Revista Brasileira de Física, Vol. 11, nº 1, 1981

On-the-Energy-Shell Approximation for the Heavy Ion Coupled-Channels Problems (II)

Consequences on the Sub-Barrier Elastic Scattering +

BRETT V. CARLSON
Department of Physics – University of Wisconsin, Madison, Madison, WI 53706 USA

and

MAHIR S. HUSSEIN
Department of Physics – University of Wisconsin, Madison, Madison, WI 53706 USA
and
*Instituto de Fisica – Universidade de São Paulo, CP 20516, São Paulo, Brasil**

Recebido em 7 de Agosto de 1980

The on-energy-shell-approximation is used for the channel Green's function to solve the coupled channels equations for sub-barrier multiple Coulomb excitations in heavy ion collisions. A set of recursion relations is derived which permits a simple algebraic solution for the S-matrix elements. The resulting excitation probabilities satisfy the unitarity condition exactly. Cornparison of aur results with those of the exact quantum mechanical and semi- classical treatments is made.

A aproximação "on-energy-shell" foi usada para as funções de Green dos canais para resolver as equações acopladas de excitações múltiplas Coulombianas para íons pesados de baixas energias. Achou-se um conjunto das equações de recorrência que permite uma solução algébrica

+ Supported in part by the Tinker Foundation, the U.S.-NSF, the CNPq--Brazil and FAPESP-Brazil.

* Permanent address.

simples para os elementos da matriz-S. As probabilidades de excitação resultantes, satisfazem exatamente a condição de unitariedade. Foram feitas também comparações dos nossos resultados com os obtidos nos tratamentos quânticos e semi-clássico.

1. INTRODUCTION

In a previous paper[1] (referred to as I), we studied the consequences, on the sub-barrier elastic scattering between heavy ions, of using the on-energy-shell approximation for the channel Green's function in the solution of the coupled channels equations for multiple Coulomb excitation. A simple expression was obtained for the Coulomb polarization potential which was then used to calculated the sub-barrier elastic cross section. Another method, the Born series summation, was also considered and it gave very close results for the elasticcross section as the WKB approximation. By adjusting the Coulomb polarization potential so that it reproduces, on the average, the Alder-Winther potential we were able to account, to some extent, for the off-energy-shell effects. The resulting adjusted potential, seems to describe well the effect of multiple Coulomb excitation on the heavy ion sub-barrier elastic scattering.

In the present paper we apply the method developed in I tothe sub-barrier heavy ion inelastic scattering. This application is, in our view, an important testing ground for any approximation schemes devised to simplify the heavy ion coupled channels problem and should be considered in conjunction with the application to elastic scattering. Although the semiclassical theory of multiple Coulomb excitation as used, e.g., in the de Boer-Winther[2] code is adequate for not too many coupled channels, the need for alternative approximation becomes apparent when the number of the coupled channels increases. Several methods[3] have been developed to calculate the inelastic cross sections for heavy ions in the case of strong coupling. We mention in particular, the methods based on the work of Miller which allows some feedback onto the trajectory due to the coupling to the different excited states (channels). However, these methods have been, so far, restricted to back-angle scattering only. Our aim in this work is to solve the coupled-channels.

Multiple Coulomb excitation problem exactly within the on-energy-shell approximation adopted for the channels' Green's function. This procedure allows for the construction of the different inelastic amplitude in a simple closed form and thus permits the introduction of improvements in a simple way. The numerical evaluation of the amplitudes involves only the inversion of finite matrices which was discussed in I. Furthermore, the introduction of the nuclear effects, which becomes important at higher energies, is straightforward and will be discussed in part 3 of this series. Although we do not expect our final results, which are based on the OESA, to be very close to the exact coupled channels or de Boer-Winther results, we do believe, however, that such an OESA procedure could serve as a reasonable starting point for a more precise, yet simpler, calculation of the effects on the different heavy ion processes due to strong coupling to many other channels.

Furthermore, as was discussed in I, the formalism we develop allows for a very clear separation between reorientation effects and coupling effects and thus allows for a simple exploratory study of the change in the nature of the channeis, i.e., vibrational vs. rotational. Although the energy loss associated with the different excitations processes was accounted for in I, using the semiclassical energy loss factors, in the present paper the energy loss is accounted for exactly, albeit semiclassically. After some preparatory developments in Section 2 we derive, in the OESA, general expressions for the inelastic amplitudes which contain multiple Coulomb excitation effects to all orders. In Section 3 we evaluate the sub-barrier inelastic cross sections for the system $^{40}Ar + ^{238}U$ at E_{lab} = 240 MeV assuming for the excited states a pure rotational character. Comparison of our result at back angles with those obtained by Alder and Winther[7] are discussed. Finally we give a general discussion of our results and several concluding remarks in Section 4.

2. PRELIMIINARIES

In this section we summarize some of the results obtained in I which are relevant for our discussion of the inelastic cross section. We consider the collision of a spherical nucleus 1 on a deformed target

nucleus 2, at sub-barrier energies. We study the Coulomb excitation of low-lying states IM of spin I and magnetic quantum number of which excitation energy E_I. We further assume nucleus 1 to be a point charge $Z_1 e$; then from the set of coupled radial equations givenin Section 2 of I we can extract the coefficients $T^J_{\ell I_1 \ell_0 I_0}$ which determine the amplitudes $f_{I_0 M_0 \to I_M}(\theta)$ for Coulomb excitation from the ground state $|I_0 M_0>$ to the final state $|IM>$ (in coordinate system C of Ref. 6) through

$$f_{I_0 M_0 \to IM}(\theta,\phi) = \frac{i\sqrt{\pi}}{\sqrt{k_I k_0}} \sum_{\ell \ell_0 J} \sqrt{(2\ell+1)} <\ell_0 0 I_0 M_0 | J M_0> (i)^{\ell_0 - \ell} <\ell m\ IM | J M_0> \{\delta_{I I_0}\ \delta_{\ell \ell_0} - e^{i\sigma_\ell(\eta_I)+\sigma_{\ell_0}(\eta_{I_0})}\ T^J_{\ell I, \ell_0 I_0}\}\ Y_{\ell m}(\theta,\phi) \tag{1}$$

where ℓ_0(L) is the orbital angular momentum in the incident (final) channel k_0(k_I) is the asymptotic wave number in the incident (final) channel J is the conserved channel angular momentum, and $\sigma_R(\eta)$ is the Coulomb phase shift with η being the Sommerfeld parameter. In what follows we consider the ground state to be 0^+ which fixes the value of $J=\ell_0$. The inelastic cross section is then given by

$$\frac{d\sigma_I}{d\Omega} = \frac{v_I}{v_0} \sum_M |f_{0\ 0 \to IM}(\theta,\phi)|^2 \tag{2}$$

where $v_I(v_0)$ is the relative velocities of the heavy ions after (before) the collision. By using the on-energy-shell approximation for the channel Green's function we were able to obtain the solution for the matrix $T^J_{\ell I, \ell_0 0}$ in closed form

$$\underline{T} = \frac{k^{-1/2}\ \underline{C}\ k^{-1/2}}{1 + i k^{-1/2} \underline{C}\ k^{-1/2}} \tag{3}$$

where the matrix $\underline{C}$ is given by

$$\underline{C}_{\ell'' I'', \ell I} = \frac{1}{k_{I''}}\ a_{\ell'' I'', \ell I}\ I_{\ell'' \ell}(k_{I''}, k_I) \tag{4}$$

Here the coupling matrices a have the form (assuming quadrupole coupling)

$$a_{\ell' I', \ell I} = \sqrt{a_I a_{I'} \eta_I \eta_{I'}}\, q_{I \to I'} \begin{pmatrix} I & I' & 2 \\ 0 & 0 & 0 \end{pmatrix} \sqrt{(2I+1)\,(2I'+1)} \times$$

$$\times \sqrt{(2\ell+1)\,(2\ell'+1)}\,(-1)^{\ell+I'} \begin{pmatrix} \ell & \ell' & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \ell & \ell' & I' \\ 2 & I & \ell \end{Bmatrix} \tag{5}$$

and $I_{\ell'' \ell}(k_{I''}, k_I)$ are the usual Coulomb excitation integrals given by

$$I_{\ell'' \ell}(k_{I''} k_I) = \int_0^\infty dr\, F_{\ell''}(k_{I''} r)\, \frac{1}{r^3}\, F_\ell(k_I r) \tag{6}$$

In Eq. (5) the quadrupole coupling strength $q_{I \to I'}$ are defined by

$$q_{I \to I'} = \sqrt{\frac{\pi}{5}}\, \frac{\sqrt{\eta_I \eta_{I'}}}{a_I a_{I'}}\, \frac{\langle I \| M(E2) \| I' \rangle}{Z_2 e}$$

$$\times \left[\sqrt{(2I+1)\,(2I'+1)} \begin{pmatrix} I & I' & 2 \\ 0 & 0 & 0 \end{pmatrix} \right]^{-1} \tag{7}$$

The quantity a_I is half the distance of closest approach for head-on collision in channel ■ and is given by

$$a_I = \frac{Z_1 Z_2 e^2}{2(E - E_I)} \tag{8}$$

The Sommerfeld parameters η_I are defined as usual by $Z_1 Z_2 e^2 / \hbar v_I$ with being the asymptotic relation velocity in the center of mass system in Channel I. In the special case of a pure quadrupole rotational band description of the excited states in the target the couplings $q_{I \to I'}$ as defined in (7) satisfy the simple relation

$$q_{I \to I'} = (-)^{\frac{I+I'+2}{2}}\, q_{0 \to 2} \tag{9}$$

Our expression for T (Eq.(3)) guarantees that the resulting S-matrix, defined by 1-2 iT, is unitary

In order to carry out the numerical calculation we have used Eq. (3) to obtain the following recursion equation that relates the amplitudes for the different processes. The details of the derivation of this recursion relation is given in Appendix 1:

$$R_{I0} = -iC_{II-2}\left[1 + iC_{I-2\ I-2} + C_{I-2<}\frac{1}{1+iC_{<<}}C_{<I-2}\right]^{-1} \qquad (10)$$

where

$$R_{I0} \equiv -iC_{I<}\left[1+iC_{<<}\right]^{-1}\delta_{I0} \qquad (11)$$

and $R_{00} = 1$ and the symbol < refers to states with spin less than ■ _ The matrix element $\langle I|\left[1+i\underline{C}\right]^{-1}0\rangle \equiv g_{I0}$ is given in terrns of R_{I0} by

$$g_{I0} = \left|1+iC_{II} + C_{I<}\left[1+iC_{<<}\right]^{-1}C_{<I} + C_{I>}\left[1+iC_{>>}\right]^{-1}C_{>I}\right|^{-1} R_{I0} \qquad (12)$$

where > implies all ■ is larger. The matrix elements $\langle I|T|0\rangle$ are identical to g_{I0} since for $I\neq 0$.

$$\langle I|\left[1+i\underline{C}\right]^{-1}iC'|0\rangle = \langle I|\left[1+i\underline{C}\right]^{-1}|0\rangle \ . \qquad (13)$$

Eqs. (12) and (13) constitute the basis of our calculation of the inelastic amplitudes. Implicit in our calculations is the usual semiclassical assumptions of large R and η. The results in the following form for the inelastic cross section

$$\frac{d\sigma_I}{d\Omega} = \sigma_R(\theta)\ P_I(\theta) \qquad (14)$$

where $\sigma_R(\theta)$ is the Rutherford cross section and $P_■(\theta)$ is the probability for Coulomb exciting the state ■ and is determined basically by $|g_{I0}|^2$.

3. NUMERICAL RESULTS

Before entering into the details of our numerical results we give below an account of our treatment of the energy loss. Although in I we accounted for the energy loss through the introduction of the ccnstant semiclassical energy loss factors, in this paper we try to treat the energy loss more realistically. Since in our theory energy loss comes in basically through the Coulomb excitations integrals, $I_{\ell',\ell}(k_{I'},k_I)$ of Eq. (6), we therefore used the semiclassical theory of Coulomb excitation to calculate the I's and utilized the tables given in Ref.8. Such a procedure to approximately account fortheenergy loss in the excitation processes turns out to be quite reasonable especially in the calculation of the inelastic cross section, σ_{2+}. If, on the other harid, we were to use the constant semiclassical energy loss factors of i the $\sigma_{2+}(\theta)$ would show an almost monotonic decrease with θ_{CM} and a maximum at $\theta=0$. As we show below our calculated $\sigma_{2+}(\theta)$ exhibits a maximum at some small angle and drops to zero at $\theta=0$ as it should[7].

To be specific we shall consider below the system $^{40}Ar+ ^{238}U$ at $E_{lab} = 240$ MeV. Although the center of mass energy corresponding to the above E_{lab} is slightly higher than the Coulomb barrier of 183 MeV, we shall ignore all nuclear effects in our calculation. We shall assume a quadrupole rotational band structure for the excited states in ^{238}U and shall ignore projectile excitation. Since the value of q for the above system is 9.56, [9] it is clear that we must consider in our calculation the coupling to all states with $I < 18$. This requirement is a consequence of the approximate semiclassical relation between the average angular momentum transferred, $\langle\Delta\ell\rangle$, and $q_{0\to2}$ namely[7]

$$\langle\Delta\ell\rangle = \langle I_{max}\rangle \simeq 2q_{0\to2} \tag{15}$$

We have used our recursion equations (10)-(13) using for the coupling matrix $\underline{C}$ the explicit form given in Eq. (4) with the I's calculated according to the prescription given previously. We have generated a pure rotational band for ^{238}U by taking the experimental value for the first excited states to be 44 keV [9] and used the rotational energy formula $E_I = \hbar^2 I(I+1)/2g$. The results of our calculations are shown in

Figs. 1-3. The elastic cross section normalized to the Rutherford cross section is plotted in Fig.1 . The rise of σ/σ_R back angles will, of course, be modified if nuclear effects were included. Also shown in Fig. 1 is the inelastic cross section σ_{2+} for exciting the 2^+ state. Our results for σ_{2+} is qualitatively similar to what one expects for smaller $q_{0\to 2}$. It is interesting to observe that with increasing I the peak in $\sigma_{I+}(\theta)$ is shifted to larger angles and the right wing of the cross section becomes flatter and flatter as shown in Fig. 2 until finally the peak disappears completely starting at I=16 (see Fig. 3). It is important to note that our results for σ_{el} and σ_{I+} satisfy the unitarity sum rule at every angle, i.e.,

$$\frac{\sigma_{el}}{\sigma_R} + \sum_{I=2}^{18} \frac{\sigma_I}{\sigma_R} = 1 \qquad (16)$$

It is worth noting that if we mock up some of the off-shell effects by multiplying the reorientation matrices by a factor 2-3 and the coupling matrices by 4.6, as was done in I, we would obtain peaking, in all

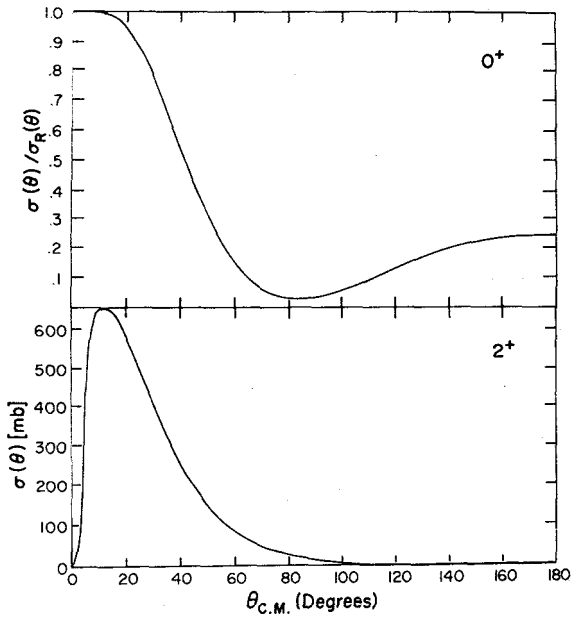

Fig.1 - The sub-barrier elastic cross section, normalized to σ_R, for the system $^{40}Ar + ^{238}U$ at E_{lab} = 240 MeV plotted vs. θ_{CM}. Also shown is the inelastic cross section σ_{2+}.

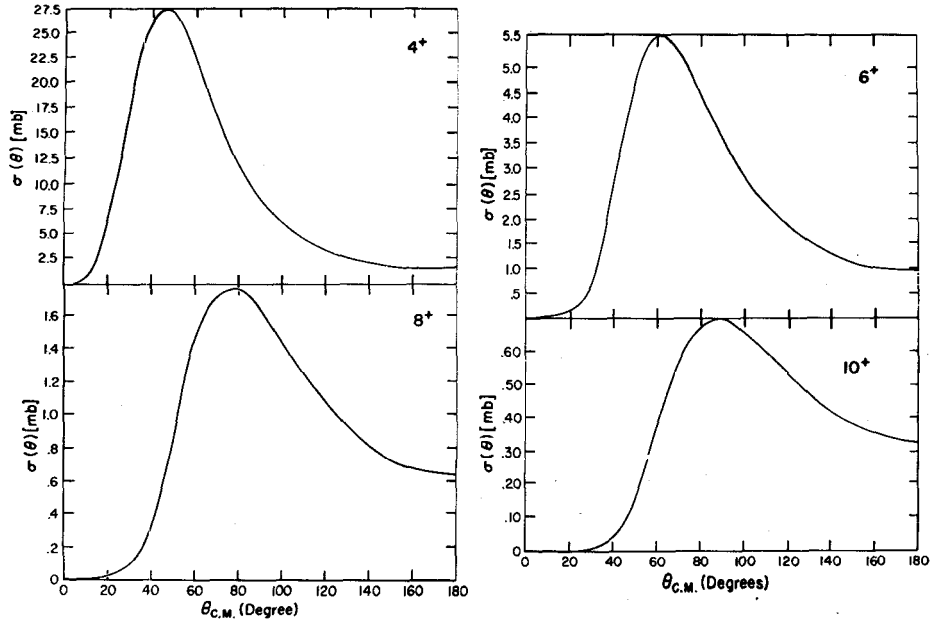

Fig. 2 - The inelastic angular distributions σ_{4+}, σ_{6+}, σ_{8+} and σ_{10+}.

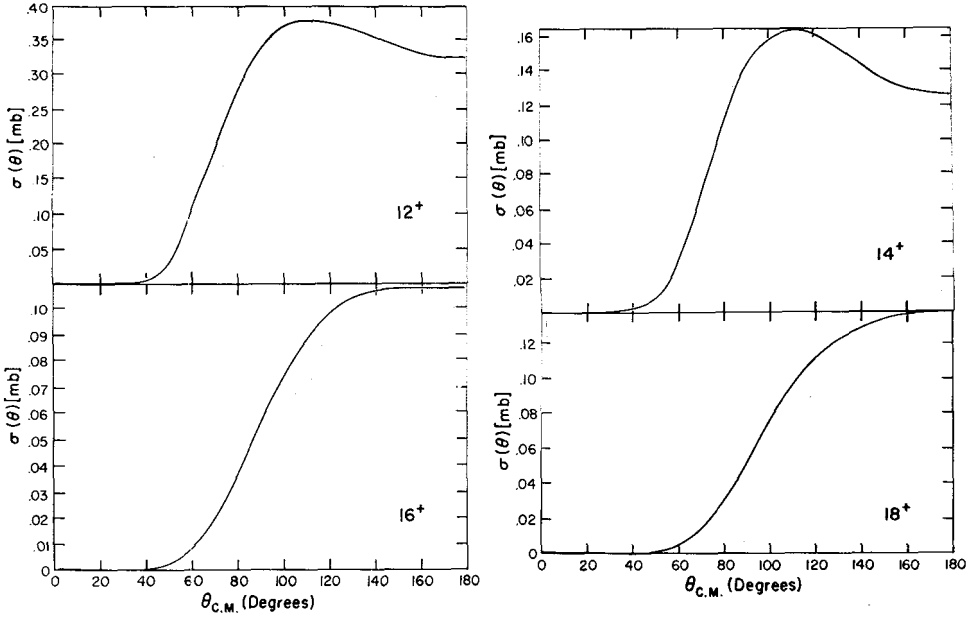

Fig.3 - The inelastic angular distributions σ_{12+}, σ_{14+}, σ_{16+} and σ_{18+}.

$\sigma_{I\neq 0}$ at some nonzero angles. It is clear that the spin dependence of our resulting Coulombexcitation probabilities $P_I = \frac{\sigma_I}{\sigma_R}$ evaluated at back angles shows amarked difference from the results of semiclassical theories . Whereas exact quantum mechanical and semiclassical theories show an initial rise of P_I as a function of increasing I and then, after passing through a maximum, drops precipitantly as the excitation of states with higher spins become classlcally forbidden, our results show, on the other hand, a rather smooth decrease of P_I with increasing I. This failure of our theory in reproducing the exact quantum mechanical and semiclassical results is a clear indication of the limitations of the on-energy-shell-approximation. It is hoped, however, that one might be able to account, approximately, for the off-shell effects by usirig the simple OESA for the wave function (or amplitude) as an input inhomogeneous term in an integral equation whose kernel contains the off-shell parts of the channel function. It is further hoped that íteratíve, and therefore less time-consuming, solutions of this equation would suffice to bring the results closer to the exact quantum mechanical or semiclassical results. In a future publication we shall present an account of our endeavor mentioned above.

4. DISCUSSION AND CONCLUSION

In this paper we have discussed the consequences of adopting the on-energy-shell approximation for the channel Green's function in the coupled channels description of multiple Coulomb excitation in heavy ion collisions. Our work in this paper is a natural exterision of the results reported in the first part (I) of this series which dealt with sub-barrier elastic scattering. Although in the present paper we have improved upon our treatment in i of the energy loss in the different excitation processes, the overall qualitative behavior of our results differ considerably from those of the exact quantum mechanical and semiclassical treatments. This shortcoming of our approximations is certainly a consequence of our neglect of all off-shell effects. These off-shell effects manifest themselves in the nonseparable form of the r- and r' dependence of the channel Green's function. It is hoped, however, that by isolating tne separable part of channel Green's function and calculating the corresponding wave function and amplitude, a

simple estimate of the inelastic cross section at sub-barrier energies becomes available. Furthermore, such an OESA wave function might serve as an input inhomogenous part in an integral equation whose kemel contains the principal value of the Green's function. It is our expectation that iterative and less time consuming solutions of such an integral equation could be obtained which would contain most of the off-shell effects and thus would give comparable results to those obtained from exact quantum mechanical and semiclassical treatments. In the third part of this series[5] we shall present the results of our program above as well as develop a method for inclusing nuclear excitations.

APPENDIX 1

In this appendix we derive the recursion relation Eq. (12), and in our calculation. Our aim is to calculate the matrix element $\langle I|(1+i\underline{C})^{-1}|0\rangle \equiv g_I$. First we write

$$(1 + i\underline{C})\, g_I = \delta_{I0} \tag{1.1}$$

We now define the projection operator P_I which projects onto the I-state. We also define $P_<$ and $P_>$ which project onto the space spanned by all states with spin smaller than I and larger than I, respectively. Since the quadrupole nature of the coupling requires that $P_< \underline{C} P_> = P_> \underline{C} P_< = 0$ we, therefore, have for $I \neq 0$

$$(1 + iC_{II})g_I + C_{I<}g_< + iC_{I>}g_> = 0 \tag{1.2}$$

$$(1 + i\underline{C}_{>>})g_> + iC_{>I}g_I = 0 \tag{1.3}$$

$$(1 + i\underline{C}_{<<})g_< + iC_{<I}g_I = \delta_{I0} \tag{1.4}$$

Solving (1.3) and (1.4) for $g_>$ and $g_<$ and substituting into Eq. (1.2) and solving for g_I we obtain

$$g_I = -[1 + iC_{II} + C_{I<}(1 + iC_{<<})^{-1} C_{<I} + C_{I>} (1 + iC_{>>})^{-1} C_{>I}]^{-1}$$

$$\times C_{I<}(1 + iC_{<<})^{-1} \delta_{I0} \qquad (1.5)$$

In the special case of $I = 0$ we have $P_< \underline{C} = 0$ and $g_< = 0$. Therefore,

$$(1 + iC_{00})g_0 = - iC_{0>}g_> + \underline{1} \qquad (1.6)$$

$$(1 + iC_{>>})g_> = -iC_{>0}\, g_0 \qquad (1.7)$$

Solving for g_0 we obtain

$$g_0 = [1 + C_{0>} (1 + iC_{>>})^{-1} C_{>0}]^{-1} \underline{1} \qquad (1.8)$$

Since the operator $(1 + iC_{<<})^{-1}$ appearing in Eq. (1.5) acts only in the $<$ - subspace we can rewrite it in terms of operators acting in smaller sub-spaces. Specifically since the maximum spin in the $<$ - subspace is 1-2, we have for $\langle I\text{-}2|(1 + i\underline{C}_{<<})^{-1}|0\rangle \equiv \hat{g}_{I-2}$ the following

$$\hat{g}_{I-2} = [1 + iC_{I-2\ I-2} + C_{I-2<}(1 + iC_{<<})^{-1} C_{<I-2}]^{-1}$$

$$[- iC_{I-2<}(1 + i\underline{C}_{<<})^{-1} \delta_{I0} \qquad (1.9)$$

where $<$ implies smaller than 1-2. Defining now the matrix element

$$R_I = - iC_{I<}(1 + iC_{<<})^{-1} \delta_{I0} \qquad (1.10)$$

with $<$ implying smaller than I. We can combine (1.9) and (1.10) into one equation namely

$$R_I = - i\, C_{I\ I-2}\, \hat{g}_{I-2}$$

$$= - i\, C_{I\ I-2}\, [1 + i\, C_{I-2\ I-2} + C_{I-2<}(1 + i\, C_{<<})^{-1} C_{<I-2}]^{-1} R_{I-2}$$

$$(1.11)$$

Eq. (1.11) is to be solved with the "boundary condition" $R_0 = 1$, and the matrix element g_I is obtained from Eq. (1.5) which are' rewritten as

$$g_I = \left[1 + i\,C_{II} + C_{I<}(1 + i\,C_{<<})^{-1}\,C_{<I} + C_{I>}(1 + i\,C_{>>})^{-}\,C_{>I}\right]^{-1} \times R_I \tag{1.12}$$

REFERENCES

1. B.V.Carlson and M.S.Hussein, Preceding paper.

2. J.de Boer and Aa. Winther, in *Coulomb Excitations*, edited by K.Alder and Aa. Winther (Academic Press, New York, 1966).

3. S.Levit, U.Smilansky and D.Pelte, Phys.Lett. *53*B, 39 (1974); H.Massmann and J.O.Rasmussen, Nucl. Phys. A*243*, 155 (1975); R.Donangelo et *al*., Phys.Lett. *46*B, 377 (1977).

4. W.H.Miller, Adv. Phys. Chem. 25, 67 (1974).

5. B.V.Carlson and M.S.Hussein, to be published.

6. J.de Boer, G.Dannhauser, H.Massmann, F.Roesel and Aa. Winther, J. of Phys. G:Nucl. Phys. 3, 889 (1977).

7. K.Alder and Aa. Winther, *Electromagnetic Excitation* (North Holland Publishing Co., Amsterdam (1975)).

8. K.Alder, A.Bohr, T.Huus, B.Mottelson and Aa.Winther, Rev.Mod. Phys. 28, 1432 (1956).

9. Nuclear Data Sheets.