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Deuteron-, Triton- and Alpha- Clusters in Nuclear Matter+

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We study with a simple model how deuteron-, triton- and α-clusters behave concerning their cluster structure identities during the scattering process and just after reaching nuclear matter of finite size.

Se estuda, através de um modelo simples, como aglomerados deuteron-, triton e alfa comportam-se, com respeito às suas estruturas de aglomerados durante o processo de espalhamento e logo após atingirem uma matéria nuclear finita.

1. INTRODUCTION

It is interesting and instructive to consider what will arise with a nuclear cluster when it is put in infinite nuclear matter with a certain density. The presence of nuclear matter will affect the cluster through the Pauli exclusion effect and the potential field acting on it. This problem was first investigated by one of us (H. B.) and A. Kuriyama for deuteron cluster in connection with the transition between the deuteron-like super state and the deuteron-like boson gas

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state[1]. This deutercn cluster problem has been subsequently studied in more detail[2].

In this paper we extend our consideration to triton and alpha in nuclear matter. In the sense of local density approximation, this study could tell us hcw clustering can grow on a nuclear surface of finite nuclei and also how a cluster can keep Its identity during the course of scattering process.

2. MODEL AND EQUATIONS

i) Deuteron

A deuteron in nuclear matter is described essèntially by the Bethe-Goldstone equation[1,2]. We use a simple Yamaguchi form[3], separable in momentum space, as an s-state nucleon-nucleon interaction

$$\langle k|V|k'\rangle = - G\, g(k)\, g(k') \; , \qquad g(k) = 1/(k^2 + \mu^2) \; . \tag{2.1}$$

Calculations have been done[2,4] by using more realistic interactions including repulsive core and strong tensor component. Although thestrong tensor component gives rise to an interesting polarization effect on deuteron[2], in average it gives a quite similar result to that obtained frorn the simple interaction like Eq. (2.1). The simple force, Eq.(2.1), will be used in the following discussion. With the interaction (2.1), the BG-equation for deuteron with energy E is reduced to the dispersion equation

$$\int_0^\infty \frac{g(k)^2\, Q_{12}(k,K;k_F)}{E - \frac{\hbar^2 k^2}{M^*}}\, k^2\, dk = -\frac{1}{G} \tag{2.2}$$

where the effective mass M* takes care of the effect of the potential field generated by nuclear matter and Q_{12} is the angle-averaged Pauli operator defined by

$$Q_{12}(k,K;k_F) = \begin{cases} 1 \quad , & |\frac{K}{2} - k| \geq k_F \quad , \\ 0 \quad , & k^2 + \frac{K^2}{4} \leq k_F \quad , \\ (k^2 + \frac{K^2}{4} - k_F^2)/ \quad , & \text{otherwise} \quad , \end{cases} \qquad (2.3)$$

where k, K and k_F are respectively the relative momentum, the center-of-mass momentum and the Fermi momentum of nuclear matter. The Fermi momentum k_F is related to the density ρ by $\rho = \frac{2}{3\pi^2} k_F^3$. The binding energy E of the deuteron cluster is solved as a function of k_F, K and M^*/M.

ii) Triton and Alpha

Now for triton and alpha in nuclear matter, it is not intended to solve the problem exactly, since the present calculation is rather directed at getting a physical picture. We set up a trial wave function in momentum space

$$\Psi_A = \hat{Q} \left[\exp\left\{ - \frac{b^2}{2} \sum_{i=1}^{A} \vec{k}_i^2 \right\} . \exp\left\{ - \frac{b^2}{2A} (\vec{K} - \vec{K}_g)^2 \right\} \right] , \qquad (2.4)$$

(A = 3 for t and A = 4 for a)

where the Pauli-projection operator $\hat{Q}$ is given by

$$\hat{Q} = \prod_{i=1}^{A} \hat{Q}_1(\vec{k}_i ; k_F) \quad , \qquad (2.5)$$

$$\hat{Q}_1(\vec{k}_i ; k_F) = \begin{cases} 1 \ , & |\vec{k}_i| > k_F \quad , \\ 0 \ , & |\vec{k}_i| \leq k_F \quad . \end{cases} \qquad (2.6)$$

In Eq. (2.4), the first expanential represents the internal wave function of cluster with $\vec{k}_i$, $\vec{k}_i \equiv -\vec{K}/A$, $\vec{K} \equiv \Sigma \vec{k}_i$ and the second exponential represents the center-of-mass wave function of the cluster which is moving with momentum $\vec{K}_g$ an average. The size parameter of cluster b is treated as a variational parameter.

The internal energy of cluster in nuclear matter will be given by the expectation value of the Hamiltonian

$$H = \sum_{i=1}^{A} T_i - T_{CM} + \sum_{i>j=1}^{A} V_{ij} \tag{2.7}$$

with respect to Ψ_A of Eq. (2.4). Some algebra lead to

$$\begin{aligned} & E_A(b,\ k_F,\ K_g,\ M^*/M) \\ & = \frac{\hbar^2}{2M^*}(A-1)\left\{ \frac{t_A}{n_A} - \left(\frac{t_A^{CM}}{n_A}\right)^2 \right\} + \frac{A(A-1)}{2}\ \frac{v_A}{n_A^{av}}\ , \end{aligned} \tag{2.8}$$

where

$$n_A = \int_{k_F}^{\infty} \exp\ \{-b^2(k^2+K_g/A^2)\}\ \tilde{J}_0(2b^2K_g k/A)k^2dk\ , \tag{2.9}$$

$$t_A = \int_{k_F}^{\infty} \exp\ \{-b^2(k^2+K_g^2/A^2)\}\ \tilde{J}_0(2b^2K_g k/A)k^4\,dk\ , \tag{2.10}$$

$$t_A^{CM} = \int_{k_F}^{\infty} \exp\ \{-b^2(k^2+K_g^2/A^2)\}\ \tilde{J}_0(2b^2K_g k/A)k^3dk\ , \tag{2.11}$$

$$\begin{aligned} v_A = -G_A \int_0^{\infty} dK\ \exp\{-b^2(K^2+4K_g^2/A^2)/2\}\tilde{J}_0(2b^2K_g K/A)K^2 \\ \times \left[\int_0^{\infty} \exp\{-b^2k^2\} g_A(k) Q_{12}(k,K;k_F)k^2dk \right]^2\ , \end{aligned} \tag{2.12}$$

$$\begin{aligned} n_A^{av} = \int_0^{\infty} dK\ \exp\{-b^2(K^2+4K_g^2/A^2)/2\}\tilde{J}_0(2b^2K_g K/A)K^2 \\ \times \int_0^{\infty} \exp\{-\ b^2k^2\}Q_{12}(k,K;k_F)k^2dk\ , \end{aligned} \tag{2.13}$$

with $\tilde{J}_0(Z)$ and $\tilde{J}_1(Z)$ being spherical Bessel functions with imaginary argument. In Eqs. (2.12) and (2.13) we have used the angle-average approximation for the Pauli operator. The mass number A is put on the interaction, G_A and $g_A(k)$, because the interaction parameters g and μ of Eq. (2.1) need be adjusted to reproduce the energy and radius of free triton (A=3) and alpha (A=4). It is known that the effective interaction, say G-matrix, changes quite strongly depending on massnumber in very light nuclei, because of the shielding effect of the strong tensor force[5]. Thus the choice of different interaction parameters in difference nuclei is natural in such a simple effective interaction as Eq. (2.1).

3. RESULTS AND DISCUSSION

Calculated binding energies of the deuteron-, triton- and alpha-clusters in nuclear matter are shown respectively in Figs. 1, 2 and 3 with respect to the Fermi momentum of nuclear matter k_F. A value of K_g, center-of-mass momentum carried by the cluster, is attached to each curve. The nucleon effective mass M^* has been put equal to M throughout.

Every curve starts at k_F=0 with the binding energy of a free d, t or a. With increasing k_F the binding of a cluster becomes weaker and at a critical value k_F^c the cluster ceases to be bound. The critical k_F value for a cluster "at rest" (K_g=0) is seen to be 0.25, 0.40 and 0.55 fm^{-1} for d, t and a, respectively. The Pauli exclusion principle which nuclear matter imposes on the clusters is responsible for all the above behaviors. More explicitly stated, momentum components occupied by nucleons in nuclear matter cannot be available for a cluster to construct its bound wave function, as is explicit in the equations given in 2. We thus naturally understand that as the nuclear matter density increases, the longest-surviving cluster is the a-cluster, and then t and d. The a-cluster is most compactly bound in configuration space and therefore most extended in momentum space, thus being least subject to the momentum cutoff due to the Pauli principle. It is noted that if we take a more appropriate tail behavior for t and α-cluster wave functions, the critical k_F values will becomes somewhat larger than those obtained here by using the simple Gaussian form. In this sense the result for the d-cluster is most reliable.

The above results for K_g=0 can be interpreted in the local. density approximation. A cluster with bound state structure can appear on the nuclear surface only in the reqion with density lower than a critical value. The critical density ρ_c is estimated by $\rho_c=(k_F^c/k_F^N)^3\rho_N$ with k_F^N and ρ_N being the normal Fermi momentum and density ($k_F^N \simeq 1.4$ fm^{-1}). We have thus

$$\rho_c/\rho_N \sim \begin{cases} 1/170 \rightarrow d \\ 1/43 \rightarrow t \\ 1/16 \rightarrow \alpha \end{cases} \quad (3.1)$$

With a Fermi-type density distribution in mind, ρ_c given in Eq. (3.1), corresponds to a nucleus of a quite sharp edge. Even in such a low density the Pauli principie can inhibit a subgroup of nucleons from constituting a bound state cluster.

The situation changes, if the clusters run through nuclear matter with a non-zero momentum K_g. With increasing K_g the clusterscan remain as bound states more easily and the critical Fermi momentum k_F^c increases, as is evident in Figs. 1-3. The value of k_F^c reaches the normal value $k_F^N = 1.4$ fm^{-1} at $K_g \simeq 4.0$, 4.5 and 5.0 fm^{-1} for d-, t- and a-clusters, respectively. Recall that K_g is the center-of-mass momentum of a cluster, hence its share per nucleon is K_g/A with $A=2$, 3 or 4. We thus see that the d-, t- and a-cluster can remain internally bound even enough inside a nucleus, if they run with the momentum per nucleon of 2.9, 1.5 and 1.25 fm^{-1} respectively. For a cluster running

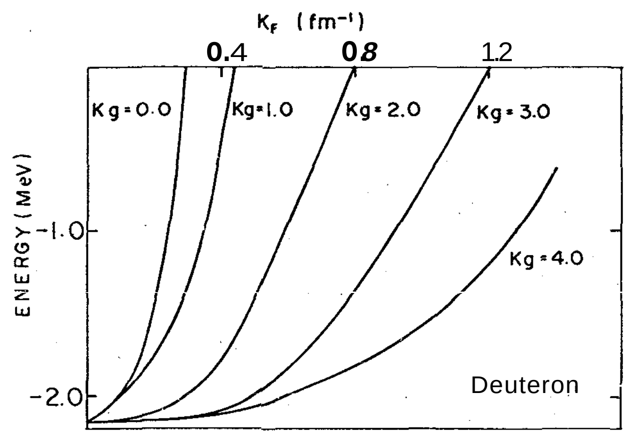


Fig. 1 - The binding energy E of the deutercn cluster versus the Fermi momentum of nuclear matter k for different values of the center-of--mass momentum K (fm).

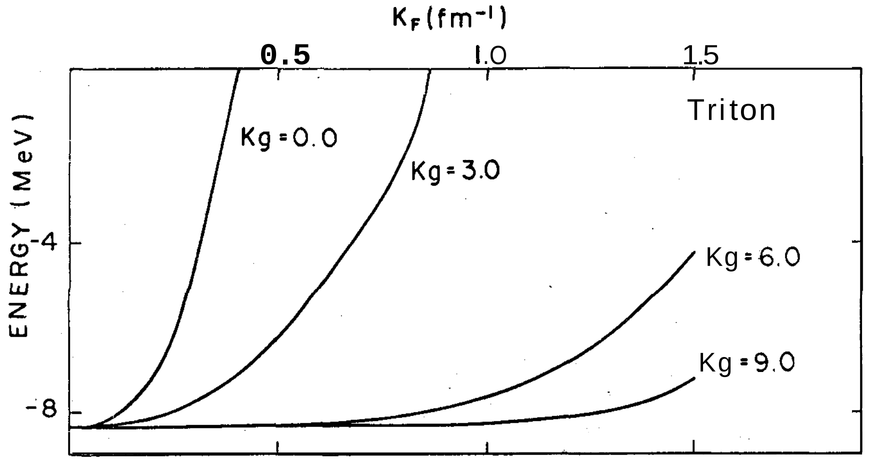


Fig.2 - The binding energy E of the triton cluster versus k_F.

relative to nuclear matter, the Pauli principle can only cut off an unimportant part of the momentum component needed to construct the internally bound structure.

Let us now consider a cluster scattered by a nucleus. In this case the cluster feels the potential field generated by the nucleus and gains a certain amount of kinetic energy in addition to its incident energy E_0. In the sense of the local density approximation, when the cluster passes through a portion of the nucleus with density ρ, each nucleon constituting the cluster gains a kinetic energy of $(\hbar^2 k_F/2M + \Delta)$, where k_F corresponds to ρ and h is the separation energy of the least bound nucleon, ∿8 MeV. Thus the effective center-of-mass momentum K_g of the cluster reads

$$K_g \simeq \sqrt{\frac{2AM}{\hbar^2}\,\{E_0 + A\left(\frac{\hbar^2 k_F^2}{2M}\right) + \Delta)\}} \quad . \tag{3.2}$$

For the very low incident energy $(E_0 \simeq 0)$, Eq. 0.2) leads to $K_g \simeq Ak_F$ by neglecting Δ. Regarding Figs. 1–3 as defining the binding energy E as a functions of k_F and K_g, we can trace the cluster passing though a nucleus by following the points given by $E(k_F, K_g{=}Ak_F)$. We find that an a-particle with zero incident energy maintains its boundness all the way through the nucleus, i.e. $E_\alpha(k_F, K_g{=}4k_F)$ is always negative for $k_F \leqslant k_F^N$. For d and t, $E_d(k_F, K_g{=}2k_F)$ and $E_t(k_F, K_g{=}3k_F)$ vanish at

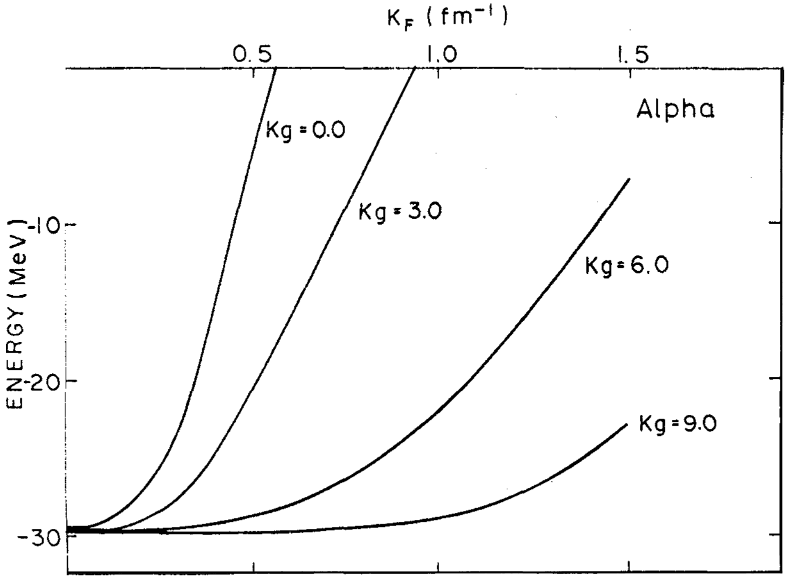


Fig.3 - The binding energy E of the alpha cluster versus k_F

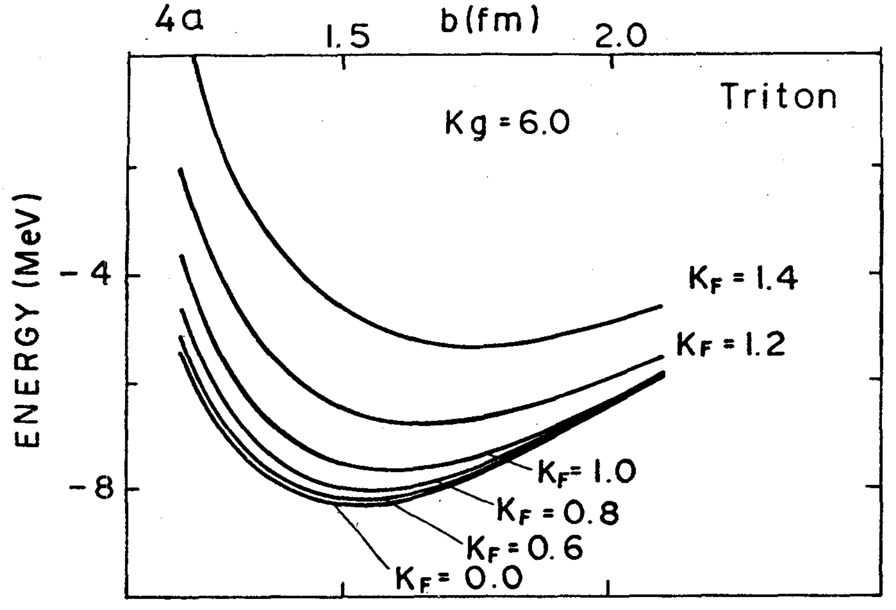
k_F~0.4 and ∿0.8 fm^{-1}, respectively. In order for a deuteron and a triton to be able to pass the highest density part of the nucleus without losing their boundness, their incident energy E_0 must be high enough to satisfy

$$E_A(k_F^N, K_g^A(E_0, k_F^N) < 0 \qquad (3.3)$$

with $K_g^A(E_0, k_F)$ being just Eq.(3.2). We find such E, values to be ∿ 80 MeV for d and ∿ 20 MeV for t.

Finally in Figs. 4 and 5 the binding energies of the t- and a-clusters are drawn with respect to the variational parameter b specifying the harmonic oscillator size. (See Eq.(2.4)). Energy minima shift to larger b side with increasing k_F. This feature is consistent with weaker binding energies and hence less compact wave functions. Note however that this b does not literally characterize the extention of the wave function, but Ψ_A of Eq.(2.4) should actually be much more extended because of the Pauli operator $\hat{Q}$.

The deuteron-, triton- and alpha-clusters in nuclear matter have been studied with a rather simple and crude model. We are aware



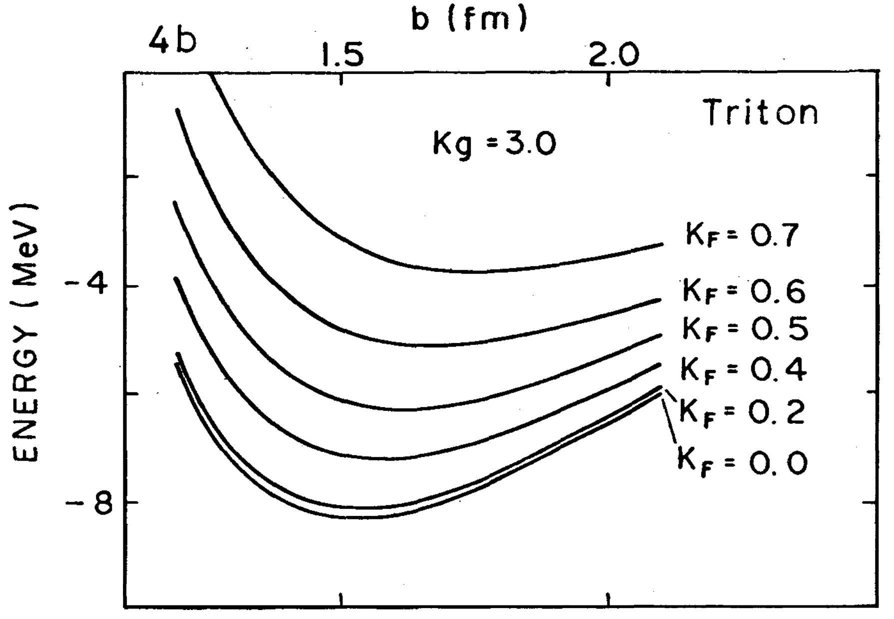


Fig.4 - The binding energy E of the triton cluster versus the size parameter of cluster b, for center-of-mass momentum of triton cluster K_g equal to $6fm^{-1}$ (a) and $3fm^{-1}$ (b).

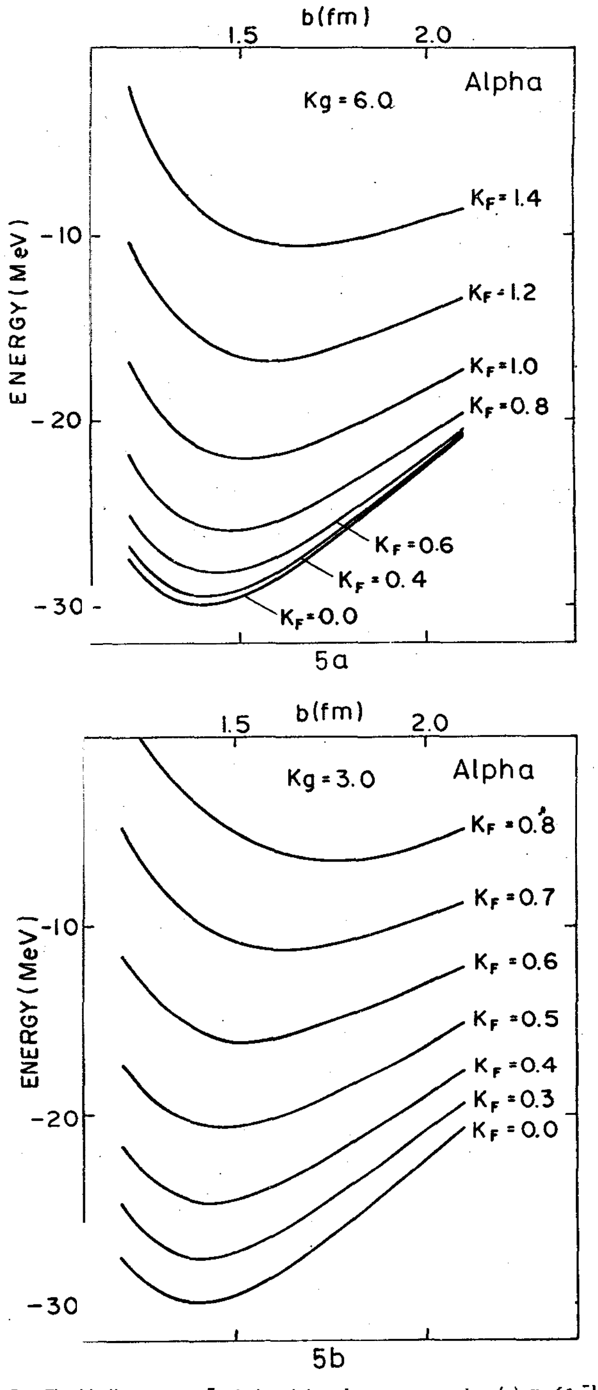


Fig.5 - The binding energy E of the alpha cluster versus b. (a) $K_g = 6fm^{-1}$ and (b) $K = 3fm^{-1}$.

that this simple minded approach soniehow did not take into account properly various importanf effects which play an important role in this are of physics. For instsnce, the separable nucleon-nucleon interaction (eq.2.1) used in this work is not so realistic since interactions including repulsive core somehow affect the results. By also making the approximation $M^*=M$, we did not take into considerations carefully the effect of the average potential field generated by the nuclear matter. However it is still a difficult task to treat simultaneously all these effects. Our idea was basically to treat the problem in a more introductory way, hoping that something instructive could be achieved. Even with our approximations the results obtained are not so far fran reality.

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REFERENCES

1. H.Bando and A. Kuriyama, Pi-og.Theor.Phys.Suppl.Extra No. (1968), 69.
2. B.L.Gambhir and J.J.Griffin, Phys.Rev. C5, 1856 (1972); C7, 590 (1973). A.A. toannides and R.C.Johnson, Phys.Rev. C17, 1331 (1978).
3. Y.Yamaguchi, Phys.Rev. 95, 1628 (1954).
4. H.Bando and A.Kuriyama, Soryushiron-Kenkyu, 36, 362 (1967).
5. Y.Akaishi and S.Nagata, Prog.Theor.Phys. 48, 133 (1972). Y.Akaishi, H.Bando and S.Nagata, Prog.Theor.Phys. Supple. No. 52, 339 (1972).