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Causality, Faster than Light Signals and Clock-Synchronization

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The synchronization of clocks is revisited. It follows that when signals of V are used there is nothing paradoxical in the resu'lt $t_0 \cdot t'_0 < 0$. The real paradox would be to get $t_0 \cdot t'_0 > 0$.

A sincronização de relógio é reexaminada. Segue que, quando sinais de $V>c$ são usados, nada há de paradoxal no resultado $t_0 . t' < 0$. O paradoxo real seria a obtenção de $t_0 . t'_0 > 0$.

1. INTRODUCTION

The causal "paradoxes" which arise in the special theory of relativity when signals of speed $V>c$ are allowed are very old and well known[1-2]. Recently this subject has received considerable attention[1-3], the present *opinion being* that the *causality hypothesis should* notbe mixed or confused with the temporal reordering of events when observed from different inertial frames. It follows that the existence of superlight signals *is* compatible with the special relativity.

We give here further arguments leading to the same conclusion. The novelty is that on the one hand our arguments are much simpler than the standard ones. On the other hand our approach is based on the analysis of the clock-synchronization prescriptions which are necessary in order to give an operational meaning to the symbol $\underline{t}$ appearing in the standard Lorentz transformations.

2. CLOCK-SYNCHRONIZATION AND CONSEQUENCES

It is well known[4] that in order to synchronize the clocks lying on an arbitrary inertial frame S one can use the light signals starting from an arbitrary point P_0 of S. If a light-ray is sended from P_0 at t_0, the clock situated at P has to show a time $t=t_0 + |P_0-P|/c$ when the light-ray arrives to P. When this operation has been performed for any of the clocks of S we say that they are synchronized with the clock in P_0. Under the standard hypothesis of space-isotropy and space - time homogeneity[2] in any inertial frame one can show that the mathematical relation of "being synchronized" is an equivalence relation and, therefore, we can speak of synchronized clocks, with independence of theciock P_0 employed in the above mentioned process.

Once the clocks of the inertial frames S_1 and S_2 have been synchronized, one has to adjust or "connect" the measures of the t variable measured by the clocks cf S_1 and S_2. If c_1 and c_2 are clocks situated on S_1 and S_2 which pass over the same space point then we *impose* that at the instant of the pass over, both clocks have to mark cr show the same time (for instance, $t=0$). When this operation has been accomplished for all the frames S_1, S_2, S_3,... which might be irivolved we say that S_1,S_2,S_3, ... have a ccinmon origin of time. Only when this troublesome operations have been performed there is any sense in writing the standard transforrnation formulae connecting the readings of the meters and clocks of S and S':

$$x' = (x-vt)\ \gamma(v) \ ; \quad t' = (t-vx/c^2)\ \gamma(v)$$
$$z' = z;\ y' = y\ ; \quad \gamma(v) = (1-v^2/c^2)^{-1/2} \tag{1}$$
$$v < c$$

Since for the clock situated at $x=0$, and when $t=0$, we get from eq. (1) $t'=0$, S and S' have a ccinmon t-origin.

Assume now a physical signal propagating with speed V on S, that is:

$$x = V\tau\ , \quad t = \tau\ , \quad V \gtrless C\ . \tag{2}$$

The signal will reach $x_0=L$ in a time $t_0= L/V$. For S' the time employed is,

$$t'_0 = t_0(1 - vV/c^2) \cdot \gamma(v) \quad . \tag{3}$$

Therefore, for $V > c^2/v$, $(t_0 \cdot t'_0) < 0$, which is (in substance) the causal "paradox" connected with the physical signals (2) propagating with $V > c^2/v > c$ since $v < c$.

In particular, if V is increased, t'_0 is more and more negative. Since the value of t'_0 can be written as,

$$t'_0 = L/V(1 - vV/c^2) \cdot \gamma(v) \quad , \tag{4}$$

when $V \to \infty$, t'_0 reaches the minimum value $\bar{t}'_0$ given by,

$$\bar{t}'_0 = \lim_{V \to \infty} t'_0 = -Lv/c^2 \cdot \gamma(v) \quad . \tag{5}$$

Our contention is that there is no paradox implicit in the negative value taken by t'_0 or $\bar{t}'_0$. On the contrary, we shall see that eq. (5), and the minus sign of eq. (4) when $V > c^2/v$, are natural if one *has not forgotten* the operational way used in order to define a common t- origin for S and S'.

Indeed, from eq.(1) we get that the local time t_0 shown by the clock situated (on S') in front of $x_0=L$ when $t=0$ is,

$$t_0 = - vL/c^2 \cdot \gamma(v) \quad , \tag{6}$$

which coincides with $\bar{t}'_0$ as given by eq. (5). This is natural, as even *if* the clocks of S and S' situated at $x=x'=0$, when $t=0$, do show a *common* time, those clocks of S' situated in front of $x_0 > 0$ at $t=0$ do show a *negative* time, as a consequence of (1). In the same way those clocks of S' situated in front of $x_0 < 0$ when t=0 must show a positive time. On the other hand, since a signal like (2), when $V \to \infty$, reaches the point $x_0=L$ in a $\tau_0 = 0$, the clock in front of $x_0=L$, $t_0=0$ must show, according to (1), a time $-vL/c^2 \cdot \gamma(v)$, which is nothing more that the time defined by eq. (5).

Therefore, there is nothing paradoxical about being $t_0 \cdot t'_0 < 0$ when $V > c^2/v$, or in the extreme negative value given by (5) when $V \to \infty$. The only real trouble is to *forget* the operational way in which the clocks are synchronized in special relativity, implying that even if the clocks situated at the common origin $x=x'=0$ do show the common time $t'=$ $=t=0$, this *does not* happen for the clocks situated at $x_0 \neq 0$. The *real* logical contradiction would be to get $t'_0 > 0$ when signals of velocity $V > c^2/v$ are used. Therefore the reason for the turning up of negative values of t'_0 (when signals of $V > c$ are used) is of kinematical origin, has nothing to do with causality[2] and *does not* contradict Einstein's ideas.

REFERENCES

1. R.Mignani, E.Recami: Riv.N.Cimento 4, 209-398 (1974) and the numerous references therein,

2. Ya.P.Terletskii: *Paradoxes in the theory of Relativity*. Plenurn Press NY 1968.

3. P.Csonka: Phys. Rev. 180, 1266 (1969); ibid. Nucl. Phys. *B21*, 436 (1976); J.Parmentola, D.Yee: Phys. Rev. *D4*, 1912 (1971). L.Schulman: Am. Journ. Phys. 39, 481 (1971).

4. A.Einstein: *The Meaning of Relativity*, Princeton 1955, 4th. ed. Chapt. I. M.Becquerel: *Le principe de Relativité et la theorie de la Gravitation*. Pergamon Press, London 1966. Chapt. I. H.Arzelies: *Relativistic Kinematics*. Pergamon Press, Oxford 1966, pg.19.