2 \left[1 + x(\beta^{-1} - 1)\right]^{-1} \ . \tag{10}$$

Substituting in Eq. (5) the on-mass-shell form factors by form factors varying like (9) the resulting mass shift can be split according to

$$\delta M^N = \delta M_c^N + \delta M_v^N \tag{11}$$

where δM_c^N is the mass shift one would get from Eq.(5) if the form factors were constant, namely

$$\delta M_c^N = -\frac{i8e^2}{M}\int\frac{d^4k}{(2\pi)^4}\frac{\left[2M^2k^{-2}q_N^2+\mu_N^2\right]\left[M^2k^2-(pk)^2\right](2M^2-kp)}{k^2(k^2-2pk)(k^2-4M^2)^2}, \tag{12}$$

where q_N ir the nucleon charge in units of e. δM_V^N results from the variation of the form factors and is given by

$$\delta M_v^N = -i\frac{4}{3}\frac{e^2}{M}\frac{\partial^3}{\partial\beta^3}\frac{1}{\beta(1-\beta)}\int\frac{d^4k}{(2\pi)^4}\frac{\left[2M^2k^{-2}q_N^2+\mu_N^2\right]\left[M^2k^2-(pk)^2\right](2M^2-kp)}{(k^2-4M^2)^2(k^2-2pk)\left[k^2-\frac{\beta}{1-\beta}(M^2-2pk)\right]}. \tag{13}$$

δM_c^N can be obtained without much difficulty; it is

$$\delta M_c^N = \frac{\alpha M}{8\pi}\left[q_N^2(8\ \ell n\ 2+1)-\mu_N^2(7-4\ \ell n\ 2)\right]. \tag{14}$$

This part alone contributes to the *p-n* mass difference with

$$\begin{aligned}\Delta_c &= \delta M_c^p - \delta M_c^n = \frac{\alpha M}{8\pi}\left[8\ \ell n\ 2+1-(\mu_p^2-\mu_n^2)(7-4\ \ell n\ 2)\right]\\ &= -2.99\ \text{MeV}\end{aligned} \tag{15}$$

Unfortunately, δM_v^N is not that simple. It has the form

$$\delta M_v^N = -\frac{\alpha M}{2\pi}\beta^4\frac{\partial^3}{\partial\beta^3}\left[\left(\mu_N^2+\frac{1}{2}q_N^2\right)A-\frac{1}{2}q_N^2B\right], \tag{16}$$

where

$$A = \frac{1}{(4-\beta)}\left[\frac{1}{6}+\frac{\beta(\ell n\beta-1)}{12}+\frac{(7/9)+(4/3)\ell n 2}{\beta}+\frac{(1/9)-(1/8)(\beta-2\ \ell n\ \beta)}{(4-\beta)}\right.$$
$$\left.-\frac{8\ \ell n\ 2-(14/9)}{\beta(4-\beta)}\right]+\frac{1}{12}\int_0^1 dx\ \ell n\left[x^2+\beta(1-x)\right], \tag{17}$$

and

$$B = \frac{167}{288\beta} + \frac{2\,\ell n\, 2-(167/288)}{\beta(4-\beta)} + \frac{2-\ell n\,\beta}{(4-\beta)} - \frac{\ell n\,\beta}{12} - \frac{(4-\beta)\,\ell n\,\beta}{24} -$$

$$- \frac{(4-\beta)}{24} \int_0^1 \frac{dx}{x^2 + \beta(1-x)} \quad , \qquad (18)$$

After taking the derivatives, I calculated everything analytically except for integrals like the one in Eq.(18) which were done numerically in a desk calculator. The result depends on $\beta=(m^2/M^2)$ in Eq. (7). Data up to about $(-k^2)$ ~5 GeV^2 used to give $m^2=0.71$ (GeV^2)[8] corresponding to a $\beta\approx 0.8$ with yields

$$A_V = 2.01 \text{ MeV} \rightarrow A = A_C + A_V = -0.98 \text{ MeV} \,, \qquad (19)$$

to be compared to the experimental value of 1.29 MeV.

The calculation reported here was done in the Minkowski region using Eq. (5) with form factors as in Eq. (7). The reason for doing things that way, instead of working with the Cottingham rotated Eq. (6), is that I could do most of the integrals analytically.

In the Cottingham region, on the other hand, delicate points like having to deal with time-like form factors and the behaviour at the pair production threshold of the integrand in Eq.(5), are avoided. Presently, I am doing the calculation of the mass difference in the Cottingham region starting with Eq. (6) and properly rotated form factors, and expect to be able to report on that work shortly.

Meanwhile, I hope to have shown in this paper that by taking their off-mass-shell variations into account, the form factors cut off the integrals less abruptly allowing for the negative magnetic energy to overtake the positive Coulomb energy. The net result is a negative proton-neutron mass'difference. If that is correct, the oldest puzzle in hadron physics would have ceased to be such.

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