Revista Brasileira de Física, Vol. 10, Nº 2, 1980

Electron Temperature Spatial Oscillations and the Electrothermal Instability in the Diffuse Pinch

A. TOMIMURA

Instituto de Física, Universidade Federal Fluminense, C.P. 296, 24000 Niterói, RJ

and

M. G. HAINES

Blackett Laboratory, Imperial College, London SW7

Recebido em 26 de novembro de 1979

Energy and momentum equilibria of magnetically confined plasmas in the diffuse pinch and tokamak configurations have been found with scalar classical tensor classical and neo-classical transport models. A radial oscillation of the electron temperature has the same wavelength ~ $a_e(m_i/m_e)^{1/2}$ (a_e = electron Larmor radius) as the fastest growing electrothermal instability. Its non-linear amplitude enhances electron energy loss by equipartition. The plasma-wall boundary conditions examined in each case; and, in particular, the scalar classical transport model is found to give a finite pressure here.

Os estados de equilíbrio de um "pinch" difuso e configurações do tipo Tokamak, magneticamente confinados, foram estabelecidos mediante o uso dos balanços de pressão e energia em modelos clássicos e neo-clássicos de transporte com condutividade escalar e tensorial. Uma oscilação radial na temperatura de eletrons tem o mesmo comprimento de onda ~ $a_e(m_i/m_e)^{1/2}$ (a_e = raio de Larmor do eletron) que o modo associado com o crescimento mais rápido de uma instabilidade eletrotérmica.

* Partially supported by Conselho Nacional de Desenvolvimento Científico (CNPq).

Sua amplitude (não-linear) amplifica a energia perdida por eletrons por equipartição. As condições de contorno plasma-parede são examinadas em cada caso; em particular, o uso da condutividade escalar no modelo clássico de transporte resulta numa pressão finita ($\neq 0$) naquele contorno.

INTRODUCTION

Three steady state models for the two-temperatures diffuse pinch and tokamak have been studied under conditions representing momentum and energy equilibria. In the first model, a scalar electric conductivity is assumed and numerical solutions of the transport equations show that it is possible for the electron temperature to have a radially oscillating component with a wavelength of the order $(m_i/m_e)^{1/2}$ a_e where a_e is the electron Larmor radius, which is also approximately the wavelength of the fastest growing electrothermal instability in a spatially homogeneous plasma with $\underline{k}$ perpendicular to $\underline{B}$. This results in enhanced electron energy loss to the ions.

It has also been shown that in this model an acceptable boundary condition (low pressure and temperatures at the plasma-wall boundary) is not possible, posing a serious question on the plasma confinement. A second model where an electric tensor conductivity is assumed with thermoelectric effects however yields equilibria profiles with the desired boundary conditions provided severe restrictionsare imposed upon its free parameters. A third model employing neoclassical transport has similar properties to the second. A strong axial magnetic field and a two temperature plasma is included in all models.

1. A SCALAR CONDUCTIVITY MODEL

The transport equations for a steady-state cylindrical equilibrium of a magnetically confined plasma with only radial dependence of parameters can be written in a straight-forward manner if diffusion of particles is reduzed to zero by prescribing an appropriate axial electric field. Thermoelectric and viscosity effects are neglected and a scalar electric conductivity is assumed as follows:

Electron energy: $\frac{j_z^2}{a} = \frac{\qquad)}{m_i \, a} - \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{r\, K_e \frac{\partial T_e}{\partial r}}{1 + \omega_e^2 \tau_e^2}\right]$, (1.1)

Ion energy: $\frac{3n^2 e^2 k_B (T_e - T_i)}{m_i \, \sigma} = -\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{r\, K_i \frac{\partial T_i}{\partial r}}{1 + \omega_i^2 \tau_i^2}\right]$, (1.2)

Pressure balance: $\frac{\partial}{\partial r} n k_B (T_e + T_i) = -\frac{j_z B_\theta}{c}$ (1.3)

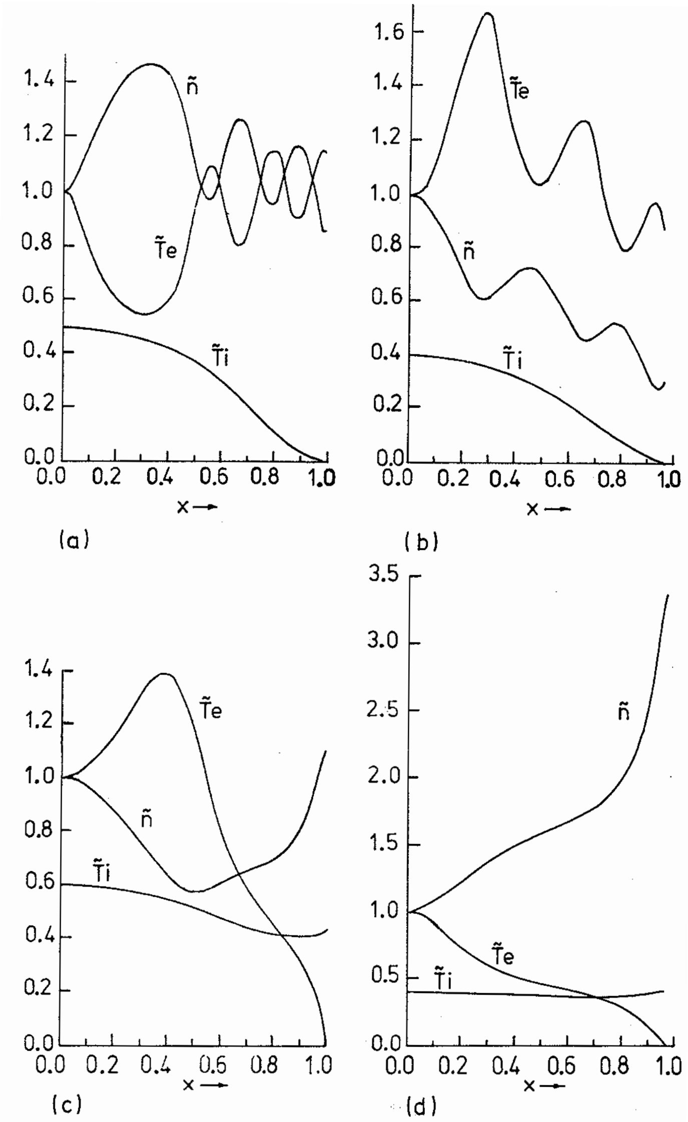
Ampere's law: $\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \frac{4\pi}{c} j_z$ (1.4)

Ohm and Faraday's laws: $\sigma E_z = j_z$, $\frac{\partial E_z}{\partial r} = 0$ (1,5) , (1.6)

where the classical temperature dependence[1] of a, K_e and K_i are assurned. In this model the axial magnetic field does not contribute to momentum balance, but does affect the' radial thermal conduction.

By prescribing the free paramaters E_z, B_z, T_{i0}, T_{e0} and n_0 where we have the on axis boundary conditions $T_e(0) = T_{e0}$, $T_i(0) = T_{i0}$, $n(0) = n_0$, $(\partial T_e/\partial r = \partial T_i/\partial r = B)_{r=0} = 0$, eqns. (1.1)-(1.6) have been numerically solved employing a Runge-Kutta technique to obtain the dimensionless parameters $\tilde{T}_e = T_e/T_{e0}$, $\tilde{T}_i = T_i/T_{e0}$, $\tilde{n} = n/n_0$ in terms of $x = r/r_0$, the subscript 0 referring to values at $r = 0$ except for r_0 which is an arbitrary scale factor. The integration conducted from the axis proceeds until one of the parameter $\tilde{T}_e$ or $\tilde{T}_i$ goes to zero. Iteration methods fail to find solutions for which $\tilde{T}_e$, $\tilde{T}_i$ and $\tilde{n}$ all become zero at some radius which would define the plasma-wall boundary. The introduction of radiation and viscosity in the model fails to relax the wall boundary conditions. Indeed it can be shown by analytical expansion of the parameters that the equations only permit two types to solutions according to their behaviour at the boundary, viz., (i) $T_e \to 0$ with $T_i \neq 0$, $n \neq 0$ as in figs. 1 (c) and 1 (d); (ii) $T_i \to 0$ with $T_e \neq 0$, $n \neq 0$ as in figs. 1 (a) and 1 (b).

It could be that inclusion of some further physical processes (e.g., neutrals) might relax these conditions but as can be seen in



figs.1	Teo (°K)	$\tilde{T}_i(0)$	$n_o(cm^{-3})$	Bz(Teslas)	Ez(esu)	r_o(cm)
(a)	10^7	0.5	10^{14}	2	1.1×10^{-5}	2.3
(b)	10^8	0.4	„	„	5×10^{-7}	5.0
(c)	3×10^8	0.6	„	„	1.7×10^{-7}	6.4
(d)	5×10^7	0.4	„	„	3.2×10^{-6}	1.6

Fig.1 (a), (b), (c) and (d). Four typical radial profiles of dimensionless electron temperature $\tilde{T}_e$, number density $\tilde{n}$ and ion temperature $\tilde{T}_i$ for the scalar conductivity model.

the figures, the parameters that remain finite are not small and so there is no obvious reason for excluding the validity of the model. The solutions imply the tendency for the plasma to have a finite pressure at the wall, i.e., the plasma is not confined. Whilst one might question the conclusions of the wall-boundary solutions because of the assumption cif steady-state and the neglect of other physical processes that might arise say, due to the presence of neutra1 particles, the interesting spatial variations of T_e and n are of more general validity. The spatial oscillations are probably the non-linear steady amplitude of an electrothermal istability and their wavelength is approximately $a_e(m_i/m_e)^{1/2}$ where a_e is the electron Larmor radius[2]. Their amplitude can be estimated by equating Ohmic heating at one peak with the equipartition in the adjacent trough and is given by

$$\tilde{T}_{ep}^{3/2} \simeq 5.15 \times 10^{-26}(n_0^2/E_z^2\ T_{e0}^2)(1 + \tilde{T}_{i0})^2/\tilde{T}_{it}^{5/2} \qquad (1.7)$$

where $\tilde{T}_{it}$ is the average value of $\tilde{T}_i$ at the trough. The amplitude of the spatial temperature oscillations here is comparable with that of the temporal oscillations measured on the PLT Tokamak[3] and the electrothermal instability might well be the explanation of this latter phenomenon.

One effect of the spatial oscillations of electron temperature and the π out of phase oscillations of electron density is to give an enhanced electron energy loss which can be described as follows. Ohmic heating is a maximum (through σE^2) in the electron temperature peaks. The heat is transported by electron thermal conduction across the magnetic field lines, but only over the short radial scale length $a_e(m_i/m_e)^{1/2}$ which is the scale length of the electron temperature gradient, to the electron temperature troughs. . Equipartition of energy to the ions has a maximum at T $\simeq$ 1.58 T. for a given pressure, and this is close to the minimum of T_e in the spatial oscillations. In comparison with a spatially averaged monotonic profile of temperature, the spatially oscillatory profile leads to an enhanced electron energy lossdue to the increased equipartition to ions in the temperature minima. This could perheps contribute to the apparent anomalous electron energy loss in Tokamaks[4].

2. TENSOR CONDUCTIVITY MODEL

Neglecting diffusion and viscosity, assuming an anisotropic electric conductivity and considering thermo-electric effects, the transport equations for the two-temperature fully ionized diffuse pinch can now be written with the help of Braginskii's classical transport theory formulation[5] and in his own notation as follows:

Electron energy: $$\frac{1}{r}\frac{\partial}{\partial r}(rq_r^e) = j_z E_z - 3\frac{m_e}{m_i}\frac{n_e(T_e-T_i)}{\tau_e} \tag{2.1}$$

with $$q_r^e = -\left(\chi_\perp^e - a_\perp \frac{\beta_\wedge^{Tu}\beta_\wedge^{uT}}{(en)^2}\right)\frac{\partial T_e}{\partial r} - \frac{\beta_\wedge^{Tu}}{en}\,\frac{\sigma_\perp}{e}\, E_z \varepsilon \tag{2.2}$$

Ion energy: $$\frac{1}{r}\frac{\partial}{\partial r}(rq_r^i) = 3\frac{m_e}{m_i}\frac{n_e(T_e-T_i)}{\tau_e} \tag{2.3}$$

with: $$q_r^i = -\chi_\perp^i\,\partial T_i/\partial r \tag{2.4}$$

Pressure balance: $$\frac{\partial p}{\partial r} = \frac{B\sigma_\perp}{c}\left(\frac{\beta_\wedge^{uT}}{en}\frac{\partial T_e}{\partial r} - \varepsilon_\theta E_z\right), \tag{2.5}$$

Ampere's law: $$\frac{\partial B_z}{\partial r} = -\frac{4\pi}{c} j_\theta\,, \quad \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \frac{4\pi}{c} j_z\,, \tag{2.6), (2.7}$$

Faraday's law: $$\partial E/\partial r = 0\,, \tag{2.8}$$

Ohm's law:
$$j_z = (\varepsilon_\theta^2\sigma_\perp + \varepsilon_z^2\sigma_{||})E_z - \varepsilon_\theta\sigma_\perp\frac{\beta_\wedge^{uT}}{en}\frac{\partial T_e}{\partial r} \tag{2.9}$$

$$j_\theta = (\sigma_{||} - \sigma_\perp)\varepsilon_\theta\varepsilon_z E_z + \varepsilon_z\sigma_\perp\frac{\beta_\wedge^{uT}}{en}\frac{\partial T_e}{\partial r} \tag{2.10}$$

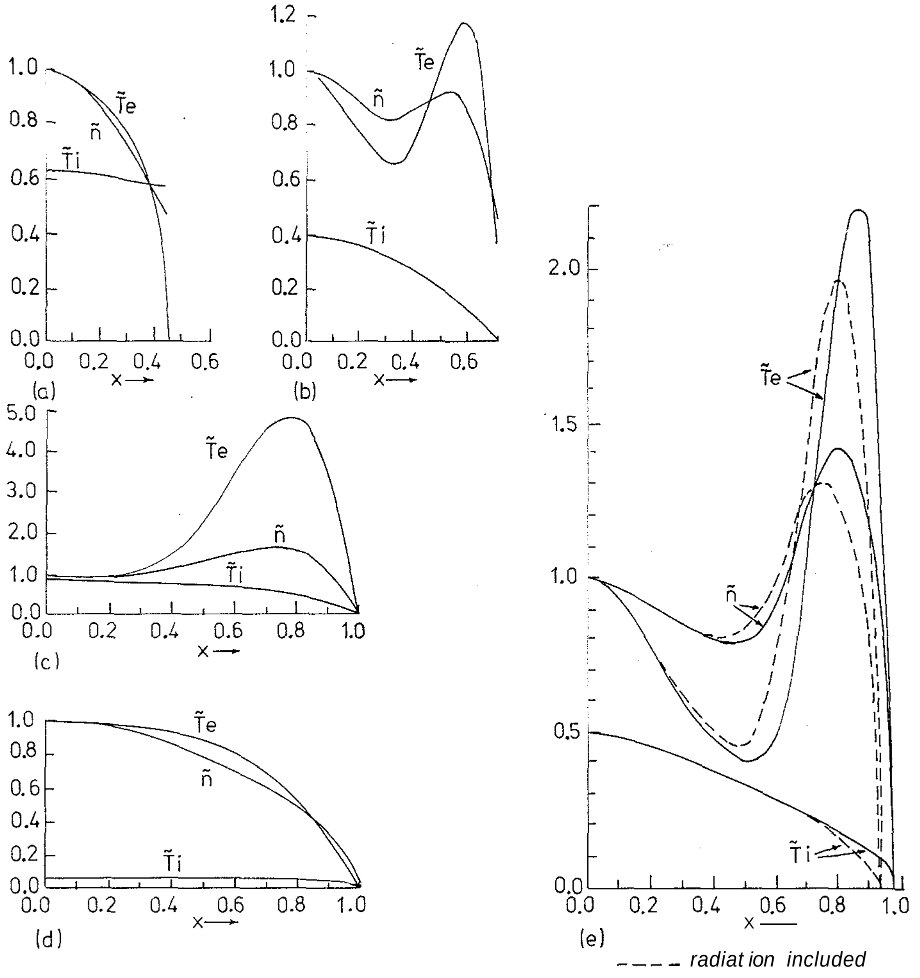
where $\varepsilon_\theta = B_\theta/B$, $\varepsilon_z = B_z/B$, $B = (B_\theta^2 + B_z^2)^{1/2}$.

In terms of the same dimensionless parameters defined before and using the same boundary conditions at r=0 and numerical integration routine, the set of equations (2.1)-(2.8) above are solved yielding equilibria profiles such as in figs. 2(a) - 2(e). The profiles presented in figs. 2(c) - 2(d) are obtained by an iteration scheme based on a fine partitioning of some range of values of the free parameter T_{i0}/T_{e0} (fig. 2(c)) and E_0 (the axial electric field) (figs. 2(d) and 2(e)) end were found as a limiting case of profiles like those presented in figs. 2(a) and 2(b). In the uni-dimensional parameter space of E_0 (or T_{i0}/T_{e0}) however, only one discrete value of E_0 (or T_{i0}/T_{e0}) gives profiles where the appropriate boundary conditions are met (i.e., zero pressure and temperatures at the wall) such as in figs. 2(c), (d) and (e). Bremsstrahlung radiation has little effect on the overall profiles as can be seen by the broken lines of the plots in fig.2(e). The question of whether in practice the plasma profiles adjust to the very precise condition on, say, the parameter E_0, so as to satisfy the boundary, condition of zero plasma pressure at the wall, or whether a finite pressure parsists as in the scalar conductivity model cannot be answered at this stage.

It can be shown that thermo-electric effects in this model are resporisible for the desired boundary conditions being met but unlike in the previous model, the equilibria profiles now present fewer spatial oscillations for the electron temperature. Indeed it is possible to obtain hollow electron temperature profiles (fig. 2(c)) as has been observed experimentally[6] without recourse to neutrals or impurities.

3. NEOCLASSICAL TRANSPORT MODEL

An equilibrium energy and momentum balance can be set up employing neoclassical transport coefficients[7] in a similar way to MacMahon and Ware[8]. As in the tensor transport model above it is found that by iterating one of the free parameters (e.g. E_0) it is possible to reduce the 'pressure to a very small value at the wall. However a small deviation in the now defined value of a free parameter leads .to



figs 2	Teo(°K)	T̃i(0)	no(cm^{-3})	Bo(Teslas)	Eo(esu)	ro(cm)
(a)	2.47×10^{8}	0.64	10^{14}	1	1.765×10^{-7}	10.0
(b)	5×10^{7}	0.4	,,	2	1.3×10^{-6}	4.5
(c)	10^{7}	0.8471	,,	2	3.052×10^{-6}	2.4
(d)	5×10^{7}	0.07	5×10^{14}	2	7.322×10^{-6}	1.06
(e)	10^{7}	0.5	10^{14}	1	6×10^{-6}	4.0

Fig. 2 (a), (b), (c), (d) and (e). Five typical radial profiles or dimensionless electron temperature $\tilde{T}_e$, number density **Z** and ion temperature $\tilde{T}_i$ for the tensor conductivity model.

large changes in the profile of the electron temperature and a strong violation of the wall boundary condition. This sensitivity of the steady state profiles to a small perturbation leads us to doubt whether in practice the pressure will be small at the wall.

4. THE ELECTROTHERMAL INSTABILITY FOR k PERPENDICULAR TO B AND j

Electrothermal instabilities in a fully ionized gas have been previously considered for $\underline{k}$ parallel to $\underline{B}$[9] and a simple correction of the optimum wavelength for growth to the case of $\underline{k}$ perpendicular to B[10] gives a wavelength of $a_e(m_i/m_e)^{1/2}$, where a_e is the electron Larmor radius, because the electron thermal conduction is reduced by a factor of $(1 + \omega_e^2\tau_e^2) \gg 1$. Thus we can consider the wavelength to be small compared to the scale length of the equilibrium profile and, for ease of analysis, we perturb a homogeneous equilibrium with uniform pressure p and with $\underline{j}_0 \times \underline{B}_0 = 0$. As argued previously the ion motion should not be neglected, and, as a result a dispersion equation is derived for $\underline{k}$ perpendicular to $\underline{B}_0$ and $\underline{j}_0$ that is quintic in the growth rate and quartic in the square of the perpendicular wave number. (Details will be published elsewhere). For $\underline{k}$ parallel to $\underline{B}$ the dispersion equation[9] is quartic in the growth rate. The growth rate is close to $m_e/(m_i\tau_e)$ where τ is the electron-ion collision time. The optimum wavelength arises because shorter wavelengths are damped by electron thermal conduction whilst longer wavelengths are energetically less unstable because of the increase in the accompanying perturbed magnetic energy included through Faraday's law. In principle the model employed is not dissimilar to that of Furth et al[11], in part III B, C of their paper, but they did not include ion inertia and also failed to find the optimum wavelength. Similarly through neglect of electron thermal conduction (in part II) and assuming $n \propto T_e$ (instead of a momentum equation) none of the features of our equilibrium profiles were found.

We have carried out the stability analysis for a homogeneous plasma for both scalar and tensor conductivity models, the optimum wavelength being slightly larger in the latter. Comparison of these optimum wavelengths with the finite amplitude mode structures found in the cylindrical equilibria indicates that indeed the current filamentation and electron temperature oscillations are electrothermal in origin.

REFERENCES

1. L.Spitzer, Jr., *Physics of Fully Ionized Gas* (Interscience, 1962).
2. M G. Haines and A. Tomimura, *Seventh European Conference on Controlled Fusion and Plasma Physics,* 1-5 Sept. 1975, p. 105 (CRPP, École Polytechnique Federa! de Lausanne, Switzerland).
3. V.Arunasalam, R.Cano, J.C.Hosea, E.Mazzucato, Phys.Rev.Lett., 39 , 888 (1977).
4. L.A.Artsimovich *et al.*, in *Plasma Physics and Controlled Nuclear Fusion Research* (IAEA, Vienna, 1969) vol.1.
5. S. I.Braginskii, Reviews of Plasma Physics (ed. M.A.Leontovich) (Consultant Bureau, N.Y.) *1*, 205 (1965).
6. See for example, J.Hosea, Bull.Am.Phys.Soc., 15, 116 (1972).
7. M.N.Rosenbluth, R.D.Hazeltine, F.L.Hinton, Phys. Fluids *25*, 116 (1972).
8. A.B.MacMahon and A.A.Ware, Nucl. Fusion 13, 413 (1973).
9. M.G.Haines, J. Plasma Phys. 12, 1 (1974).
10. M.G.Haines, *Proc. 2nd Int. Congress on Waves and Instabilities in Plasma,* p.1. Survey Lectures, ed. G.Auer, F.Cap, (Inst.Th.Phys., Innsbruck).
11. H.P.Furth, M.N.Rosenbluth, P.H.Rutherford, W.Stodiek, Phys. Fluids 13, 3020 (1970).