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On a Relativistic Model for Baryons with a Scalar Raising Potential

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The Bogoliubov model for baryons permits us to include the relativistic effects in a first approximation when we regard quarks as Dirac particles interacting among themselves through a central potential. Further, based on gauge theories and the results of charmonium model, we have assumed that the potential is linear and raising. In this work, we have shown that for several processes involving baryons, the results obtained by taking relativistic effects into account are better than the non-relativistic ones.

O modelo de Bogoliubov para os barions permite que efeitos relativísticos sejam introduzidos numa primeira aproximação quando os quarks são considerados como partículas de Dirac interagindo através de um potencial central. Além disso, assumimos que o potencial é linear e crescente baseados nas teorias de gauge e no modelo do charmonium. Mostramos, neste trabalho, que nos vários processos envolvendo barions, os resultados obtidos levando em conta efeitos relatívisticos são melhores do que os resultados não relativisticos.

1. INTRODUCTION

We suppose that a baryon is formed by the system qqq assuming that there are four quarks (p,n,λ,p') each of them appearing in three colours so that the known baryons are colour singlets. In otherwords we suppose that the symmetry group of strong interactions is $SU(4) \otimes SU(3)$ colour.

In spite of many good predictions of the quark model, no free quark has been observed. So, it must have a mechanism that confines the quark inside hadrons (string, bags, etc.) . Here we use a raising potential between the quarks to confine them. The shape of potential, assuggested by quantum electrodynamics[1] and the lattice gauge theory[2], is taken to be linear[3].

However, the three-body problem is rather complex and little progress has been made for the relativistics system *qqq*. As we are interested in a first approximation we shall suppose that the quarks interact with one another through a mean central potential (Bogoliubov model)[4]. Further, we shall make use of the additivity hypothesis. Of course, the results are strongly dependent on the shape of the potential.

According to these ideas, the Dirac equation for the quark reads as

$$(m_q + V + \vec{\gamma}\vec{p})\Psi_q = E\gamma_0 \Psi_q , \tag{1}$$

so that

$$m_q^* = m_q + V \tag{2}$$

can be taken as an effective quark mass. This is very interesting if we do not want a great anomalous magnetic moment for the quarks[4]. In Eq. (1) we shall use

$$V(r) = - V_0 + \alpha r . \tag{3}$$

We shall study several processes involving baryons[5] by solving Eq. (1) and compare the results with the non-relativistic ones.

2. THE WAVE FUNCTION

In accordance with §1, we suppose that each quark is subject to a scalar potential, given in Eq. (3), so that the quark wave function $\Psi(\vec{r})$ satisfies the Dirac equation

$$(\vec{\alpha}\vec{p} + \beta(m - V_0 + \alpha r)\,\Psi(\vec{r}) = E\,\Psi(\vec{r}) \tag{4}$$

By using the usual decornposition in spherical coordinates, it resultsin

$$\Psi = \begin{pmatrix} g(r)\, Y^{j_3}_{j\,\ell_a} \\ \\ i f(r)\, Y^{j_3}_{j\,\ell_a} \end{pmatrix} \tag{5}$$

so that, we get the following system of coupled differential equations

$$\begin{aligned} f' + \frac{1-\kappa}{r} f + (E - m + V_0 - \alpha r)\, g &= 0 \\ g' + \frac{1+\kappa}{r} g - (E + m - V_0 + \alpha r)\, f &= 0 \end{aligned} \tag{6}$$

For convenience, we define the linear combinations

$$\begin{aligned} X(r) &= r(f(r) + g(r)) \\ Y(r) &= r(f(r) - g(r)) \ , \end{aligned} \tag{7}$$

so that, far the ground state ($\kappa = -1$), we have

$$\begin{aligned} X' - (a + \alpha r)\, x &= (E - \frac{1}{r})Y \\ Y' + (a + \alpha r)\, y &= (E + \frac{1}{r})X \ , \end{aligned} \tag{8}$$

where $a = m - V_0$.

For convenience, we multiply the asyrnptotic solution $\exp(-\alpha r^2/2)$ by the factor $\exp(-ar)$ so that

$$\begin{aligned} X(r) &= x(r)\, \exp(-\alpha r^2/2 - ar) \\ Y(r) &= y(r)\, \exp(-\alpha r^2/2 - ar) \ , \end{aligned} \tag{9}$$

where $x(r)$ and $y(r)$ are series in powers of r. The differential equations for $x(r)$ and $y(r)$ are given by

$$x' - 2\alpha r x - 2ax = (E - \frac{1}{r})y$$
$$y' = -(E + \frac{1}{r})x \quad . \tag{10}$$

Writing

$$x(r) = \sum_{n=1}^{\infty} p_n r^n$$
$$y(r) = \sum_{n=1}^{\infty} q_n r^n \tag{11}$$

we get the recursion relations for p_n and q_n ,

$$p_{n+1} = A_n p_n + B_n p_{n-1} \tag{12a}$$

and

$$nq_n = -Ep_{n-1} - p_n \tag{12b}$$

where

$$A_n = \frac{2an(n+1)-E}{n^2(n+2)}$$
$$B_n = \frac{(n+1)(2\alpha n-E^2)}{n^2(n+2)} \tag{13}$$

Supposing that the solutions are quadratically integrable, the coefficient p_n must tend to zero when n goes to infinity. Defining

$$b_n = \frac{p_n}{p_{n-1}} \tag{14}$$

and dividing Eq.(12a) by p_n we get

$$b_n = \frac{B_n}{-A_n + b_{n+1}} \tag{15}$$

The last equation is an infinite continued fraction for b_n. When n goes to infinity, we have

$$b_n \to \pm \sqrt{2\alpha/n}\ , \quad n \to \infty \tag{16}$$

It can be shown that only the alternating series is a converging one and the convergence is like

$$\sum_n \frac{(-1)^n}{n!} (\sqrt{2\alpha}\ r)^n \tag{17}$$

In order to apply the infinite continued fraction, Eq. (15), we must truncate it in a given term b_N so that $b_N = b_{N+1}$. This condition gives the solution for b_N.

$$b_N = \frac{1}{2} (A_N^2 - \sqrt{A_N^2 + 4B^2}) \ . \tag{18}$$

Using this b_N in Eq. (15) we obtain the p_n and we have the condition which the parameters a, a and E must satisfy so that the series $x(r)$ and $y(r)$ are quadratically integrable. Of course, if p_n converges, q_n also converges.

3. APPLICATIONS OF THE MODEL

3.1 – Magnetic Moments

Using the Bogoliubov model for hadrons, we work with the energy of bouncled quark instead of its free mass so that we have a decrease in the anomalous magnetic moment of the quark. In this model we have"

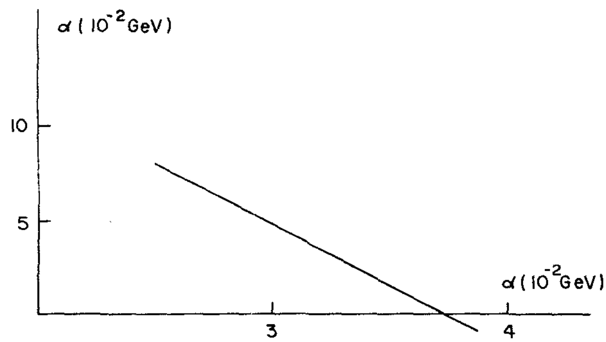
$$\mu_q = \frac{eQ_q}{2F_0} (1 - \delta) \tag{19}$$

where

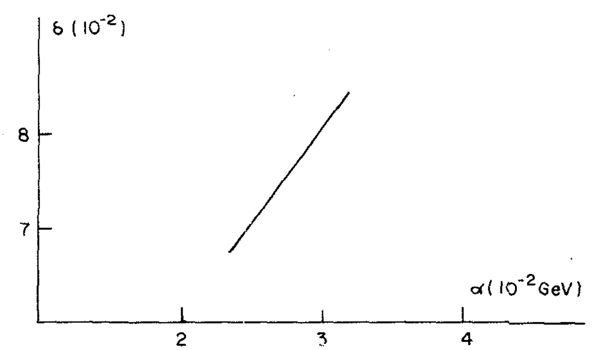
$$\delta = \frac{2}{3} \frac{\int F^2(r) dr}{\int (F^2(r) + G^2(r)) dr} \ . \tag{20}$$

It must be observed that the positive term 6 is a relativistic correction. So, the non-relativistic magnetic moment is decreased and it tends to the experimental value.

In this work, in order to calculate the quantity 6 we fix the baryon mass and use the continued fraction, Eq.(15), thus obtaining a relation between the parameters a and a. For example, for the proton we get the graph in Fig.1. Then, using Eq. (20), we can calculate the parameter δ, and obtain the graph in Fig.2[6]. Fitting the magnetic moment of proton (μ_p = 2.793μ), we get 6 6.9 $\times 10^{-2}$ and α = 2.3463$\times 10^{-2}$ GeV^2. So, we assume that a, that is the inclination of the potential, has the same value for all representation 20 of SU(4). The quantity a is the symmetry breaking parameter and varies with the particle mass. With this value of α we can calculate 6 and a for all particles getting the results shown in Table 1. The magnetic moments are presented in Table 2. Note the decrease of the magnetic moment compared with the non-relativistic calculations. We also observe that the correction decreases with the increase in the particle mass.



Graph 1 - Relation between parameters a and a for the proton



Graph 2 - Relation between parameters δ and a for the proton.

	$a(10^{-1}$ GeV)	δ (10^{-2})
N (939)	0.8469	6.9000$^{(1)}$
Δ (1232)	2.0355	5.0016
Λ (1116)	1.5756	5.6291
Σ (1193)	1.8820	5.2587
Ξ (1317)	2.3665	4.6139
Ω (1672)	3.7103	3.4397
Λ_c^+(2260)	5.8529	2.3907

Table 1 - Values of a and δ. (1) - This value is fitted using proton magnetic moment.

	μ_B(calc)	μ_B(NR)	μ_B(exp)	μ_B(Rein)
P (939)	2.793$^{(1)}$	1	2.793	2.85
N (939)	-1.862	-2/3	-1.913	-1.90
Λ (1116)	-0.793	-1/3	-0.67 $\pm$ 0.06	-0.64
Σ^+(1193)	2.235	1	2.62 $\pm$ 0.4	2.75
Σ^-(1193)	0.745	-1/3	-1.48 $\pm$ 0.37	-1.05
Ξ^-(1317)	-0.680	-1/3	-1.85 $\pm$ 0.75	-0.53
Ω (1672)	-1.626	-1	-	-
Λ^+(2260)	-0.811	2/3	-	-

Table 2 - Magnetic Moments. a) μ_B(calc) - are the magnetic moments calculated in this work. b) μ_B(NR) - are the non-relativistic values. c) μ_B(exp) - ai-e the experimental values[9]. d)- μ_B(Rein) are the values calcuted in ref.(5). (1) - This value is fitted using the proton magnetic moment. Magrietic moments are given in units of n.m.

3.2 — Leptonic Decays

Using the additivity hypothesis, we consider the leptonic decay $B \to B' + \ell + \bar{\nu}$ taking into account the constituent quark decay $q \to q' + \ell + \bar{\nu}$. Using Eq.(20), we get for the ratio G_A/G_V [7]

$$\frac{G_A}{G_V} = \left(\frac{G_A}{G_V}\right)_{NR} (1 - 2\delta) \tag{21}$$

where $(G_A/G_V)_{NR}$ is its non-relativistic value. If we use the values of δ shown in Table 1, we get the values of Table 3. Note the tendency of the results to approach the experimental values, in comparison with the non-relativistic ones.

Process	G_A/G_V(calc)	G_A/G_V (NR)	G_A/G_V(exp)	G_A/G_V(well)
$N \to P + e + \nu$	1.437	1.667	1.250 ± .009	1.40
$\Lambda \to P + e + \nu$	0.887	1	0.66 ± 0.05	0.66
$\Sigma \to N + e + \nu$	0.895	1	0.435 ± .035	-

Table 3 - Ratio G_A/G_V. a) G_A/G_V(calc) are the values calculated in this work. b) G_A/G_V(NR) are the non-relativistic values. c) G_A/G_V(exp) are the experimental values.[9] d) G_A/G_V(well) are the results using a well potential.[10]

3.3 — Strong Decays

Considering that the emmited pion in the strong process $B \to B' + \pi$ has a srnall moment, we get the relativistic correction for the width[7]

$$\Gamma = \Gamma_{NR} \left(\frac{1 - 2\delta_B}{1 - 2\delta_N}\right)^2 \tag{22}$$

where Γ_{NR} is the non-relativistic width and δ_B is the value of δ for the fixed baryon B. Using the values of 6 frorn Table 1, we get the widths shown in Table 4. Once more the relativistic results are rnodified in the correct direction.

Process	Γ(cal)	Γ (NR)	Γ(exp)
$\Delta^{++} \to P + \pi^+$	80.747	74.078	110. - 120
C-(1386) → A +a-	35.598	31.876	35 ± 2
$\Sigma^0(1530) \to \Xi^0 + n$	7.968	6.966	9.1 ± .5

Table 4 - Widths for strong decays. a) Γ(calc) are the widths calculated in this work. b) Γ(NR) are the non-relativistic values. c) Γ(exp) are the experimental values[q].

3.4 — Mean Square Ratio

If $\vec{r}_q$ is the quark position in relation to a reference frarne then the center of rnass of baryon is given by

$$\vec{R} = \frac{1}{3} (\vec{r}_1 + \vec{r}_2 + \vec{r}_3) \qquad (23)$$

and the distance of quark 1 from the center of rnass is

$$\vec{r}_1 - \vec{R} = \frac{2}{3} \vec{r}_1 - \frac{1}{3} \vec{r}_2 - \frac{1}{3} \vec{r}_3 \ . \qquad (24)$$

Then, the mean square ratio is

$$\langle r^2 \rangle = \frac{1}{3} \langle (\vec{r}_1 - \vec{R})^2 + (\vec{r}_2 - \vec{R})^2 + (\vec{r}_3 - \vec{R})^2 \rangle . \qquad (25)$$

Since the wave function for the ground state is isotropic and symmetric we have

$$\langle r^2 \rangle = \frac{2}{3} \langle r_1^2 \rangle \qquad (26)$$

or

$$\langle r^2\rangle = \frac{2}{3}\ \frac{\int r^2(F^2(r) + G^2(r))\,dr}{\int (F^2(r) + G^2(r))\,dr} \tag{27}$$

Using the last equation, we have obtained the numericai results presented in Table 5. Note the near constancy of the msr with mass. The experimental value for the proton is 0.92 ± 0.03 Fermi.

	$\langle r^2\rangle^{1/2}$ (Fermi)
P (939)	0.9941
Δ (1232)	0.8881
Λ (1116)	0.9245
Σ (1193)	0.8960
Ξ (1317)	0.8647
Ω (1672)	0.7879
Λ_c^+ (2260)	0.7164

Table 5 - Mean Square Ratio.

4. CONCLUSION

The model presented here is attractive because of its simplicity and retains the ideas of confinement, linear raising potential and relativistic effects, considering quarks as Dirac particles. Of course, it must be considered only as a first correction. The results indicate that the relativistic corrections improve the agreement of the results with the experimental values, as compared to the non-relativistic ones.

APPENDIX

Here we give the integrals of $F(r)$ and $G(r)$ that appears so often in the calculations. Using Eq. (11) and Eq. (9) in Eq. (7) we get

$$\left.\begin{matrix} F \\ G \end{matrix}\right\} = \exp(-\alpha r^2/2 - ar) \sum_{n=1}^{\infty} \left\{\begin{matrix} s_n \\ t_n \end{matrix}\right\} r^n \tag{A.1}$$

Where

$$s_n = \frac{p_n + q_n}{2}, \quad t_n = \frac{p_n - q_n}{2} . \tag{A.2}$$

Squaring Eq. (A.1) and grouping terms with the same power, we get

$$\left.\begin{matrix} F^2 \\ G^2 \end{matrix}\right\} = \exp(-\alpha r^2 - 2ar) \left[\left\{\begin{matrix} s_1^2 \\ t_1^2 \end{matrix}\right\} r^2 + 2 \left\{\begin{matrix} s_1 s_2 \\ t_1 t_2 \end{matrix}\right\} r^3 + \dots \right] \tag{A.3}$$

Therefore, the integrals to be calculated are

$$I_n = \int_0^{\infty} r^n \exp(-\alpha r^2 - 2ar)\, dr , \quad n = 2, 3, \dots \tag{A.4}$$

which are cylindrical parabolic functions, $U(a,z)$,[8] so that

$$I_n = \frac{\Gamma(n+1)}{(2\alpha)^{(n+1)/2}} \exp(a^2/2\alpha) \; U(n + 1/2, \sqrt{2}\; a/\sqrt{\alpha}\,) . \tag{A.5}$$

The normalization integrals are

$$\int_0^{\infty} \left\{\begin{matrix} F^2 \\ G^2 \end{matrix}\right\} dr = \left\{\begin{matrix} s_1^2 \\ r_1^2 \end{matrix}\right\} I_2 + 2 \left\{\begin{matrix} s_1 \;\; s_2 \\ t_1 \;\; t_2 \end{matrix}\right\} I_3 + \dots \tag{A.6}$$

To evaluate the mrs, we must use the integrals

$$\int_0^{\infty} r^2 \left\{\begin{matrix} F^2 \\ G^2 \end{matrix}\right\} dr = \left\{\begin{matrix} s_1^2 \\ t_1^2 \end{matrix}\right\} I_4 + 2 \left\{\begin{matrix} s_1 \;\; s_2 \\ t_1 \;\; t_2 \end{matrix}\right\} I_5 + \dots . \tag{A.7}$$

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