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Magnetie Properties of an Interacting Electron Gas under De Haas--Van Alphen Conditions

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Using the propagator technique in the grand ensemble method, properties of an interacting electron gas are investigated under de Haas -van Alphen conditions. Considering the electron density to be a constant and taking the first order exchange effects into account, the dHvA oscillations of the magnetic susceptibility and sound velocity show only one phase with the Dingle factor, in agreement with experimental results. This improves results and interpretations made in previous works. Appearance of the Dingle factor due to the electron-electron collision is confirmed and interpreted to be a dragging effect from z-directional exchange interaction of electrons. New interpretations of the ring diagram are also presented.

Usando a técnica dos propagadores no grande "ensemble", as propriedades do gás de elétrons interagentes foram investigadas nas condições de Haas-van Alphen (dHvA). Considerando constante a densidade dos elétrons e levando-se em conta efeitos de troca de primeira ordem, as oscilações dHvA da susceptibilidade e da velocidade do som exibem apenas uma fase com o fator Dingle, em acordo com os resultados experimentais. Isto melhora os resultados e interpretações obtidos em trabalhos anteriores. O aparecimento do fator Dingle devido às colisões elétron-elétron é confirmado e interpretado como um efeito de arrastamento ocasionado pela interação de troca z-direcional dos elétrons. Novas interpretações dos diagramas em anel são também apresentadas.

1. INTRODUCTION

In the past years, many-body theories have been successful in investigating electron gas systems without the appliration of an external magnetic field, or with a small one. In special the ground state energy was calculated in many works[1], and it was even developed up to the third order terms[2]. But this theory was not developed for the case of medium-size magnetic field, under which the magnetic susceptibility showed the oscillatory character.

In 1930, oscillations of the magnetic susceptibility in Bi were first observed by de Haas and van Alphen[3] (dHvA), when comparatively strong magnetic field was applied. Since then, several theoretical works were made using the free electron model to explain qualitatively the dHvA phenomena, and based on these works the phenomena has been utilized[5] to determine geometrical structures of Fermi surfaces of various metals.

Dingle[6] introduced in a semi-empirical way an exponential factor (Dingle factor) in the amplitudes of oscillations of the susceptibility to involve the collision broadening effects on the Landau levels, modifying the ideal gas results.

Ichimura and Tanaka[7], and Isihara *et al*[8] studied the susceptibility of an interacting electron gas in the dHvA condition, using the propagator technique in the grand ensemble developed by Montroll and Ward[9]. The former did not.observe the existence of the Dingle factor but the latter did. However both works encountered the same problem of appearance of two phases of oscillations differing by $\pi/2$, when eletronic interaction was introduced.

In this article, deeper analyses and interpretations of the results shall be made and we shall see the agreement with experiments and with Dingle's semi-empirical result.

We shall use the same method used by Ichimura and Tanaka[7], and Isihara et *al*[8] (we refer hereafter to the paper of Isirara et *al*[8] as (I)). This method permits a unified treatment of the constant and oscillating part of the susceptibility for the interacting system with arbitrary strength of an external magnetic field, without separation of spin effects.

We shall follow the same notations as (I) in this article:

$$\gamma \equiv \frac{\hbar\omega_0}{E_F}, \quad \delta \equiv \frac{kT}{E_F}, \quad r_s \equiv \frac{(9\frac{\pi}{4})^{1/3}\hbar}{p_F\, a_0}, \quad \alpha \equiv \frac{\hbar\omega_0}{kT}$$

where $\frac{\hbar}{2}\omega_0 = \mu_B H$ is the energy associated with the magnetic field, μ_B is the Bohr magneton, a_0 is the Bohr radius, T is the temperature and k is the Boltzinann constant. Here E_F and p_F are the Fermi energy and momentum before renormalization, respectively.

The logarithrnof the grand partition function (G.P.F.) can be separated in accordance with the order of interactions involved as

$$\ln \Xi = \ln \Xi_0 + \ln \Xi_{1x} + \ln \Xi_r + \ln \Xi_{2x} + \dots, \qquad (1.1)$$

where the first term is the free gas contribution, the second isthe first order exchange, the third is the ring diagram, and so on. The magnetic susceptibility is calculated by the statistical formula

$$\chi = \frac{1}{\beta H V} \left.\frac{\partial \ln \Xi}{\partial H}\right|_{z,V,\beta}, \qquad (1.2)$$

where z is the absolute activity defined as $z = \exp(\beta E_F)$, V is the volume of the system and β is the reciproca1 temperature. The density ofthe system is derived as

$$\eta = \frac{1}{V}\left.\frac{\partial \ln \Xi}{\partial \ln z}\right|_{V,\beta,H} = -\frac{\delta^2}{V}\left.\frac{\partial \ln \Xi}{\partial \delta}\right|_{V,\beta,\alpha} \qquad (1.3)$$

The density derived above is a function of β, H and E_F as well as the interaction parameter. We shall renormalize this E_F in terms of the absolute Fermi eiiergy or momentum of the ideal gas at absolute zero. This is also given by

$$\eta = \lim_{\beta\to\infty} \frac{1}{V} \left.\frac{\partial \ln \Xi_0}{\partial \ln z^*}\right|_{V,\beta,H} \qquad (1.4)$$

with

$$z^* = \exp\left(\beta \frac{p_0^2}{2m}\right)$$

By dHvA condition we mean

$$\delta < \gamma << 1 \quad \text{or} \quad E_F >> \hbar\omega_0 > kT \ .$$

In Chapter 2, we shall construct the grand partition function of the system and introduce the propagator we shall use.

In Chapter 3, the susceptibility is calculated and some important interpretations of the Dingle factor are given.

Many experiments were conducted to observe dHvA phenomena using the sound velocity technique[10]. We shall then calculate[11] the change of the sound velocity under the dHvA condition in Chapter 4.

Finally, the ring diagram contribution is discussed in appendix A.

2. GRAND PARTITION FUNCTION

a) Propagator of the systsm

In this model metals are treated as an interacting three dimensional electron gas under the influence of an externa1 magnetic field of medium size. The effects of lattice structure are take into consideration as an effective mass (m^*) of an electron, but the density of states are assumed to be spherically symmetric and quadratic in momentum space.

The free propagator of the Bloch equation is given as:

$$K_0(\vec{r}_2\beta_2,\vec{r}_1\beta_1) = \sum_{n,\sigma} \exp\left[-(\beta_2-\beta_1)\varepsilon_{n,\sigma}\right] \psi_{n,\sigma}(\vec{r}_2)\psi^*_{n,\sigma}(\vec{r}_1)$$

$$= \prod_{\text{spin}} (S) \otimes \frac{b^2 \, e^{i\phi - z^2/4S} \exp\left[-\dfrac{b^2(x^2+y^2)}{4\tanh(Sb^2)}\right]}{8\,\pi^{3/2}\,S^{1/2}\sinh(Sb^2)} \tag{2.1}$$

where

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \ ; \ S = \beta_2 - \beta_1 ; \ \phi = \frac{1}{2} b^2 (x_2 - x_1)(y_1 + y_2)$$

and

$$\underset{\text{spin}}{\Pi} (S) = \exp(-\tfrac{1}{2} gSb^2) |\uparrow\rangle\langle\uparrow| + \exp(\tfrac{1}{2} gSb^2) |\downarrow\rangle\langle\downarrow|$$

with g being the Lande's g-factor. Here we have used the units $\hbar = 1$ and $2m^* = 1$, so that, for example βb^2 is in regular units $\beta eH\hbar/2m^*c$. The conversion to the regular units are given as follows;

$$\beta \to \frac{\beta\hbar^2}{2m^*} \ , \quad e^2 \to \frac{2m^* e^2}{\hbar} \ , \quad H \to \frac{H}{\sqrt{2m^*}} \ ,$$

and

$$g \to g \frac{m^*}{m}$$

b) Ideal gas contribution

The ideal gas contribution to the magnetic susceptibility is well known and may need no explanation. The explicit expression of the grand partition is needed for later considerations of the Fermi momenta and the other quantities of the system. It is given by

$$\ell n \ \Xi_0 = \mathrm{Tr}_\sigma \sum_{\ell=1}^{\infty} (-)^{\ell+1} \frac{z^\ell}{\ell} \int_V K_0(\vec{r}, \ell\beta; \vec{r}, 0) d\vec{r} = \frac{2V\beta p_F^5}{15\pi^2} \Bigg[1 + \frac{5}{8}\pi^2\delta^2 + \frac{15}{8}\gamma^2 \left[(\tfrac{1}{2} g)^2 - \tfrac{1}{3} \right] + \frac{15}{4}\gamma^{3/2}\delta \sum_{\ell=1}^{\infty} (-)^{\ell+1} \frac{\cos(\frac{g\ell\pi}{2}) \cos(\frac{\ell\pi}{\gamma} - \frac{\pi}{4})}{\ell^{3/2} \sinh(\ell\pi^2/\alpha)} \Bigg] \tag{2.2}$$

where a, y, 6 and z were defined in the introduction of this work.

c) First order exchange contributions

The first order exchange contribution to the G.P.F. was first treated by making use of the propagator method in (1). Use of the Mellin transformation is introduced to sum up all graphs of the first order exchange. The trace of spin part is improved in this article so that after a travel of a R-toron, the spin part yields

$$2 \cosh\left[\frac{g\alpha}{2}\ (n+t)\right]$$

where $n+t$ = R, and it is not separated into two hyperbolic cosines of n and t by the introduction of an interaction line.

It is then given by

$$\ln \Xi_{1x} = \frac{\beta}{2} \mathrm{Tr}_\sigma \int d\vec{r}_1\ d\vec{r}_2 \phi(|\vec{r}_1-\vec{r}_2|) L(\vec{r}_1\beta;\vec{r}_2 0) L(\vec{r}_2\beta;\vec{r}_1 0) \tag{2.3}$$

where

$$L(\vec{r}_2\beta,\vec{r}_1 0) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{e^{t/\delta}}{\sin\pi t} K_0(\vec{r}_2 t\beta;\vec{r}_1,0)\,,\ 0 < c < 1 \tag{2.4}$$

and $\phi(r) \equiv \dfrac{e^2}{r}$.

After some calculations Eq.(2.3) yields an oscillating and a non-oscillating part. The non-oscillating part is given as

$$\ln \Xi_{1x}^{\mathrm{non}} = \frac{V\omega_0^2 e^2}{32\pi^2\alpha^2} \left\{ I_1 + \left[\frac{\pi^2}{3} + \left(\frac{g\alpha}{2}\right)^2 - \frac{\alpha^2}{3}\right] \frac{1}{2} I_2 - \left[\frac{\pi^2}{3} - \frac{\alpha^2}{9}\right] I_3 \right\} \tag{2.5}$$

where I_1, I_2 and I_3 are the same as those obtained in (I). In the last square bracket, however, the term with g-factor does not appear. This indicates that there is no strong dependence of the paramagnetic susceptibility on the tempeture variable, while the diamagnetic susceptibility depends on the temperature rather strongly in the case of medium size magnetic field. It is not our present interest to investigate further the non--oscillating part of the susceptibility. The oscillating part becomes,

$$\ln \Xi_{1x}^{\mathrm{osc}} = \frac{V\omega_0^2 e^2}{64\pi} \left[\frac{2\beta}{\alpha\pi}\right]^{3/2} p_F \sum_{\ell=1}^{\infty} \frac{(-)^\ell \cos(\frac{\ell\pi g}{2})}{\ell^{1/2}\ \sinh(\ell\pi^2/\alpha)} \times I \tag{2.6}$$

where, being $J_\nu(r)$ the Bessel function of v–order

$$I = \int d\vec{r}\, \frac{1}{r^{5/2}} \sin\left(\frac{\ell\pi}{\gamma} + \frac{\alpha z^2}{4\beta\pi\ell E_F} - \frac{\pi}{4}\right) J_{3/2}(r) \quad J_0[(x^2+y^2)^{1/2}] \tag{2.7}$$

with the conversion $\vec{r} \to \vec{r}/p_F$. The above expression is separated into two parts so that it becomes:

$$I = \sin\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right) I_c(\ell,\gamma) + \cos\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right) \quad I_s(\ell,\gamma) \tag{2.8}$$

where

$$I_c(\ell,\gamma) = \int d\vec{r}\, \frac{\cos(\gamma z^2/4\pi\ell)}{r^{5/2}} J_{3/2}(r)\, J_0(\sqrt{x^2+y^2}) \simeq 4\sqrt{\pi/2}\,, \tag{2.9}$$

and

$$I_s(\ell,\gamma) = \int d\vec{r}\, \frac{\sin(\gamma z^2/4\pi\ell)}{r^{5/2}} J_{3/2}(r)\, J_0(\sqrt{x^2+y^2}) \tag{2.10}$$

The above equation $I_s(\ell,\gamma)$ is not easy to calculate but convergent. It is zero for $\gamma \to 3$ and it may also be very small if γ is brought tolarge number since the sine part oscillates very rapidly. These arguments thus imply that the result will have small positive plateau when we change γ. The change of γ is very small for the experimental region of magnetic field. Thus $I_s(\ell,\gamma)$ will behave in this region like a function of only R-variable or more or less constant for small R's. Of course, it must be treated together with the ring diagram for more complete calculations (see Appendix A). It is also noted that the fact that

$$I_c(\ell,\gamma) \simeq 4\sqrt{\frac{\pi}{2}} \tag{2.11}$$

is not an accidental value, but it plays an important role when one performs the interation to renormalize the Fermi momentum. After all it yields the foliowing result:

$$\ell n\, \Xi_1^{osc} = \frac{V\omega_0^2 e^2}{16\pi} \left[\frac{2\beta}{\alpha\pi}\right] p_F\left(\frac{\pi}{2}\right) \sum_{\ell=1}^{\infty} (-)^{\ell} \frac{\cos\left(\frac{\ell\pi g}{2}\right)\sin\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right)}{\ell^{1/2}\, \sinh(\ell\pi^2/\alpha)}$$

$$+ \frac{V\omega_0^2 e^2}{64\pi} \left[\frac{2\beta}{\alpha\pi}\right]^{3/2} p_F \sum_{\ell=1}^{\infty} (-)^{\ell} \frac{\cos\left(\frac{\ell\pi g}{2}\right)\cos\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right)}{\ell^{1/2}\, \sinh(\ell\pi^2/\alpha)} \tag{2.12}$$

3. MAGNETIC SUSCEPTIBILITY

Using (Eq.(1.2)), we obtain the following expression for the magnetic susceptibility as a function of p_F, which is the square root of the chemical potential.

$$\chi^{\text{non}} = \frac{p_F}{2\pi^2} \left(\frac{e}{c}\right)^2 \left\{ \left[\left(\frac{g}{2}\right)^2 - \frac{1}{3} \right] \left(1 + \frac{S}{\pi}\right) - \frac{S}{\pi} \frac{\ln \delta}{18} \right\} \tag{3.1}$$

$$\chi^{\text{osc}} = \frac{p_F}{2\pi^2} \left(\frac{e}{c}\right)^2 \left\{ \frac{\pi\delta}{\gamma^{3/2}} \sum_{\ell=1}^{\infty} (-)^{\ell+1} \frac{\cos\left(\frac{\ell\pi g}{2}\right) \sin\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right)}{\ell^{1/2} \sinh(\ell\pi^2/\alpha)} \right.$$

$$\times \left[1 - \frac{S}{2\pi} \left(1 + \sqrt{\pi/2}\, \frac{\ell I_s}{\gamma}\right) \right]$$

$$\left. + \left(\frac{S}{\pi}\right) \frac{\pi^2 \delta}{\gamma^{5/2}} \sum_{\ell=1}^{\infty} (-)^{\ell+1} \frac{\cos\left(\frac{\ell\pi g}{2}\right) \cos\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right)}{\sinh(\ell\pi^2/\alpha)} \ell^{1/2} \right\} \tag{3.2}$$

where $S \equiv \frac{e^2}{p_F} - 2\, r_s \left(\frac{4}{9\pi}\right)^{1/3}$.

Here one must recall that p_F is a function of the temperature, interaction parameter and density of the system as well as the external magnetic field. There were some attempts[6,12] in the past to interpret the ideal gas results so that the Fermi energy was kept constant and the electrons were transfered out of and into the dHvA band, specially in the case of bismuth. In this interpretaticn p_F was kept constant. If interactions among electrons do not exist, it does not show any contradiction in itself.

At the same time the result could also be interpreted soas for the total number of electrons in the band to be fixed.

The introduction of the interaction reveals that the former interpretation is not suitable. As can be seen from Eq. (3.2), the last term in the bracket may dominate the ideal gas contribution, and it even changes the phase of the oscillation from sine to cosine under the dHvA conditions.

New interpretation is made as follows: (1) The total number of electrons in the system does not change; (2) E_F, p_F and other quantities should oscillate with the same phase as those of the ideal gas, and thus not constant; (3) small lipplings wich are observable may be attributed to the lattice structure and are directional dependente.

Fine lipplings due to the detailed relation of the electron--electron interactions are very small, and this must be characterized by products of two oscillations such as

$$\sin\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right) \times \cos\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right)$$

this means they must be the higher harmonics.

The calculations are perfomed satisfying all the above requirements. One must also assume that the interaction parameter r_s is very small but not extremely small, so that the p_F appearing in the sine and cosine arguments can be expanded in terms of p which is defined as

$$p_0 \equiv \left(\frac{N}{3\pi^2 V}\right)^{1/3} \tag{3.3}$$

Using the relation

$$\frac{N}{V} = \eta = -\frac{\delta^2}{V} \frac{\partial\ \ell n\ \Xi}{\partial\delta}\bigg]_{V,\beta,\alpha} \tag{3.4}$$

one finds the following expression for the Fermi monientum

$$p_F = p_0 \left\{ 1 + \frac{3S}{2\pi} + \frac{\pi^2\delta^2}{8}\left(1 - \frac{S}{\pi}\right) + \frac{3}{8}\gamma^2\left(\frac{1}{2}g\right)^2 - \frac{\gamma^2}{8}\left(1 - \frac{S}{3\pi}\right) \right.$$
$$- \frac{3\pi^2}{2}\left(\frac{S}{\pi}\right)\frac{\delta}{\gamma^{1/2}}\sum_{\ell=1}^{\infty}(-)^{\ell+1}\frac{\cos\left(\frac{g\ell\pi}{2}\right)\cos\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right)\ell^{1/2}}{\sinh(\ell\pi^2/\alpha)}$$
$$\left. - \frac{3\pi}{2}\gamma^{1/2}\delta\sum_{\ell=1}^{\infty}(-)^{\ell+1}\frac{\cos\left(\frac{g\ell\pi}{2}\right)\cos\left(\frac{\ell\pi}{\gamma} - \frac{\pi}{4}\right)}{\ell^{1/2}\sinh(\ell\pi^2/\alpha)}\left[1 + \frac{S}{2\pi}\left(1 - \frac{\sqrt{2\pi}\ell I_s}{2\gamma}\right)\right]\right\}^{-1/3} \tag{3.5}$$

We can solve Eq. (3.5) by an iteration procedure. After some iterations we obtain:

$$p_F = p_0 \left\{ 1 - \frac{S_0}{2\pi} - \frac{\pi^2\delta_0^2}{24}\left(1 - \frac{S_0}{2\pi}\right) - \frac{\gamma_0^2}{8}\left(\frac{1}{2}g\right)^2\left(1 + \frac{S_0}{2\pi}\right) + \frac{\gamma_0^2}{24}\left(1 + \frac{S_0}{6\pi}\right) \right.$$

$$\left. + \frac{\pi}{2}\gamma^{1/2}\delta_0 \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \frac{\cos\left(\frac{g\ell\pi}{2}\right)\sin\left(\frac{\ell\pi}{\gamma_0} - \frac{\pi}{4}\right)}{\ell^{1/2}\sinh(\ell\pi^2/\alpha)} \left[1 + \frac{S_0}{2\pi}\left(1 - \frac{\sqrt{2\pi}\ell I_s^0}{2\gamma_0}\right)\right] \right\} \tag{3.6}$$

where the index zero in the parameters means that all the p_F are replaced by p_0 for example:

$$\delta_0 = \frac{kT}{p_0^2}$$

After the replacement of p_F in the magnetic susceptibility in terms of p_0, the oscillating magnetic susceptibility is given by

$$\chi^{osc} = \frac{p_0}{2\pi^2}\left(\frac{e}{c}\right)^2 \frac{\pi\delta_0}{\gamma_0^{3/2}} \sum_{\ell=1}^{\infty} (-)^{\ell+1} \frac{\cos\left(\frac{g\ell\pi}{2}\right)\sin\left(\frac{\ell\pi}{\gamma_0} - \frac{\pi}{4}\right)}{\ell^{1/2}\sinh(\ell\pi^2/\alpha)} \left[1 - \frac{\sqrt{2\pi}\ell I_s^0 S_0}{4\pi\gamma_0}\right] \tag{3.7}$$

The above expressions only have one phase and other phases are at most the products of the two phases and the coefficients are smaller.

The above result are to be compared with others.

Comparing the results in Eq.(3.2), two terms which are proportional to $S_0(= e^2/p_0)$ disappeared after iteration, but the term which is characterized by I remained.

As has been mentioned previously, the disappearance of the other first order terms in the oscillating susceptibility is deeply related with the approximation made for $I_c(\ell,\gamma)$. If I_c had another value the effect would remain.

With the regular units the above expressions is rewritten as

$$\chi^{osc} = \frac{p_0^2 \sqrt{m}\, kT\mu_B^{1/2}}{\hbar^3\pi\sqrt{2}\, H^{3/2}} \sum_{\ell=1}^{\infty} (-)^{\ell+1} \frac{\cos(\frac{g}{2}\ell\pi\frac{m^*}{m})\sin(\frac{\ell\pi E_F^0}{\mu_B^* H} - \frac{\pi}{4})}{\ell^{1/2}\sinh(\frac{\ell\pi^2 kT}{\$H})}$$

$$\times \left[1 - \frac{v_F^* \ell I_s^0}{\sqrt{2}\pi\, a_0^* \omega_0^*}\right] \qquad (3.8)$$

where

$$\mu_B^* = \frac{e\hbar}{m^*c} \;;\; \omega_0^* = \frac{eH}{m^*c} \;;\; v_F^* = \frac{p_0}{m^*} \quad \text{and} \quad a_0^* = \frac{H^2}{m^*e^2}$$

Recognizing the last bracket in the last equation as the expansion of an exponential, one sees

$$\left[1 - \frac{v_F^* \ell I_s^0}{\sqrt{2}\pi\, a_0^* \omega_0^*}\right] \sim e^{-\frac{2\pi^2 kT_D}{\hbar\omega_0^*}} \qquad (3.9)$$

where T_D is a constant which depends on materials used. Dingle first derived the existence of the collision broadeningof the Landau levels but the classifications of origins were left unexplained.

The magnetic susceptibility of the **2-D** Metals was calculated by Kojima and Isihara[13]. They did not find the same kind of term, that is, the Dingle factor do not appear in two dimensions. This gives an important information concerning the electronic part of the collision broadening.

The first order exchange effect appears as the dragging (viscous like) effect of the z-direction, since the first order exchange effect appears to be attractive.

In 2-D case phase factor $-\frac{\pi}{4}$ does not appear. Thus change of sizes of samples will result in slight changes of both the collision broadning and the phase factor.

One must also note that the collision broadening also originates from impurities and from other factors. These changes may be observable in experiment in future.

The hyperbolic term in the denominator of the oscillating magnetic susceptibility in Eq. (3.7) also decreases the amplitude exponentially when $kT \gtrsim \mu_B^* H$, and it is understood to be the thermal agitation part of the broadenings. As well known, the electron-electron interaction plays similar effects as the thermal agitation and broadens the cliff of the Fermi distribution at the Fermi energy.

4. VELOCIW OF SOUND

In some experiments[10] velocities of sound were measured to obtain the dHvA effects.

Change in the sound velocity due to the introduction of the external magnetic field is given as[14]

$$\frac{v(H)}{v} = \frac{v(H) - v(0)}{v(0)} \quad \frac{B_e^{osc}}{2\rho v^2(0)} \tag{4.1}$$

where B_e^{osc} is the bulk modulus of oscillating part, and ρ is the mass density of the system.

The grand partition function is directly related to the pressure of the eiectron cloud by

$$PV = kT \ln \Xi \tag{4.2}$$

and the bulk modulus of the system is given as

$$B = - V \frac{dP(N,T,H,V)}{dV} \tag{4.3}$$

Thus after reexpressing the grand partition function in terms of number density of the system, or p_0, one can obtain the bulk modulus through the equation (4.3), the oscillating part of it is then

$$B_e^{osc} = \frac{2\delta_0 P_0^5}{9\gamma_0^{1/2}} \sum_{\ell=1}^{\infty} (-)^{\ell+1} \frac{\cos(\frac{\ell\pi g}{2})\,\ell^{1/2}\cos(\frac{\ell\pi}{\gamma_0} - \frac{\pi}{4})}{\sinh(\ell\pi^2/\alpha)} \left[1 - \frac{\sqrt{2}\,\pi}{4} (\frac{S_0}{\pi}) \frac{\ell I_s^0}{\gamma_0}\right] \tag{4.4}$$

By Eq. (4.1) we obtain the dominant term for the change in sound velocity:

$$\frac{\Delta v}{v} = \frac{4m^{*3/2}\ kT\ E_F^{0\,2}}{9v^2(0)\rho\hbar^3(\hbar\omega_0^*)^{1/2}} \sum_{\ell=1}^{\infty} (-)^{\ell+1}$$

$$\times \frac{\cos(\frac{g\ell\pi}{2})\cos(\ell\pi E_F^0/\mu_B^* H - \frac{\pi}{4})\,\ell^{1/2}}{\sinh(\ell\pi^2 kT/\mu_B^* H)} \qquad (4.5)$$

Again the Dingle factor appears as the same manner as befure and other first order interactions were completely canceled out. Appearance of the same factor through the complicated statistical mechanical procedure is not obvious fact although both the sound velocity and magnetic susceptibility are obtained from the same G.P.F.

The energy and specific heat of the system are also observed but they are not given here, because to obtain them with the theoretical consistency., one needed even to evaluate terms of two oscillations, wich should cancel in principle. Neglecting these terms we observed they also oscillates like ideal gas with the same exponential factor.

APPENDIX A – THE RING DIAGRAM

In the case of weak field the ring diagram contribution is calculated without much difficultles, combining partly with the first order exchange to remove a divergence involved in a magnetic field dependent part.

On the other hand when the dHvA condition is satisfied, the contribution becomes extremely complicated and also the first order exchange becomes convergent since expansion with respect to the magnetic field is not performed.

The ring diagram contribution to the G.P.F. is given, as well known, as:

$$\ln \Xi_r = \frac{V}{2(2\pi)^3} \sum_j \int d\vec{q} \, \{U(q)\lambda_j(q) - \ln[1 + U(q)\lambda_j(q)]\} , \qquad (A.1)$$

where $U(q) = \frac{4\pi e^2}{q^2}$

$$\lambda_j(q) = \frac{b^2}{4\pi^2} \int_{\infty}^{\infty} dp_z \int_0^{\beta} d\alpha' \sum_{\ell=1}^{\infty} \sum_{s=0}^{\infty} (-)^{\ell+1} z^{\ell} \cosh(\frac{\ell}{2} g\pi\beta b^2)$$

$$\times \left\{- \frac{2\pi i j\alpha'}{\beta} - (s\beta+\alpha')(p_z^2+q_z^2)^2 - [(\ell-s)\beta-\alpha']p_z^2 \right\} \times A(s,\ell-s) \qquad (A.2)$$

where

$$A(s,\ell-s) = \exp\left\{- \frac{(q_x^2+q_y^2)}{b^2} \frac{\sinh[((\ell-s)\beta-\alpha')b^2]\sinh[(s\beta+\alpha')b^2]}{\sinh(\ell\beta b^2)}\right\} \qquad (A.3)$$

It is found that the approximation made on $A(s,\ell-s)$ in (1) is not sufficient when approaches either zero or β. Also on the contrary to the result in (1) we found that the sine oscillation dominates the cosine oscillation as in the case of the first order exchange. This is the same tendency as the first order exchange contribution.

It is known that when one calculates

$$-\int \sum_j \lambda_j(q) \; U(q) \; d\vec{q} \qquad (A.4)$$

it contains the First order exchange (Eq.2.3), which appears from the region of a' being both zero and β.

This implies us that the cosine oscillation of the ring diagram will contribute to correct the collision broadening but the sine term may be canceled after proper iteratian. The cosine term will be extremely small.

Taking into account all the above considerations one obtains the following results:

$$\lambda_j(q) = \lambda_j^{\text{non}}(q) + \lambda_j^{\text{osc}}(q) , \qquad (A.5)$$

$$\lambda_j^{\text{non}}(q) = \frac{p_F}{4\pi^2} \left\{ e^{-(q_x^2+q_y^2)/2\gamma} F(q_z,j) + \frac{8}{3}\left[1 + \frac{3}{4}\delta^2 \left(\frac{g^2}{8} - \frac{\alpha^2}{6} + \frac{\pi^2}{6}\right)\right] \left[\frac{q^2}{q^4 + (2\pi j\delta)^2} - \frac{q_z\, e^{-\frac{q_x^2+q_y^2}{2\gamma}}}{q_z^4 + (2\pi j\delta)^2}\right]\right\} \quad (A.6)$$

and

$$\lambda_j^{\text{osc}}(q) = \frac{\gamma^{1/2}}{\pi\beta p_F} \sum_{\ell=1}^{\infty} (-)^{\ell} \frac{\cos(\frac{\ell\pi g}{2})\sin(\frac{\ell\pi}{\gamma} - \frac{\pi}{4})}{\sinh(\ell\pi^2/\alpha)} \left\{ \frac{q_z^2\, e^{-\frac{(q_x^2+q_y^2)}{2\gamma}}}{q_z^2 + (2\pi j\delta)^2} + K(\vec{q},j)\right\}$$

+ (cosine phase which is very small), (A.7)

where

$$F(q_z,j) = 1 - \left(\frac{1}{8} q_z - \frac{4}{q_z} - \frac{(2\pi j\delta)^2}{q_z^3}\right) \ln\left|\frac{(2\pi j\delta)^2 + (q_z^2+2q_z)^2}{(2\pi j\delta)^2 + (q_z^2-2q_z)^2}\right| - \frac{2\pi j\delta}{2q_z}\left[\tan^{-1}\left(\frac{q_z^2+2q_z}{2\pi j\delta}\right) - \tan^{-1}\left(\frac{q_z^2-2q_z}{2\pi j\delta}\right)\right] \quad (A.8)$$

and

$$K(\vec{q},j) = \frac{q^2}{q^4+(2\pi j\delta)^2} - \frac{q_z^2\, e^{-(q_x^2+q_y^2)/2\gamma}}{q_z^2 + (2\pi j\delta)^2} - e^{-q^2/2\gamma}\left\{\cos\left(\frac{2\pi j\delta}{2\gamma}\right) \times \left[\frac{q^2}{q^4+(2\pi j\delta)^2} - \frac{q_z^2}{q_z^4+(2\pi j\delta)^2}\right] + \sin\left(\frac{2\pi j\delta}{2\gamma}\right)\left[\frac{2\pi j\delta}{q^4+(2\pi j\delta)^2} - \frac{2\pi j\delta}{q_z^4+(2\pi j\delta)^2}\right]\right\} \quad (A.9)$$

The first parts in both λ_j^{non} and λ_j^{osc} correspond to the results in (1), but the coefficient of λ_j^{osc} differs in detail. Further approximation reveals that the sine oscillatory term is more or less spherically symmetrical in q variable, and the term in the curiy bracket of (A-7) may be approximated within the dHvA condition as

$$\frac{q^2}{q^4 + (2\pi j\delta)^2}$$ 7

The smallness of z-variable corresponds to the largeness of p_z variable, and in this limit cosine phase disappears but sine phase rernains. This situation is exactly the same as the first order exchange. We hope that the explicit calculation of this term will be performed in near future.

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