Revista Brasileira de Física, Vol. 9, Nº *2,* **1979**

Viscosity Tensor of 3 ~ e - ~ *

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Recebido em 12 de Maio de **1978**

The components of the viscosity tensor in the axial phase of superfluid. 3 ~ a are studied in the generalized paramagnon model. It is found that, while the components of the viscosity tensor decrease like B/T for $T \cong T_c$ which is consistent with other predictions. At low temperatures, however, we find a temperature dependence which contains one power of T more than previous estimates.

O modelo generalizado de pararnagnons foi utilizado para se estudarem as componentes do tensor de viscosidade do 3 ~ e superfluido na fase axial, prevendo-se os seguintes resultados: para T 2 T_c as componentes do tensor de viscosidade decrescem como Z/T, o que é consistente com outras previsões; a baixa temperatura, entretanto, estas possuem um fator extra, dependente da temperatura.

1. INTRODUCTION

Our purpose in this paper is to investigate the behaviour of the viscosity tensor in the axial phase of superfluid ^{3}He, in the generalized paramagnon model[1,2]. We follow here a model deviced by A. Houghton and K. Maki[3], given by the authors the name of "Spin Fluctuation Dominance Model". The idea behind this variation of the paramagnon model (in which it is assumed that the scattering of the ^{3}He-*A* quasiparticles is dominated by spin fluctuations exchange[4]) is to leave the partial wave scattering amplitudes associated with the spinfluctuations as free parameters. These free parameters can be identified with the empirical

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* Work supported in part by CNPq.

amplitudes of the normal state. This isdone in order to avoid the over-estimation of the effect of forward scattering in the paramagnon model.

In section 2 we outline the method of calculation of a transport coefficient in ^3He in the generalized paramagnon model. In section 3 the building blocks of the theciry; the quasiparticles propagator, is presented and the spin fluctuation propagator is calculated. In section 4 and 5 the quasiparticle relaxation time and the relaxation rateof external perturbation are derived from the model. Finally in section 6 we calculate the components of the viscosity tensor at low ($T << T_c$) and high ($T \cong T_c$) temperatures.

2. THE METHOD

The coefficients of the viscosity tensor η_{ij} are given in terms of the retarded product of the stress tensor τ_{ij}

$$\eta^{ij} = \lim_{\omega\to 0} \frac{1}{\omega} \, \mathrm{Im} \langle | \tau_{ij}, \tau_{ij} | \rangle (\omega + i\delta) \tag{1}$$

For an anisotropic liquid η is a fourth rank tensor, but due to the cylindrical symmetry of the A phase, it has only five independent components[5]. We can also write Eq.1 as

$$\eta^{ij} = \lim_{\omega\to 0} \frac{1}{\omega} \, \mathrm{Im}\, \chi_{\tau_{ij}, \tau_{ij}} (\omega + i\delta) \tag{2}$$

Here $\tau_{ij} = p_i p_j \rho_3$, p_i is the i^{th} component of the momentum, ρ_3 is a Pauli matrix operating in particle-hole space. $\chi_{\tau,\tau}$ is a retarded susceptibility, and is obtained by analytit continuatíon of the corresponding Matsubara propagators[6]. We can write for the imaginary part of the susceptibility

$$\mathrm{Im}\,\chi(\omega + i\delta) = \frac{1}{4\pi i} \int_{-\infty}^{+\infty} dz \left[\tanh\left(\frac{z+\omega}{2T}\right) - \tanh\frac{z}{2T}\right] \Phi(z^-, z^+ + \omega) \tag{3}$$

and

$$\Phi(z^-, z^+ + \omega) = \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\left[G_{z^-}(\vec{p})\, V(z^-, z^+ + \omega)\, G_{z^+ + \omega}(\vec{p})\right] \tag{4}$$

Here $z^- = z - i6$, $z^+ = z + i\delta$, where 6 is an infinitesimal, $G_z(\vec{p})$ is the matrix Green's function describing the propagation of ^{3}He-*A* quasiparticles and 5 is the renormalized external vertex.

The most important part of the problem is the determination of the self energy and the vertex function corresponding, to the stress tensor operator. The Dyson equation for them are shown graphically in Fig.1 and 2 respectively. We shall only need to consider the lifetimes associated with the self energy and renormalized vertex which can be handled rigorously in the generalized paramagnon model. The real part of the self energy and vertex corrections can be absorbed into the quasiparticle mass and energy gap.

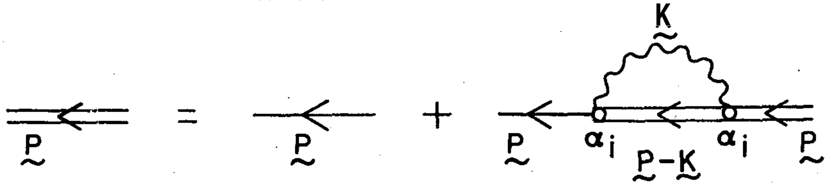

Fig.1 - Dyson equation for the single particle Green's function in the spiri fluctuation model.

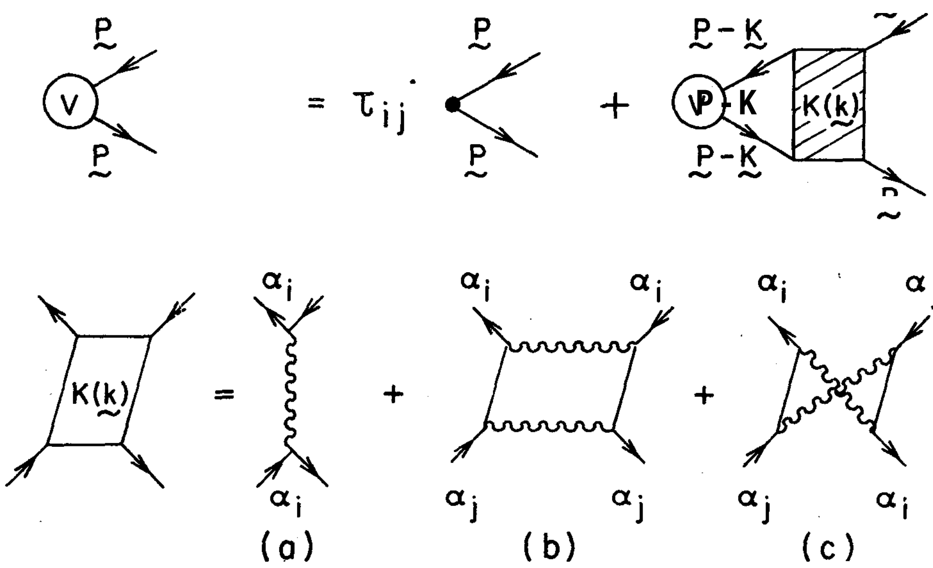

Fig.2 - Dyson equation for the external vertex V in the spin fluctuation model.

3. THE SPIN FLUCTUATION PROPHGATOR

It is assumed that the ground state of superfluid ^3He in the A phase is described by an order parameter with its $\vec{\ell}$ vector (and consequently $\vec{d}$ vector) along the y axis[4]. Whenever it is convenient the directions x, y and z will be denoted 1, 2 and 3 respectively, without further explanation. To estimate the effect of scattering due to spin fluctuations we calculate the spin fluctuation propagator. To that effect we introduce the bare Green's function for the quasiparticle:

$$G_0^{-1}(\vec{p},\omega+i\delta) = (\omega+i\delta) - \xi_p\rho_3 - \tilde{\Delta}(\rho_1\hat{p}_1 + \rho_2\hat{p}_3) \tag{5}$$

Here

$$\tilde{\Delta} = \sqrt{3/2}\ \Delta,\ \xi_p = \frac{p^2}{2m^*} - \mu \quad ,$$

A is the order parameter, m^* is the effective mass of the quasiparticles, $\hat{p}_i$ are unit momentum vectors, ρ_i are Pauli matrices operating in particle-hole space and μ is the Fermi energy. The spin fluctuation propagator is then given by

$$\chi_{ij}(\omega,\vec{k}) = \frac{\delta_{ij}N^*(0)\ \chi_{ij}^{(0)}(\omega,\vec{k})}{1 + \frac{Z_0}{4}\chi_{11}^{(0)}(\omega,\vec{k})} \tag{6}$$

Here[3]: $N^*(0) = N(0)(1 + \frac{F_1}{3})$, $N(0)$ is the free particle density of states at the Fermi level, and F_i and Z_i are "Landau Parameters"[4].

In ^3He-A we must distinguish two susceptibilities, one which describes response perpendicular to $\vec{R}$ and the other describing response along $\vec{\ell}$.

$$\chi_\perp^{(0)}(m,\vec{k}) = T\int \frac{d^3p'}{(2\pi)^3}\sum_{n'} \mathrm{Tr}\{\rho_3G^0(\vec{p}',n')\rho_3G^0(\vec{p}'-\vec{k},n'-m)\} \tag{7a}$$

$$\chi_{\parallel}^{(0)}(m,\vec{k}) = T\int \frac{d^3p'}{(2\pi)^3}\sum_{n'} \mathrm{Tr}\{G^0(\vec{p}',n')G^0(\vec{p}'-\vec{k},n'-m)\} \tag{7b}$$

After analytic continuation we obtain, for example:

$$\mathrm{Im}\,\chi_{\perp}^{0}(\omega,\vec{k}) = \frac{1}{2\pi}\int dz'\left(\tanh\frac{\omega+z'}{2T} - \tanh\frac{z'}{2T}\right)\times$$
$$\times\int \text{———}\ \mathrm{Tr}\{\rho_3\ \mathrm{Im}\,G^{0R}(\vec{p}',z')\,\rho_3\,\mathrm{Im}\,G^{0R}(\vec{p}+\vec{k},z'+\omega)\}\ , \tag{8}$$

where the label R on the Green's function means *retarded*. To be able to go on with the calculations we assume $\mu \gg v_F k \gg (\Delta,T)$, then we can perform the energy integrals and the integrals over polar angle and we find

$$\mathrm{Im}\,\chi_{\parallel}^{\perp}(\omega,\vec{k}) = \frac{\pi}{2v_F k}\left\{\left\langle\int_{\tilde{\Delta}(\Omega')}^{\infty} dz'\left\{\tanh\frac{z'+\omega}{2T} - \tanh\frac{z'}{2T}\right\}\right.\right.$$
$$\times\left.\frac{z'(z'+\omega)\mp\tilde{\Delta}^2(\Omega')}{\sqrt{z'^2-\tilde{\Delta}^2(\Omega')}\ \sqrt{(z'+\omega)^2-\tilde{\Delta}^2(\Omega')}}\right\rangle \tag{9}$$
$$\left.+\ \theta(\omega-2\tilde{\Delta}(\Omega'))\left\langle\int_{\tilde{\Delta}(\Omega')}^{\infty} dz'\tanh\frac{z'}{2T}\ \frac{z'(z'+\omega)\mp\tilde{\Delta}^2(\Omega')}{\sqrt{z'^2-\tilde{\Delta}^2(\Omega')}\ \sqrt{(z'+\omega)^2-\tilde{\Delta}^2(\Omega')}}\right\rangle\right\}$$

Here, $\theta(x)$ is the step function and $\langle\ \rangle$ means average over the azimuthal angle, ϕ', of $\vec{p}'$ in the plane perpendicular to $\vec{k}$. The previous assumption on the magnitude of % implies that only perpendicular to % contribute to the integrals of $\chi_{\perp}$ and $\chi_{\parallel}$. Further $\tilde{\Delta}(\Omega') \equiv \Delta(1-\sin^2(\widehat{k,\ell})\cos^2\phi)^{1/2}$ where $(\widehat{k,\ell})$ is the angle between $\vec{k}$ and $\vec{\ell}$. Finally, for reasons which will become obvious in the next section, we introduce

$$\mathrm{Im}\,\chi^{+} = (2\,\mathrm{Im}\,\chi_{\perp} + \mathrm{Im}\,\chi_{\parallel})/3 = \frac{\pi}{2v_F k} f_0(\omega) \tag{10}$$

and

$$\mathrm{Im}\,\chi^{-} = (2\,\mathrm{Im}\,\chi_{\perp} - \mathrm{Im}\,\chi_{\parallel}) = \frac{\pi}{2v_F k} f_1(\omega)\ , \tag{11}$$

where

$$f_{\binom{0}{1}}(\omega) = \langle \int_{\tilde{\Delta}(\Omega')}^{\infty} dz' \left\{ \tanh \frac{z' + \omega}{2T} - \tanh \frac{z'}{2T} \right\} \begin{Bmatrix} g(z',\omega) \\ h(z',\omega) \end{Bmatrix}$$

$$+ \theta(\omega - 2\tilde{\Delta}(\Omega')) \int_{\tilde{\Delta}(\Omega')}^{\infty} dz' \tanh \frac{z'}{2T} \begin{Bmatrix} g(z',\omega) \\ h(z',\omega) \end{Bmatrix} \rangle \tag{12}$$

and

$$g(z',\omega) = \frac{z'(z' + \omega) - \tilde{\Delta}^2(\Omega')/3}{\sqrt{z^2 - \tilde{\Delta}^2(\Omega')} \sqrt{(z' + \omega)^2 - \tilde{\Delta}^2(\Omega')}}$$

$$h(z',\omega) = \frac{z'(z' + \omega) - 3\tilde{\Delta}(\Omega')}{\sqrt{z'^2 - \tilde{\Delta}\ (\Omega')} \sqrt{(z' + \omega)^2 - \tilde{\Delta}^2(\Omega')}}$$

At low temperatures, $T \ll T_c$, we find

$$f_0(\omega) = \langle \frac{4}{3} \tilde{\Delta}(\Omega') e^{-\tilde{\Delta}(\Omega')/T} \sinh \frac{\omega}{2T} K_0\left(\frac{\omega}{2T}\right) \rangle, \quad f_1(\omega) = -3f_0(\omega), \tag{13}$$

where $K_0(z)$ is the modified Bessel function, while for high, $T \cong T_c$, temperatures

$$f_0(\omega) = \cong \omega \left[1 + \frac{14}{3} \zeta(3) \left(\frac{\Delta}{\pi T}\right)^2\right] \tag{14}$$

Here $\zeta(3)$ is the Riemann zeta function.

We may now proceed to construct the Dyson equations for the single particle Green's function and external vertices.

4. THE SINGLE PARTICLE PROPAGATOR

The single particle propagator renormalized by spin fluctuation exchange can be written (and it is diagramatically shown in Fig.1):

$$G^{-1}(n,\vec{p}) = G_0^{-1}(n,\vec{p}) - \Sigma(n,\vec{p}) ,$$

where the self energy

$$\Sigma(n,\vec{p}) = \left[\frac{z_0}{4}\right]^2 \frac{T}{N^*(0)} \sum_{m,i} \int \frac{d^3k}{(2\pi)^3} \left(\alpha_i G_{n-m}(\vec{p}-\vec{k})\alpha_i \chi_{ii}(\vec{n},\vec{k})\right) \tag{15}$$

the α_i's are the spin operators of the four dimension spinor representation $\alpha_1 = \rho_3\sigma_1$, $\alpha_2 = \sigma_2$ and $\alpha_3 = \rho_3\sigma_3$, σ_i's are the Pauli matrices in spin space. Evaluating the sum over the spin components we find

$$\Sigma(n,\vec{p}) = -\left(\frac{Z_0}{4}\right)^2 \frac{T}{N^*(0)} \sum_m \int \frac{d^3k}{(2\pi)^3} [3\omega_{n-m}\chi^+(\vec{k},n)$$

$$- \bar{\Delta}(\rho_1(\hat{p}-\hat{k})_1 + \rho_2(\hat{p}-\hat{k})_3)\chi^-(\vec{k},n)] d_{n-m}^{-1}(\vec{p}-\vec{k}) , \tag{16}$$

where

$$d_n(p) = \xi_p^2 + \omega_n^2 + \bar{\Delta}^2(\vec{p}) \tag{17}$$

Carrying out the analytic continuation and defining

$$\mathrm{Im}\, \Sigma^R(\vec{p},z) = -i(\Gamma_1 - (\rho_1\hat{p}_1 + \rho_2\hat{p}_3)\Gamma_2) \tag{18}$$

we find

$$\mathrm{Im}\, \Sigma_1(\vec{p},z) = -\frac{3}{2} \left(\frac{Z_0}{4}\right)^2 {}'< \int_{\tilde{\Delta}(\vec{p}-\vec{k})}^{\infty} dz' \frac{z'}{\sqrt{z'^2 - \tilde{\Delta}^2(\vec{p}-\vec{k})}} \mathrm{Im}\; \chi^+(z'-z)$$

$$\times \left(\coth \frac{z'-z}{2T} - \tanh \frac{z'}{2T}\right) > + z' \to -z' \tag{19}$$

and

$$\mathrm{Im}\,\Sigma_2(\vec{p},z) = \frac{\tilde{\Delta}}{2}\left(\frac{Z_0}{4}\right)^2 <(\rho_1(\hat{p}-\hat{k})_1 + \rho_2(\hat{p}-\hat{k})_3) \times$$

$$\times \int_{\tilde{\Delta}(\vec{p}-\vec{k})}^{\infty} \frac{dz'}{\sqrt{z'^2-\tilde{\Delta}^2(p-k)}}\ \mathrm{Im}\,\chi(z'-z)\left(\coth\frac{z'-z}{2T} - \tanh\frac{z'}{2T}\right)> - z' \to -z'$$

(20)

In deriving the last two equations it has been assumed, as will be assumed throughout this work, that the inverse scattering lifetime is less than $(\tilde{\Delta}(T), T)$. In liquid ^{3}He this condition is satisfied except in the vicinity of the transition temperature $(1 - T/T_c) \tilde{<} 10^{-2}$ where the quasiparticle spectrum becomes gapless. To leading order in k/p_F the k dependente on the order parameter appearing in Eqs. 19 and 20 can be neglected. Then the integral over the polar angles of $\vec{k}$ can be carried out by writing $|\vec{k}| = 2\,p_F \sin\theta_k/2$ where θ_k is the angle between $\vec{p}$ and $\vec{p}-\vec{k}$. We find:

$$\Gamma_1(z) = \frac{3}{2}A_0\ '<\int_{\tilde{\Delta}(\Omega)}^{\infty} dz'\,\frac{z'}{\sqrt{z'^2-\tilde{\Delta}^2(\Omega)}}\,f_0(z'-z)\left[\coth\frac{z'-z}{2T} - \tanh\frac{z'}{2T}\right]>+z\leftrightarrow z'$$

(21)

$$\Gamma_2(z) = \frac{A_1}{2}\tilde{\Delta}\ '<\int_{\tilde{\Delta}(\Omega)}^{\infty} \frac{dz'}{\sqrt{z'^2-\tilde{\Delta}^2(\Omega)}}\,f_1(z'-z)\left[\coth\frac{z'-z}{2T} - \tanh\frac{z'}{2T}\right]> -z'\to -z'$$

(22)

Here A_0 and A_1 are the coefficients of the s-wave and p-wave scattering amplitudes, associated with paramagnon exchange, in the normal state. For example, in the paramagnon model[3]

$$A_0 = \left(\frac{Z_0}{4}\right)^2\ '<\int \frac{d\Omega}{4\pi}\,\frac{\pi}{2v_F k}\left[1 + \frac{Z_0}{4}\left(1 - \frac{1}{12}\left(\frac{k}{p_F}\right)^2\right)\right]^{-1}> .$$

In this case $'<>$ means average over the azimuthal angle of $\vec{k}$ in ,the plane perpendicular to $\vec{p}$. Again we will need both the low and high temperature expansion of the diagonal (Γ_1) and of diagonal (Γ_2) self energies. Using the results of the previous section for the spin fluctuation propagator, we find at low temperatures

$$\Gamma_1 = 2\, A_0\, K(T) < \tilde{\Delta}(\Omega') e^{-\frac{\tilde{\Delta}(\Omega)}{T}} > \qquad (23)$$

and

$$\Gamma_2 \sin\theta = \Gamma_2(\Omega) = -\, 2A_1\, K(T) < \tilde{\Delta}(\Omega')\; e^{-\frac{\tilde{\Delta}(\Omega')}{T}} >, \qquad (24)$$

where

$$K(T) = 2\, \left(\frac{\pi}{2}\right)^{3/2} \left[\tilde{\Delta}(\Omega) T\right]^{1/2}$$

The angular average indicated in Eqs. 23 and 24 can be written

$$<\tilde{\Delta}(\Omega') e^{-\frac{\tilde{\Delta}(\Omega')}{T}}> = -\left(\frac{\partial}{\partial \frac{1}{T}}\right) \int_0^{2\pi} \frac{d\phi'}{2\pi} \int_0^{2\pi} \frac{d\chi}{2\pi}\, e^{-\frac{\tilde{\Delta}(\Omega')}{T}}$$

if we recall that $\cos\theta' \cong \sin(\widehat{k,\ell})\cos\phi'$, where ϕ' is the azimuthal angle of $\vec{p}\,'$ in the plané perpendicular to $\vec{k}$ and $(\widehat{k,\ell})$ is the angle between the $\vec{k}$ and $\vec{\ell}$ vectors, we see immediately

$$\cos(\widehat{k,\ell}) = \sin\chi\, \sin\theta \;,$$

χ is the azimuthal angle between $\vec{k}$ and the projection of $\vec{\ell}$ in the plane perpendicular to $\vec{p}$, and θ is the angle between $\vec{p}$ and the gap axis; therefore

$$\Delta(\Omega') = \tilde{\Delta}(\sin^2\phi + \sin^2\chi\cos^2\phi\, \sin^2\theta)^{1/2} \quad .$$

For low temperatures $\tilde{\Delta}/T >> 1$, the angular average is now easily evaluated and we find

$$< \tilde{\Delta}(\Omega') e^{-\frac{\tilde{\Delta}(\Omega')}{T}} > \cong \frac{4}{\pi} \frac{T^3}{\Delta^2} \frac{1}{\sin\theta} \; . \qquad (25)$$

On the other hand, for $T \cong T_c$

$$\Gamma_1(z) = 3\, \frac{A_0}{2} \left[1 + \frac{14}{3}\, \zeta(3)\; < \frac{\tilde{\Delta}^2(\Omega')}{(\pi T)^2}>\right] S_0(z) \;, \qquad (26)$$

$$\Gamma_2(z) = -\frac{A_1}{2}\left[1 + \frac{14}{3}\,\zeta(3)\ '\!\left\langle\frac{\Delta^2(\Omega')}{(\pi T)^2}\right\rangle\right] S_1(z)\ . \qquad (27)$$

$S_0(z)$ and $S_1(z)$ are easily extracted from Eqs. 21 and 22, by entering the form of $f_0(\omega)$ shown in Eq.14. After a straightforward but lengthy calculation we find

$$S_0(z) = (\pi T)^2 + z^2 - \Delta^2(\Omega)\ U_0\left(\frac{z}{2\pi T}\right)\ , \qquad (28)$$

$$S_1(z) = 2z\ \Delta(\Omega)\ U_1\left(\frac{z}{2\pi T}\right)\ , \qquad (29)$$

where

$$U_0(z) = 2\ \ln 2 + \mathrm{Re}\left[\psi(1+iz) + C + iz\psi'(1+iz)\right], \qquad (30)$$

$$U_1(z) = U_0(z) + \mathrm{Re}\left[iz\psi'(1+iz)\right] \qquad (31)$$

Here $\psi(z)$ and $\psi'(z)$ are the digamma function and its derivative respectively and C is the Euler constant.

5. THE VERTEX FUNCTION

Here we consider the vertex associated with the stress tensor operator. The diagrams which contribute to the vertex equation areshown in Fig.2. The Azlamazov Larkin diagrams of Fig.2b) and c) do not contribute to the vertex equation and are not considered here. The equation for the vertex function is simply written as:

$$\tilde{V}(\vec{p}) = \hat{p}_i\hat{p}_j\rho_3 + \left(\frac{Z_0}{4}\right)^2 \frac{T}{[N^*(0)]^2} \sum_{n',i} \int \frac{d^3p'}{(2\pi)^3}\, \chi^{ii}_{n'-n}(\vec{p}\,'-\vec{p})\ \alpha_i \times$$

$$\times\ G_{n'}(\vec{p}\,')\ \tilde{V}(\vec{p}\,')\,G_{n'+n}(\vec{p}\,')\,\alpha_i\ . \qquad (32)$$

The matrix $\tilde{V}(\vec{p})$ can be written as

$$\tilde{V}(\vec{p}) = V(\vec{p})\,\rho_3\ . \qquad (33)$$

In general there will be off diagonal contributions to the vertex function arising from the coupling of the stress tensor operator to fluctuations of the order parameter. These terms can however be shown to be of order $\Gamma/\Delta << 1$ when compared to the dominant term in the hidrodynamic limit. Multiplying Eq. 32 by ρ_3 and taking the trace we find

$$V(\vec{p}) = \hat{p}_i\hat{p}_j + 3\left(\frac{Z_0}{4}\right)^2 \frac{T}{[N^*(0)]^2} \sum_{n'} \int \frac{d^3p'}{(2\pi)^3} \chi^+_{n'-n}(\vec{p}'-\vec{p}) \cdot$$

$$\frac{\omega_{n'}\omega_{n'+n} - \xi^2_{p'} - \tilde{\Delta}^2(\vec{p}')}{d_{n'}(\vec{p}')d_{n'+n}(p')} V(\vec{p}') \tag{34}$$

Here both the frequency ω_n and the order parameter $\tilde{\Delta}$ contain the widths due to quasiparticle-quasiparticle scattering calculated in the previous section. After analytic continuation we find

$$V_{z^-,z^++\omega}(p) = \hat{p}_i\hat{p}_j + \left(\frac{Z_0}{4}\right)^2 \frac{1}{2\pi N^*(0)} < \int dz' \ \mathrm{Im}\ \chi^+_{z'-z} \times$$

$$\times \left(\coth\frac{z'-z}{2T} - \tanh\frac{z'}{2T}\right)\int d\xi_{p'} \frac{z_1(z^++\omega)+\xi^2_{p'}-\tilde{\Delta}^2(\vec{p}')}{-d_{z^-}(\vec{p}')d_{z'+\omega}(\vec{p}')} V(\vec{p}')> \tag{35}$$

The integrals over energy and polar angle are carried out as in the corresponding calculations of the self energy and we find

$$V^n = \frac{V_{z^-,z^++\omega} + V_{-(z^++\omega),-z^-}}{2} =$$

$$= 1 + 3A_2 < (p_ip_j)^2 \frac{15}{2}\int_{\tilde{\Delta}(\Omega)}^{\infty} \frac{z'dz'}{\sqrt{z'^2-\tilde{\Delta}^2(\Omega)}} \left(1 - \frac{\tilde{\Delta}^2(\Omega)}{z^2}\right) f_0(z'-z) \times$$

$$\times \left(\coth\frac{z'-z}{2T} - \tanh\frac{z'}{2T}\right) \frac{V^n}{\Gamma(z')} + z' \to -z' \tag{36}$$

Here

$$\Gamma(z) = 2\left(\Gamma_1(z) - \frac{\tilde{\Delta}(\Omega)}{z}\Gamma_2(\Omega,z)\right) \tag{37}$$

is the inverse of the quasiparticle lifetime in superfluid ^3He-A

At low temperatures we can easily verify that the vertex correction vanishes. Using the results of the previous section we find at low temperatures

$$\Gamma(z) = 4 \frac{(A_0 + A_1 \tilde{\Delta}(\Omega)/z)}{(\sin\theta)^{1/2}} T^2 \left(\frac{2\pi T}{\tilde{\Delta}}\right)^{3/2} \quad (38)$$

Anticipating that $\sin\theta \cong T$ this equation implies a relaxation time proportional to T^{-3} rather than T^{-4}, found by analysis of the collision integral in weak coupling theory[7].

At high temperatures the vertex equation reduces to

$$V = 1 + \frac{3A_2}{\Gamma(\varepsilon)} < \frac{15}{2} (p_i p_j)^2 (S_0 - S_2) > V, \quad (39)$$

where

$$S_2 = \pi\tilde{\Delta}(\Omega) z \coth \frac{z}{2T} - \tilde{\Delta}^2(\Omega) \; U_0\left(\frac{z}{2\pi T}\right) ,$$

$U_0(z)$ is given in Eq. 30. The inverse lifetime is given by

$$\Gamma(z) = 3A_0 \left[(\pi T)^2 + z^2\right] \quad (40)$$

and S_0 is defined in Eq.28. Keeping only leading terms in $\tilde{\Delta}$ we find for the vertex function

$$\frac{V^n}{\Gamma(z)} \cong \{3(A_0 - A_2)\left[(\pi T)^2 + z^2\right] + \frac{45}{64} A_2 \pi^2 \tilde{\Delta} z \coth \frac{z}{2T} \}^{-1}. \quad (41)$$

6. THE VISCOSITY TENSOR

We saw in section 2 that the components of the viscosity tensor are proportional to the imaginary part of a generalized susceptibility. Recalling Eq.4 we see that we can write

$$\Phi_{T_{ij}}(z^-, z^+ + \omega) = p_F^4 \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\{\hat{p}_i \hat{p}_j \rho_3 \; G_{z^-}(\vec{p}) V_{z, z^+ + \omega} \rho_3 G_{z^+ + \omega}(\vec{p}) \} .$$

Evaluating the trace and performing the energy integrals we find

$$\mathrm{Im}\langle[\tau_{ij},\tau_{ij}]\rangle = \mathrm{Re}\,\frac{N^*(0)p_F{}^4\omega}{2T}\times$$

$$\times\ \langle(\hat{p}_i\hat{p}_j)^2\int_{\tilde{\Delta}(\Omega)}^{\infty} dz\ \frac{\mathrm{sech}^2 z/2T}{\sqrt{z^2-\tilde{\Delta}^2(\Omega)}}\ z\left(1-\frac{\tilde{\Delta}^2(\Omega)}{z}\right)\frac{V^n}{\Gamma(z)}\rangle \tag{42}$$

At low temperature we have seen that the vertex correction vanishes; as $\mathrm{sech}^2 z/2T$ is an exponentially decreasing function, $\Gamma(z)$ can be replaced by its value at the gap edge

$$\Gamma(\Delta) = \frac{4T^2}{\pi}\left(\frac{2\pi T}{\Delta}\right)^{3/2}\frac{A_0+A_1}{(\sin\theta)^{1/2}}. \tag{43}$$

The remaining integral can be evaluated in a straightforward manner and we find for the viscosity tensor

$$\eta^{xy} = \frac{3}{8}\,\frac{N^*(0)p_F{}^4}{(A_0+A_1)}\,\frac{T}{\tilde{\Delta}^3} = \frac{135}{2}\,\eta_0\left(\frac{1-a_2}{1+a_1}\right)\left(\frac{T_c}{\Delta}\right)^2\frac{T}{\tilde{\Delta}}$$

$$\eta^{yy} = \frac{45}{2}\,\eta_0\,\frac{1-a_2}{1+a_1}\left(\frac{T_c}{\Delta}\right)^2\left(\frac{\tilde{\Delta}}{T}\right)$$

$$\eta^{xx} = \left(\frac{45}{2}\right)^2\eta_0\left(\frac{1-a_2}{1+a_1}\right)\frac{T_c}{\tilde{\Delta}}\left(\frac{T}{\Delta}\right)^3$$

$a_1 = A_1/A_0$, $a_2 = A_2/A_0$ and η_0 is the normal state viscosity[4] at the transition ternperature.

At high temperature $(T \cong T_c)$ we find

$$\eta^{xy} = \frac{N^*(0)p_F{}^4}{6(A_0-A_2)T}\,\langle(\hat{p}_x\hat{p}_y)^2\int d\xi\ \frac{\mathrm{sech}^2\sqrt{\xi^2+\tilde{\Delta}^2(\Omega)}/2T}{(\pi T)^2+\xi^2+\tilde{\Delta}^2(\Omega)}\ \times$$

$$\times\left(1-\frac{\tilde{\Delta}^2(\Omega)}{\xi^2+\tilde{\Delta}^2(\Omega)}\right)\left(1-\frac{15}{64}\,\frac{\pi\tilde{\Delta}(\Omega)\coth\sqrt{\xi^2+\tilde{\Delta}^2(\Omega)}/2T}{(\pi T)^2+\xi^2+\tilde{\Delta}^2(\Omega)}\,\frac{A_2}{A_0-A_2}\right)\rangle$$

which gives

$$\eta^{xy} = \eta_0 \left(\frac{T_c}{T}\right)^2 \{1 - \frac{\bar{\Delta}}{\pi T} (3 + \frac{90}{64} \frac{a_2}{1-a_2} (\frac{\pi^2}{2}) + 3)\}$$

Near T_c, qdecreases linearly with $\Delta/\pi T$ which is consistent with other calculations[8]. However at low temperatures η^{xy} only diverges as T^{-1} rather than T^{-2} as estimated previously. The origin of this discrepancy can be found in the temperature dependence of the effective quasiparticle relaxation time calculated in Section 4. Since no rneasurements of the viscosity tensor are, to our knowlsdge, reported in the literature it tecomes difficult to assess the relative merits of the different theories. We hope that the existing calculations will stimulate experimental research in this area.

The author is indebted to Dr. A. Houghton for calling her attention to the problem and for many interesting discussions.

REFERENCES

1. Berk N.F. and J.R. Schrieffer, Phys. Rev. Lett. 17, 433 (1966). S. Doriach and S. Elgelsberg, Phys, Rev. Lett. 27, 750 (1966).
2. S.K. Ma, M.T. Beal.Monod and D.R. Fredkin, Phys. Rev.*174*, 277(1968). M.T.Beal-Monod, S.K.Ma and D.R.Fredkin, Phys. Rev. Lett. *20*,929(1968).
3. A.Houghton and K.Maki, Phys. Rev., to be publ ished.
4. See for instance A.J. Leggett, Rev. Mod. Phys. 47, 331 (1975).
5. R. Graham, Phys. Rev. Lett. 33, 1431 (1974).
6. See for instance A.A.Abrikosov, L.P.Gorkov and I.E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics,* Prentice Hall (1963).
7. O.Valls and A.Houghton, Phys. Lett. *54A*,143 (1975).
8. R.Combescot, Phys. Rev. Lett. 35, 471 (1975).