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Vertex Function Considerations in the Scattering of α-Particles by Light Nuclei +

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Vertex considerations, such as spectroscopic coefficients and relative motion wavefunctions, are employed in the calculation of the vertex function necessary for the description of the elastic scattering of a-particles by light nuclei. It is found that the results are more sensitive to the spectroscopic coefficients rather than to any class of realistíc wave functions that may be chosen.

Considerações de vértice, tais como coeficientes espectroscópicos e funções de onda relativa, são usadas no cálculo das funções vértice necessárias para a descrição de espalhamento elástico de partículas a por núcleos leves. Foi achado que os resultados são mais sensíveis aos fatores espectroscópicos que a qualquer classe de funções de onda realísticas que se possa escolher.

1. INTRODUCTION

Not too long ago Santana, Coelho and Das[1] showed that pole diagrams with heavy particle transfer mechanism are essential to explain the anomalous backscattering of a particles by light nuclei, while triangle diagrams (with the heavy particle transfer mechanism) are the dominant ones to explain the forward elastic scattering for the same reactions. The same formalism[2] was recently used to explain a similar anoma-

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lous backscattering behavior in the elastic reactions $a + Li^6$ and $\alpha + Li^7$ at low energies. Other references[3] also treated similar problems. The choice of the main Feynman diagrams in a particular reaction comes basically from the analysis of the diagram singularities which should lie somewhat nearer the physical region[6].

In this work we are mostly interested in analysing the sensitivity of the vertex functions, to the appropriate spectroscopic coefficients and the relative importance of reasonable different classes of wavefunctions for the relative motion in each vertex. These considerations somehow effect the theoretical differential cross sections, which could be compared with the experimental ones. For definiteness we consider here the forward elastic scattering of a particles on C^{12} (treated as 3α) and O^{16} (as 4α) at energies between 30 and 41 MeV (direct mechanism can be considered in this energy interval[1]).

In section 2 we summarize the forrnalism, and in section 3 we give the results and conclusions.

2. FORMALISM

The Feynman diagram shown in fig. (1) represents the main contribution to the forward elastic scattering of the reactions $a + C^{12}$ and $a + O^{16}$. Applying Feynman rules[1,6] to the diagram of fig.1 we obtain T-matrix amplitude (one should notice that PWBA is implied):

$$T = \sum_{\nu JM} n \int d\vec{k}'' \; v_f^*(\vec{k}''-\vec{p}') \; \langle \vec{k}' - \tfrac{1}{2}\vec{k}'' | V_{\alpha\alpha}(r) | \vec{k} - \tfrac{1}{2}\vec{k}'' \rangle$$
$$\times v_i(\vec{k}''-\vec{p}) \left[\frac{\hbar^2}{2\mu}(k''-p')^2 + E_x\right]^{-1} \left[\frac{\hbar^2}{2\mu}(\vec{k}''-\vec{p})^2 + E_x\right]^{-1} \qquad (1)$$

where v_f and v_i are the vertex functions, describing the virtualdecays $A \to B + \alpha$ (and the reverse ones), defines by[1]

$$v_i(\vec{Q},\nu,JM) = -(2\pi)^{-3/2} C_{\nu J} \langle JJ,M,-M|00\rangle \times \left[\frac{\hbar^2}{2\mu} Q^2 + E_x(\nu,J)\right] \int e^{-i\vec{Q}.\vec{r}'} \phi_{\nu J,-M}(\vec{r}')d\vec{r}' \quad (2)$$

The factors $C_{\nu,I}$ are the usual spectroscopic coefficients[1], $\phi_{\nu JM}(\vec{r})$ are the relative motion wavefunctions for the α-B states, and μ is the reduced mass for α-B. In formula (1), n takes che values 3 or 4 for C^{12} and O^{16} nuclei, respectively, and $E_x(\nu,J)$ represents the separation snergy of $A \to o + B(\nu,JM)$. One should notice that $\vec{p} = (B/A)\vec{k}$ and $\vec{p}' = (B/A)\vec{k}'$. As shown in ref.1 the Fourier transform of eq.(1) will give in the configuration space, the effective interaction potential operator, $V(r)$, between the a-particle and A. Coulomb effects between α and Aaretaken into considerations by adding the Coulomb interaction to $V(r)$[1]. Effects of inelastic channels may be incorporated into the model by adding an imaginary term to the above effective potential (optical potential). We take here the same one as reference 1. The determination of the differential cross-sections is done by a standard technique, whose details can be found in reference 1. The upper value of the relative angular momentum, R, for the a-A system, is also taken from ref.1.

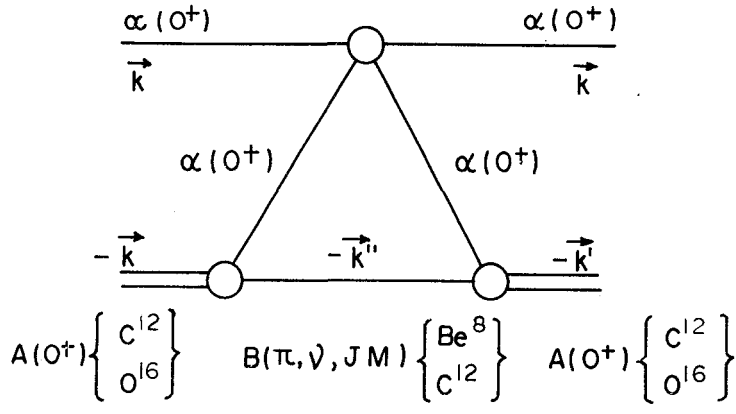

Fig.1 - The main Feynman diagram for the forward elastic scattering of a particles by $\{^{C^{12}}_{O^{16}}\}$ nuclei. The heavy particle B, $\{^{Be^8}_{C^{12}}\}$, is transferred in states described by parity a, spin J,M and extra quantum number v which label possible excited intermediate levels of B with the same n and spin.

3. RESULTS AND CONCLUSIONS

It is simple to see that the determination of the differential cross-sections depends basically on the previous knowledge of the $C_{\nu J}$ (see eq.2) arid $\phi(r)$. A rigorous way to determine $\$(r)$ and $C_{\nu J}$ could be to use the k-harmonics method[7]. Beside assuming an a-cluster structure for C^{12} and O^{16}, it would require the knowledge of their wavefunctions as well in the 0^+ ground states. This would involve a very complicate calculation and we would miss a clearer physical picture of the wavetunction. Instead we consider two phenomenological wavefunctions for $\phi(r)$: i) the Eckart function[1-5]

$$\phi_{\nu J}(r) = N(1 - e^{-r/R})^5 \frac{e^{-K_{\nu J} r}}{r} \qquad (3)$$

where N is the normalization constant; ii) the modified Eckart function[e]

$$\phi_{\nu J}(r) = N \sum_{m=0}^{J} \frac{(J+m)!}{2^m K_{\nu J}^{m+1} m! (J-m)!} (1 - e^{-r/R'})^{J+m+1} \cdot \frac{e^{-K_{\nu J} r}}{r^m} , \qquad (4)$$

where N is the normalization constant. In both equations, R and R' are parameters, and $K_{\nu J}^2 = \frac{2\mu}{\hbar^2} E_x(\nu, J)$. The Eckart function was chosen mostly because it is easy to handle analytically and has been used in many previous calculations giving reasonable results[1-5]. At the origin it has r^5 behavior and at infinity ($r>>R$) the form $e^{-K_{\nu J} r}/r$. However it was shown[8] that the J-th partial wave component should have a form more compatible with the modified Eckart function. For this reason the modified Eckart function was also chosen to be considered here. For the a-a interaction we take the d-form of the Ali-Bodmer potential[3].

Table 1 summarizes the spectroscopic coefficients used in our calculations. For lack of data their values are possibly not the best ones that can be considered but hopefully they should be a reasonable approximation for our purpose (one should notice that $\sum_{\nu J} |C_{\nu J}|^2 \leq 1$). Fig.2 shows the differential cross-section for the reaction $\alpha+C^{12}$ at E_{lab} = 40.5 MeV., using the Eckart function with R = 1.56 fm[(1)] and spectroscopic factors given in the fifth row of table 1. Si-

TABLE 1 - Square for the Spectroscopic coefficients $|C_{\nu J}|^2$. The quanturn number v stands for the first state that corresponds to J^{π}.

Vertex (ν, J^{π})	(1,0)	(1,1)	(1,2)	(1,3)	(1,4)	Reference
$O^{16} \rightarrow C^{12} + \alpha$	0.40	**	0.38	0.08	0.15	5
	.0.40	*	0.37	0.07	0.15	12
	0.36	0.	0.59	0.00	0.05	11
	0.28	**	0.48	**	**	11
$C^{12} \rightarrow Be^{8} + \alpha$	0.70	0.0	0.29	0.0	0.01	11

* not known

** too small

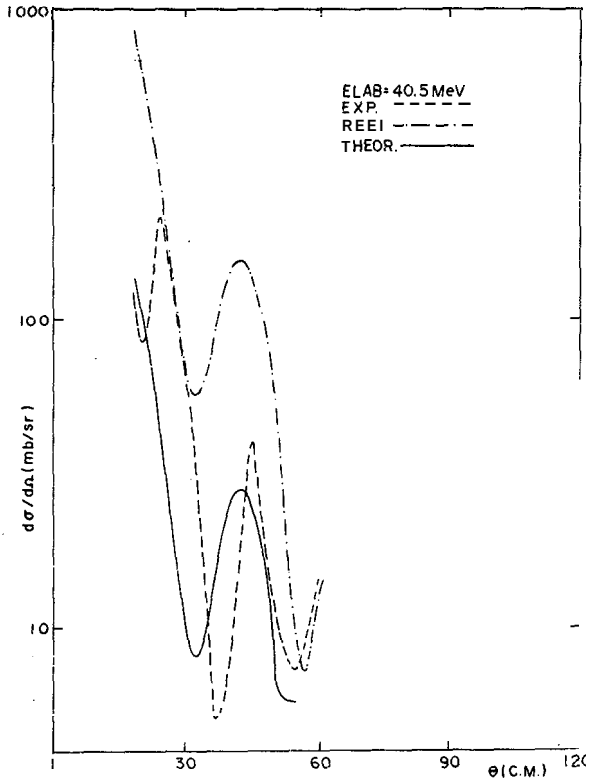

Fig.2 - Angular distribution of the differential cross-section ior forward elastic scatterirg of a-particles on C^{12} (g.s.).

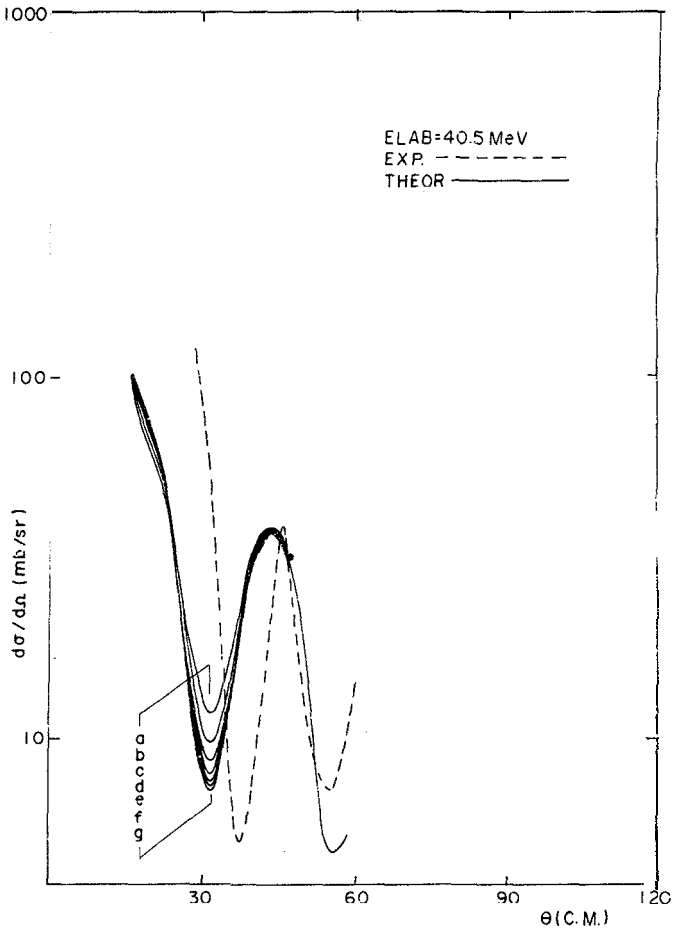

Fig.3 - Angular distributions of the differential cross-section for forward elastic scattering of α-particles by C^{12} (g.s.) using the Generalized Ecksrt function. Curves *a* to *g* correspond to different values of R', around R' = 6 fm.

milar graphs (*fig.3*) were obtained using a modified Eckart function. No appreciable change appears in the plot. Fig.4 shows a similar graph for the reaction $\alpha+O^{16}$ at $E_{lab} \simeq 40.5$ MeV, using a Eckart function with $R=$ = 2.8 fm[1] and spectroscopic coefficients given in the second row of table 1. The results for the reaction $\alpha+O^{16}$ are not so good in contrast with $\alpha+C^{12}$. In the first reaction the vertex $O^{16} \rightarrow C^{12} + a$ leads to various state of C^{12} where the knowledge of spectroscopic factors are necessary contraire to $C^{12} \rightarrow Be^{8} + a$ where basically only the spectroscopic factor for the ground state of Be^{8} is required. In the present

analysis we are not really trying to fit the data very closely. By varying the parameters more carefully we could possibly get better results. However our main goals here are to show the influence of spectroscopic coefficients and to investigate the effect that two different classes of relative motion wavefunctions have on the results.

In figs. (2) and (4) we also compare our calculations with previous ones without spectroscopic coefficients■ .

It is our conclusion that the introduction of the spectroscopic coefficients in the calculation is absulutely important. Although they are not yet precisely calculated for the considered reactions here the approximate ones used in this work are enough to guarantee the above statement. On the other hand our results are not very sensitive

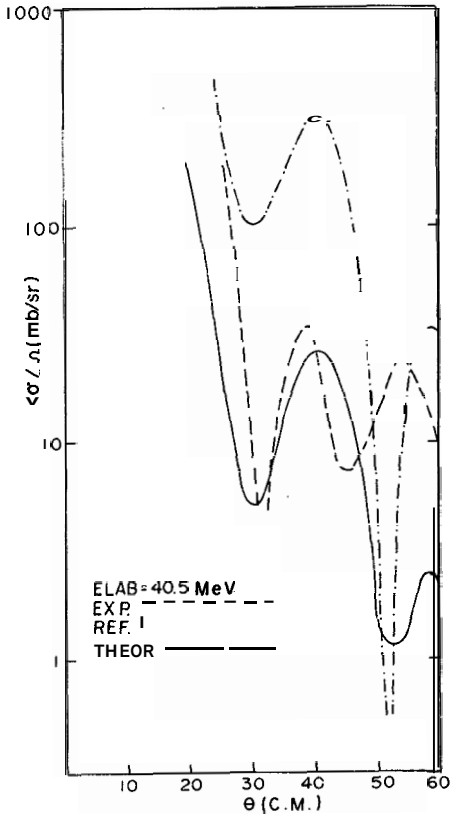

Fig.4 - Angular distributions of the differential cross-section for forward elastic scattering of a-particles by O^{16} (g.s.).

to different classes of relative motion wavefunctions, $\phi(r)$. Both wavefunctions used have basically similar behavior at the origin and at infinity, but it seems that the Eckart function has better behavior in the intermediate region.

We are aware of having omitted a $V_{\alpha\alpha}$ potential dependent in the relative angular momentum, λ, between the a's. Instead of using $\langle V_{\alpha\alpha} \rangle$ (see eq.(1)) we could substitute it by the $T_{\alpha\alpha}$-matrix, which would incorporate a natural λ-dependence on the vertex function. But this should not affect the results substantially[2,12].

We believe that our results would improve if unitsrity would be introduced more carefully in the calculation. Work is continuing in this direction. This could be done, for exarnple, in a way suggested by Giambiagi and Kibble[13].

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