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Free Monopolar Short-Circuit Discharges

C. M. LEPIENSKI

Instituto de Física, Universidade Federal do Paraná, Caixa Postal 1862, Curitiba, PR, Brasil

G. F. LEAL FERREIRA

Departamento de Física e Ciência dos Materiais, Instituto de Física e Química de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560, São Carlos, SP, Brasil

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A systematic study of the current delivered by monopolar excess charge in insulators under short circuit condition was performed. Typical distributions were studied and approximated results were derived. Attention was given to the conditions leading to the current reversal.

Um estudo sistemático da corrente liberada na descarga em curto circuito de isolantes com excesso monopolar de carga foi realizado. Distribuições típicas foram estudadas e resultados aproximados obtidos. Foi dada atenção à obtenção das condições que devem satisfazer as distribuições de carga afim de que dêem origem à reversão da corrente.

INTRODUCTION

In two preceding papers, (Refs 1 e 2) publishéd in this Review, the genera'l solution of the equations governing the motion of free excess monopolar charge in sandwiched insulator samples under given applied voltage (which may be a function of the time) were presented. Ref. (1) covers the case in which the initial charge touches only one of the electrodes, while Ref. (2) extends the solution to the general case when the charge fill up the space between the electrodes. However in order to obtain the external current, which is the usually measured physical quantity, a somewhat complicate computational work is needed and we decided to carry out a systematic study of the current delivered by different charge distributions in short circuit, such that could give to the experimentalist-confident that the model applies-means to infer from the measured current, how the charges were distributed initially. Among the effects many times observed in electrical measurements in insulators is the current reversal, that iç, the current changes sign during discharge. We tryed to understand this effect, providing a rule antecipating its ohservation.

1. THEORY

1.1. General

The free monopolar space charge motion is characterized by the fact that all charges move with a velocity proportional to the electric field. We write the equations in recluced units (unprimed), giving below the relation between them and real ones (primed).

$$\frac{\partial E}{\partial x} = \rho \qquad (1)$$

$$-\frac{\partial i}{\partial x} = \frac{\partial P}{\partial t} \qquad (2)$$

$$i = \rho E \qquad (3)$$

$$j = i + \frac{\partial E}{\partial t} \qquad (4)$$

$$V(t) = \int_0 E dx \qquad (5)$$

$$x = \frac{x'}{\ell} \;;\; \rho = \frac{\rho'}{\rho'_0} \;;\; t = \frac{\mu\rho'_0 t'}{\varepsilon} \;;\; E = \frac{\varepsilon E'}{\ell\rho'_0} \;;$$

$$i \text{(or } j) = \frac{\varepsilon}{\mu\rho'_0\ell}\, i' \text{ (or } j') \;;\; V = \frac{\varepsilon}{\rho'_0\ell^2} V' \qquad (6)$$

Eq. (1) is the Poisson's Equatinn, with $E(E')$ the field, $\rho(\rho')$ the charge density. Eq. (2) is the continuity equation, $i(i')$ being the current density, $x(x')$, the position coordinate and $t(t')$ the time. In (6), ρ'_0 may be chosen at will. Equation (3) is the definition of the current density, and the mobility μ dropped out due to the use of reduced units. For the same reason, the permittivity ε is absent in Eq.(4), defining the total externa1 current j' which, as it is well knnwn, is a only function of the time. In Eq. (6) R is the thickness of the sample. Eq. (5) specifies the boundary condition of the problem, V(V') being the applied voltage.

1.2. An approximate result

Here, we will not repeat the procedure to be used in order to transform the system of partial differential equations (Eqs.1-4) suhjected to the integral boundary condition Eq. (5) in a system of total differential equations (it is in this sense that we say that the system has been solved). This may be found in Ref. 2.

The 'system will be used to derived an approximate solution valid for nearly uniform charge distribution.

Suppose that

$$\rho(x,0) = a_0 + b_0 \ (x - \frac{1}{2}) \tag{7}$$

with $a_0 >> b_0$. Let us assume the $\rho(x,t)$ remains linear. in x, that is, $\rho(x,t) = a(t) + b(t)(x - \frac{1}{2})$ and we want to final $a(t)$ and $b(t)$. Two well known resvlts support the assumption: first, the non uniform charge density tends to became more uniform as the time goes on (3); second, the externál current is only a gross measure of the charge density (that is, as it will be shown below, depends on integrals performed over the charge density) and therefore is insensible to the exact form of the charge density.

With $V=0$, the Poisson's Equation gives

$$- E(x,t) = (a(t) - \frac{b(t)}{2})x + \frac{b(t)x^2}{2} - \frac{1}{2} (a(t) - \frac{b(t)}{2}) - \frac{b(t)}{6} \tag{8}$$

Using the continuity equation, the following results:

$$a^2(t) - \frac{3a(t)b(t)}{2} - \frac{b^2(t)}{3} + \frac{da(t)}{dt} - \frac{db(t)}{2dt} +$$
$$\left[3a(t)b(t) - \frac{3b^2(t)}{2} + \frac{db(t)}{dt}\right] x + \frac{3b^2}{2} \ x^2 = 0$$

Equating to zero the coefficients of x^0 and x^1 and taking into account that $a(t)$ remains $>> b(t)$, we get

$$\frac{da(t)}{dt} + a^2(t) = 0$$

$$\frac{db(t)}{dt} + 3a(t)b(t) = 0$$ whose solution

is $$a(t) = \frac{a_0}{1 + a_0 t} \quad \text{and} \quad b(t) = \frac{b_0}{(1 + a_0 t)}$$

The total charge inside the sample is also $a(t)$. Therefore we conclude that it decreases with the time as $a_0/(1 + a_0 t)$.

In order to obtain the external current, we integrate Eq. (4) in x (using Eq.(1)):

$$j = \frac{1}{2}\left[E^2(1,t) - E^2(0,t)\right] \tag{9}$$

Substitution of $E(1,t)$ and $E(0,t)$ from Eq. (8) gives

$$j(t) = \frac{a(t)b(t)}{12} = \frac{j(0)}{(1 + a_0 t)^4}$$

As said before, the reduced units depends on the choice of ρ_0', which will now be fixed in such a way that $\rho_0' \ell = \int_0^R \rho'(x',0)dx'$ with this choice, a, equal is to 1 and we have

$$j(t) = \frac{j(0)}{(1 + t)^4} \tag{10}$$

In real units, we have (using Eq. (6))

$$j'(t') = \frac{j'(0)}{\left(1 + \frac{\mu\rho_0' t'}{\varepsilon}\right)}$$

or

$$\left[\frac{j'(0)}{j'(t)}\right]^{1/4} = 1 + \frac{\mu\rho_0' t'}{\varepsilon} \quad , \tag{11}$$

with $\rho_0' = \frac{q}{\ell}$, q being the charge inside the sample. A plot of $\left[\frac{j'(0)}{j'(t)}\right]^{1/4}$ as a function of the time provides the value of $\frac{\mu q}{\varepsilon \ell}$.

1.3. The Total Current and Current Reversal

An expression for the total current useful in discussing the current reversal will now be derived.

We have, with V=O

$0 = \int_0^1 E(x,t)dx$ or, integrating by parts and using Poisson's Equation

$$0 = E(1,t) - \int_0^1 x\rho(x,t)dx$$

On the other hand,

$$E(1,t) - E(0,t) = \int_0^1 \rho(x,t)dx.$$

Using now Eq. (9), we get

$$j(t) = \left[\bar{x}(t) - \frac{1}{2}\right] q^2(t) \qquad (12)$$

with q the total charge, as before, and

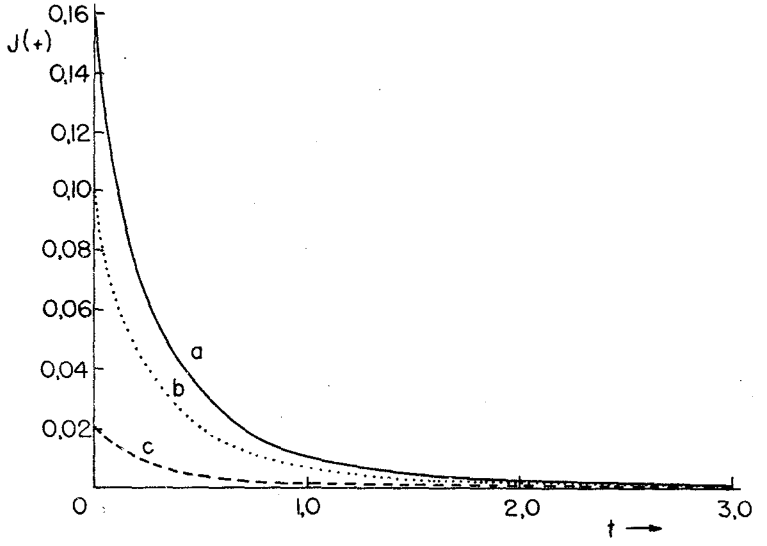
$$\bar{x}(t) = \frac{\int_0^1 x\,\rho(x,t)dx}{\int_0^1 \rho(x,t)dx}$$

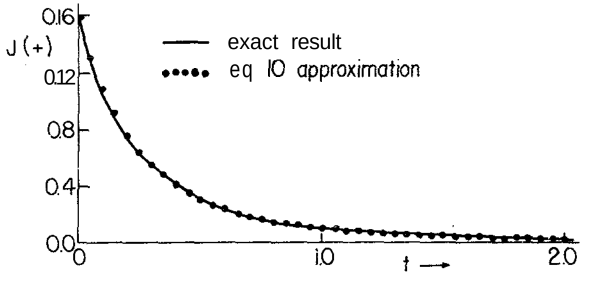
Eq. (12) shows that the sign of the current depends on the mean position $\bar{x}(t)$ of the charges; if greater than half of the sample thickness, it will positive, and negative otherwise. For instance, the linear charge density $\rho(x,t) = a(t) + b(t)(x - \frac{1}{2})$ with b > 0, will give a positive current.

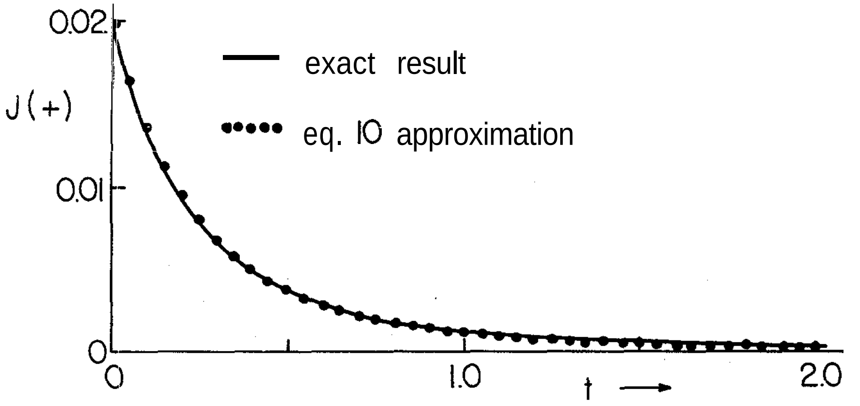
2. RESULTS

2.1. Linear and almost linear charge densities

Fig. 1 shows the exact results (that is calculated accordi'ng to Ref.2) of three discharges corresponding to linear charge densities, with the initíal mean depth equal to. 66 (curve a, .60 (curve *b*) and .52 (curve *c*). In the case of curve *a*, $a_0 = 1$ and $b_0 = 1.92$ in the notation of Eq. 7, and we could hardiy expect the approximation derived in the previous section (Eq.10) to hold for it. However, Eq.10 does provide a very good approximation as Fig. 2 shows, which refers to curve *a* calculated using Eq. 10. Fig. 3 shows the results for curve c. We have obtained that monotonically increasing (or decreasing) charge densities — unless they have a







large amount of charge near one electrode — give a current closely satisfying Eq. 10, and in all cases no current reversal is observed.

This result suggests that Eq.10 is perhaps also a good approximation even for charge densities strongly departing from a straight line.

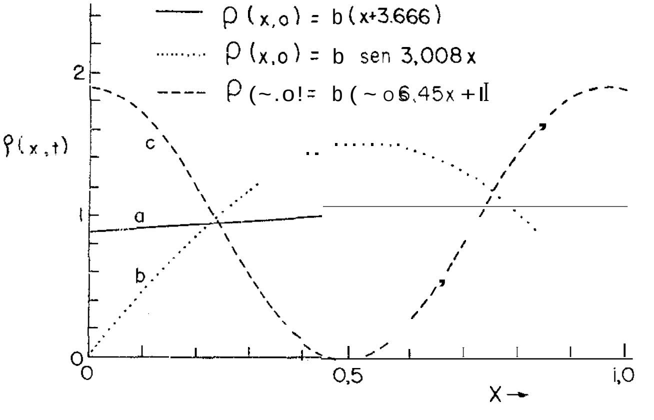
2.2. Charge diotributions with bumps or shells

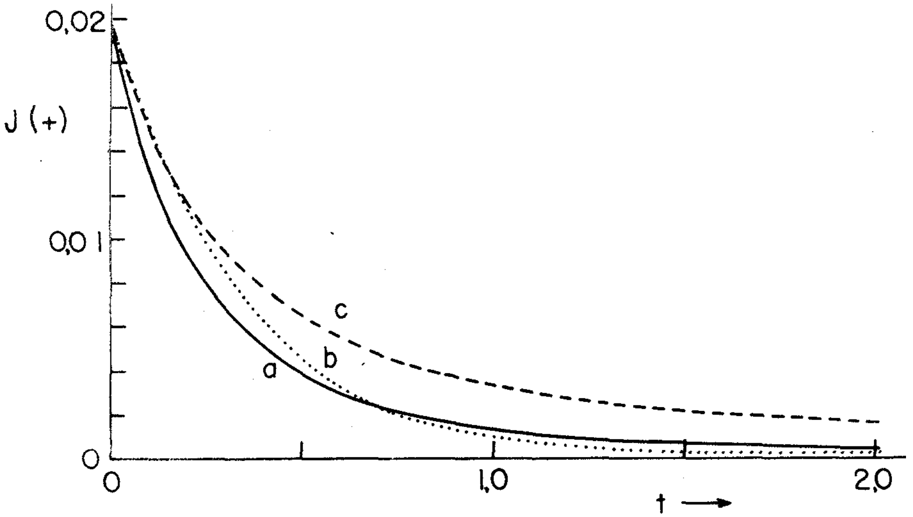
Fig. 4 shows three charges distributions:

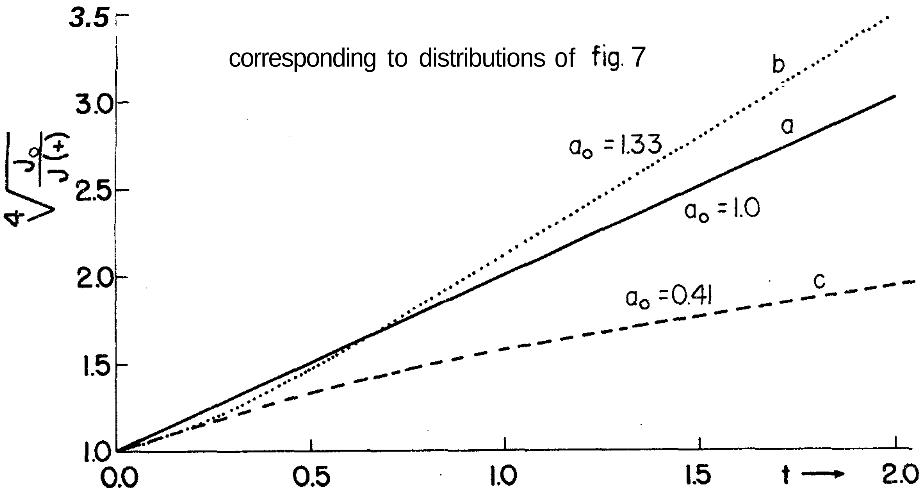
a - $\rho(x,0) = b_1(x + 3.666)$

b - $\rho(x,0) = b_2 \text{sen } 3.008x$

c - $\rho(x,0) = b_3(\cos 6,45x + 1)$;







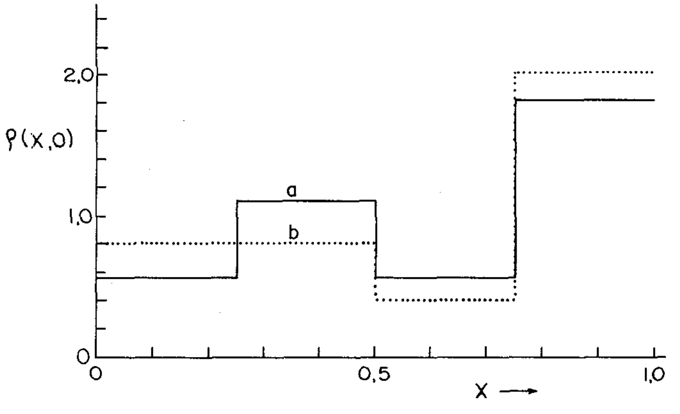
the values of b_1, b_2 and b_3 are such as to give the same initial charge. All these distributions also have the snme value of $\bar{x}$. The currents are shown in Fig. 5 and the plot according to Eq. 11 in Fig. 6. Of course, cases b and c do not give straight lines but tend to become straight at higher times. We have observed that distributions with bumps give curves with positive concavity while the opposite happens with distributions with a shell. In Fig. 6 it is also shown the angular coefficient a_0 of the straight portions of curves b and c. We note that for curve b, a, is greater than 1 and for curve c, a_0 is smaller than 1.

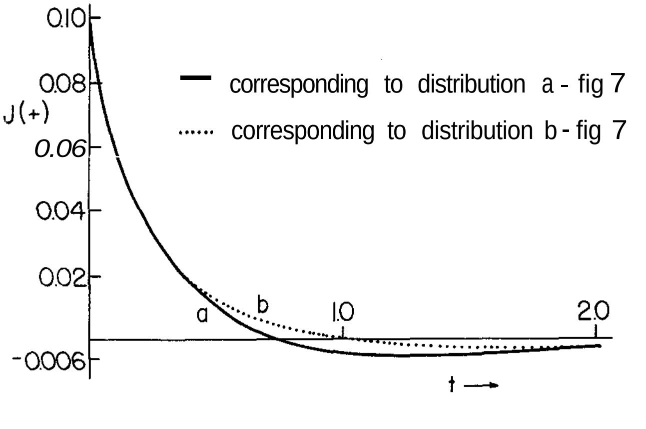
2.3. Current Reversal

According to the results of Section 1 (Fig. 2), a current reversal will be observed whenever the mean depth of the charge distribution crosses the plane $x=1/2$.

Fig. 7 shows two distributions and Fig. 8 their discharges both displaying current reversal. Of course, we would like to know if it is possible to predict such a behavior. Although only these two curves are shown in this work, the following rule was tested in many cases with great (and almost complete) success:

Take the initial charge distribution and find the position of the mean depth. Suppose $\bar{x} > 1/2$ (as in Fig. 6). Cut away equal amounts





of space near tne electrodes (for instance . 1 near $x = 0$ and $x = 1$) and consider the initial charge comprised inside this smaller distance (that is, thechargebetween .1 and .9) and find again the mean depth. Repeat the procedure again and again. If at some stage the mean depth crosses, moving to the left, the plane at $x = 1/2$, current reversal will be observed. In Fig. 6, a cut at $x = .25$ and .75 clearly shows that the remaining distribution ($.25 < x < .75$) has a mean depth smaller than .5 and therefore both distributions display current reversal. This rule, when applied to distributions of Fig.4 indicates that the reversal would not be observed, as in fact they are not.

The working reason of this rule, as we have verified, is that the time needed for charges, initially at a same distance from the electrodes at $x = 0$ and $x = 1$, to move to the nearby electrode, is approximatily the same. Therefore, the charge remaining in the sample is directly related to the initial charge distribution and despite the deformation in the pattern, due to the charge motion, its mean depth is very nearly the same as the corresponding initial one.

Using this rule as a guide, we were able to construct charge distributions with bumps and shells, displaying even three current reversals during the'ir discharges.

3. FINAL REMARKS

The important point to the experimentalist is to know what model describes the maasured external current. According to our results, of no current reversal is observed, the best test is given by the used of Eq. (11): if at large times a straight line is obtained, it may be said that we have excess charge of only one sign and that the charges behave as free.

A better test for the model is obtained if the plot according to Eq. 11 is made choosing as the zero time (and putting for $j'(0)$ the corresponding current) some time after the beginning of the discharge. This is so because the first stage of the discharge depends somewhat more closely on the shape of the charge distribution (in Ref. 4 the discharge given by the free space charge distribution following injection was studied) while latter the distribution became nearly uniform and Eq.11 should apply.

On the other hand, if a current reversal is observed, it cannot be said that our model applies. If it is known that actually the sample has excess char-ge of one sign, the rule developed in the previous section may be used to infer how the charge was distributed in the beginning of the discharge.

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