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On the Relation Between Fields and Potentials in Non Abelian Gauge Theories

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Some examples have been given in the literature of ambiguous gauge fields, i.e. those not having a unique potential (up to a gauge transformation). We examine an example given by Deser and Wilczek and find the condition (for any gauge group) that the group element generating the potentials must satisfy in order for the potentials not to be related by any gauge transformation. In three dimensions (for SU_2) there are other families of ambiguous fields characterized by arbitrary unit vector fields $\vec{n}(\vec{n})$ $(\vec{n}^2=1)$. The example given by Wu and Yang belongs to a particular family with $\vec{n} = \vec{n}.\vec{r}/\vec{r}$. We also find the sources of these fields and some interesting relations between them.

Alguns exemplos de campos de medida ambiguos, isto é, que não possuem um potencial Ünico (a menos de transformações de medida), apareceram na literatura. Examinamos aqui um exemplo devido a Deser e Wilczek e achamos a condição (para qualquer grupo de medida) que o elemento do grupo que gera os potenciais deve satisfazer afim de que os potenciais não sejam relacionados por nenhuma transformação de medida. Em três dimensões (para SU_2) há outras famílias de campos ambiguos caracterizados por campos de vetores unitários $\vec{n}(\vec{n})$ $(\vec{n}^2=1)$ arbitrários, O exemplo de Wu e Yang pertence a uma família particular com $\vec{n} = \vec{n}.\vec{r}/\vec{r}$. Determinamos também as fontes desses campos e algumas relações interessantes entre elas.

1. INTRODUCTION

T.S. Wu and C.N. Yang[1] have pointed out, by constructing a specific example, that in non-abelian gauge theories the field tensor $F_{\mu\nu}$ does not determine the potential A_μ, not even locally and not even up to a gauge. In their example one of the potentials corresponds to a magnetic monopole in three dimensions, the other being proportional to a pure gauge. One may then ask if their result reflects only a pathological situation or there are other cases as well. However, new examples have been given by S. Deser and F. Wilczek[2] and also a necessary condition for the existence of this phenomenon has been given in terms of the fields[2,3,4].

We examine here the first two examples of reference[2] and show that they are related by a gauge transformation, being therefore equivalent. Further we find an "integrability" condition, on the group element generating the fields, for the corresponding potentials being related by a gauge transformation.

For three dimensions and the group SU_2, we show that the example given in reference[1] is a particular case within a whole family of fields, each class of the family being characterized by a unit vector field $\vec{n}(\vec{r})$ ($\vec{n}^2=1$); Wu and Yang's example belongs to the class characterized by the choice $\vec{n}(\vec{r}) = (\vec{r}/r)$.

Recently Halpern[5] discussed the construction of all finite action "ambiguous" fields in the axial gauge. The examples we examine here do not belong to this class as they are "quasi-pure gauges"[6] with constant factors.

The fact that one and the same field $F_{\mu\nu}$ may correspond to physically inequivalent distributions of sources means that $F_{\mu\nu}$, although a gauge covariant quantity, does not carry all the information required to describe the physical situation.

In section 2 we present the main theorems leading to the above mentioned results. In section 3, we discuss some properties of the sources that give rise to the same field. In section 4 we show the gauge independente of the potentials which correspond to the same field. In section 5 we treat the specific case of three dimensions. Finally in section 6 we give a four dimensional example.

2. RELEVANT THEOREMS

Let us call

$$\Phi_\mu = \Phi^k_\mu \chi_k , \qquad (1)$$

χ_k being the generators of the corresponding Lie group.

Let Φ_μ and Ψ_μ be two vacuum potentials, i.e.:

$$\partial_\mu\phi_\nu - \partial_\nu\phi_\mu + [\phi_\mu,\phi_\nu] = 0 \qquad (2)$$

$$\partial_\mu\Psi_\nu - \partial_\nu\Psi_\mu + [\Psi_\mu,\Psi_\nu] = 0$$

then:

Theorem 1. The potentials[2]

$$A^{(\alpha)}_\mu = \alpha\phi_\mu + (1-\alpha)\Psi_\mu ; \qquad A'_\mu = (1-\alpha)\phi_\mu + \alpha\Psi_\mu \qquad (4)$$

(a = arbitrary constant) give exactly the same field .

Proof:

$$F^{(\alpha)}_{\mu\nu} = \partial_\mu A^{(\alpha)}_\nu - \partial_\nu A^{(\alpha)}_\mu + [A^{(\alpha)}_\mu , A^{(\alpha)}_\nu]$$

$$= \alpha(\partial_\mu\phi_\nu - \partial_\nu\phi_\mu) + (1-\alpha)(\partial_\mu\Psi_\nu - \partial_\nu\Psi_\mu) + \alpha^2[\phi_\mu,\phi_\nu]$$

$$+ (1-\alpha)^2 [\Psi_\mu,\Psi_\nu] + \alpha(1-\alpha)([\phi_\mu,\Psi_\nu] + [\Psi_\mu,\phi_\nu]) .$$

Eliminating the curls by using (2) and (3) we obtain:

$$F^{(\alpha)}_{\mu\nu} = -\alpha(1-\alpha)[(\phi_\mu - \Psi_\mu),(\phi_\nu - \Psi_\nu)] . \qquad (5)$$

As (5) is invariant under the interchange $a \rightleftarrows (1-\alpha)$, the theorem is proved.

Corollary 1. By taking $\Psi_\mu \equiv 0$ in theorem 1, we see that also[2]

$$A_\mu^{(\alpha)} = \alpha\, \phi_\mu \quad \text{and} \quad A_\mu^{(1-\alpha)} = (1-\alpha)\,\phi_\mu, \tag{6}$$

give exactly the same field

$$F_{\mu\nu}^{(\alpha)} = - a -)\,[\phi_\mu,\phi_\nu] = \alpha(1-\alpha)\,(\partial_\mu\phi_\nu - \partial_\nu\phi_\mu) \tag{7}$$

Theorem 2.

Theorem 1 is invariant under any yauge transformation.

Proof. Under a gauge transformation U:

$$A; = U^{-1} A_\mu\, U + U^{-1}\, \partial_\mu\, U \tag{8}$$

Substituting in (4) we get:

$$A_\mu'^{(\alpha)} = U^{-1}(\alpha\, \phi_\mu + (1-\alpha)\ \Psi_\mu)U + U^{-1}\partial_\mu U$$

$$= \alpha\ \phi'_\mu + (1-\alpha)\ \Psi'_\mu$$

and, from (5)

$$F_{\mu\nu}^{(\alpha)\prime} = -\alpha(1-\alpha)\ [(\phi'_\mu - \Psi'_\mu),(\phi'_\nu - \Psi'_\nu)]\ . \text{ Q.E.D.}$$

Corollary 2. It is always possible to find a gauge transformation which brings Theorem 1 into the form of Corollary 1.

Proof. Ψ_μ being a vacuum potential it has the form:

$$\Psi_\mu = V^{-1}\, \partial_\mu\, V\ , \quad \text{for some } V\ .$$

By performing the inverse gauge transformation V^{-1}, Ψ_μ is taken to zero, which together with theorem 2 completes the proof.

In this gauge, the potential $A_\mu^{(\alpha)}$ takes the form (6) and the vacuum potential derives from a certain group element U.

$$\phi_\mu = U^{-1} U_{,\mu} \tag{9}$$

$$A_\mu^{(\alpha)} = a\, U^{-1} U_{,\mu} \quad (\text{where } U_{,\mu} = \partial_\mu U) \tag{10}$$

Corollary 3. In the gauge in which the potential is given by (10), the field $F_{\mu\nu}^{(\alpha)}$ common to both $A_{\mu\nu}^{(\alpha)}$ and $A_\mu^{(1-\alpha)}$, takes the form:

$$F_{\mu\nu}^{(\alpha)} = \alpha(1-\alpha)(U_{,\mu}^{-1} U_{,\nu} - U_{,\nu}^{-1} U_{,\mu}) = F_{\mu\nu}^{(1-\alpha)} \tag{11}$$

For the proof it is only necessary to use the property:

$$U^{-1} U_{,\mu} + U_{,\mu}^{-1} U = 0 \tag{12}$$

in equation (7).

Potentials of the form (10) are called "quasi-pure gauges" by Y. Nambu[6].

3. SOURCES

It is convenient and interesting to find the sources of the potentials $A_\mu^{(\alpha)}$ and $A_\mu^{(1-\alpha)}$, as they give rise to the same field $F_{\mu\nu}^{(\alpha)} = F_{\mu\nu}^{(1-\alpha)}$. According to the general definition:

$$j_\nu = \partial_\mu F_{\mu\nu} + [A_\mu, F_{\mu\nu}] \tag{13}$$

it is easy to see that the current distributions corresponding to $A_\mu^{(\alpha)}$ and $A_\mu^{(1-\alpha)}$, in gauge (10), satisfy the relations

$$j_\nu^{(\alpha)} - j_\nu^{(1-\alpha)} = \alpha(1-\alpha)(2\alpha-1)[\phi_\mu, [\phi_\mu, \phi_\nu]] \tag{14}$$

$$j_\nu^{(\alpha)} + j_\nu^{(1-\alpha)} = 2\alpha\,(1-\alpha)\{\partial_\mu [\phi_\mu, \phi_\nu] + \frac{1}{2}[\phi_\mu, [\phi_\mu, \phi_\nu]]\} \tag{15}$$

Equation (15) can also be written as:

$$j_\nu^{(\alpha)} + j_\nu^{(1-\alpha)} = 8a\,(1-a)\, j_\nu^{(1/2)} \qquad (16)$$

From which it follows that, when $j_\nu^{(1/2)} = 0$:

$$j_\nu^{(\alpha)} = - j_\nu^{(1-\alpha)} \qquad (\text{if } j_\nu^{(1/2)} = 0) \qquad (17)$$

This means that, in some cases, (see below for an example) two equal and opposite sources can give rise to the same field. Eqs. (14) and (15) may, of course, be used to find the explicit form of $j^{(\alpha)}$.

The current (17) has the uncommon property (for non-abelian fields) of being conserved ($\partial_\nu j_\nu^{(\alpha)} = 0$) for any value of a. Thir comes about simply because when a particular current $j_\nu^{(\alpha_0)}$ is zero, (14) and (15) show that the double-commutator is proportional to the divergenceof the single commutator:

$$[\phi_\mu, [\phi_\mu, \phi_\nu]] = - \frac{1}{\alpha_0} \partial_\mu [\phi_\mu, \phi_\nu] \qquad (\text{if } j_\nu^{(\alpha_0)} = 0)$$

In that cases, from (13) we get:

$$j_\nu^{(\alpha)} = \partial_\mu F_{\mu\nu}^{(\alpha)} - \frac{\alpha}{\alpha_0} \partial_\mu F_{\mu\nu}^{(\alpha)} = (1 - \frac{\alpha}{\alpha_0})\, \partial_\mu F_{\mu\nu}^{(\alpha)} .$$

This "maxwellian" form for the current implies, of course, $\partial_\mu j_\mu^{(\alpha)} = 0$.

4. GAUGE INDEPENDENCE OF $A^{(\alpha)}$ AND $A^{(1-\alpha)}$

Up to this point we have given some general theorems on different potentials giving the same fields. One can now rise a natural question: Could it be that these potentials are not essentially different,but in fact one of them is the gauge transform of the other?

In order to answer this question let us take again

$$A_\mu^{(\alpha)} = \alpha\, \phi_\mu ,$$

where ϕ_μ is vacuum potential.

Suppose now that $A^{(\alpha)}$ and $A^{(1-\alpha)}$ are related by a gauge transformation, i.e.:

$$V^{-1}\,\alpha\,\phi_\mu\,V + V^{-1}\,V_{,\mu} = (1-\alpha)\,\phi_\mu \tag{18}$$

where V depends on *a*.

As the corresponding fields are equal, it follows that:

$$V\,F^{(1-\alpha)}_{\mu\nu}\,V^{-1} = F^{(1-\alpha)}_{\mu\nu} = F^{(\alpha)}_{\mu\nu}\ , \tag{19}$$

using (7) in (19) we get:

$$V(\phi_{\nu,\mu} - \phi_{\mu,\nu})\,V^{-1} = \phi_{\nu,\mu} - \phi_{\mu,\nu} \tag{20}$$

Putting $\alpha = 0$ in (18), we learn that

$$\phi_\mu = V_0^{-1}\,a_\mu\,V_0, \quad (\text{where } V_0 \equiv V(\alpha)\big|_{\alpha=0})\,. \tag{21}$$

With (21) and (12), equation (20) for $a=0$ can be written as:

$$V_{0,\mu}V^{-1}_{0,\nu} - V_{0,\nu}V^{-1}_{0,\mu} = V^{-1}_{0,\mu}V_{0,\nu} - V^{-1}_{0,\nu}V_{0,\mu} \tag{22}$$

which is a necessary condition for the potentials to be connected by a gauge transformation.

We will now show that there exist vacuum potentials which do not fulfill (22). We shall particularize with the group SU(2), for which the general form of V_0 is:

$$V_0 = e^{if\sigma} = \cos f + i\,a\,\text{sen}\,f\,, \qquad (f = f(\vec{r})) \tag{23}$$

where $a = \vec{\sigma}\cdot\vec{n}$, $|\vec{n}| = 1$.

From (23) we have:

$$V_{0,\mu} = i\,V_0\,\sigma\,f_{,\mu} + \text{sen}\,f\,\sigma_{,\mu} \tag{24}$$

$$V^{-1}_{0,\mu} = -i\, V^{-1}_{0}\, \sigma\, f_{,\mu} - i\, \text{sen} f\, \sigma_{,\mu} \tag{25}$$

from which we deduce:

$$(V_{0,\mu} V^{-1}_{0,\nu} - V_{0,\nu} V^{-1}_{0,\mu}) - (V^{-1}_{0,\mu} V_{0,\nu} - V^{-1}_{0,\nu} V_{0,\mu}) = 4\, \text{sen}^2 f (f_{,\mu}\sigma_{,\nu} - f_{,\nu}\sigma_{,\mu}). \tag{26}$$

Eq. (26) shows that the condition (22) is only satisfied if (33) fulfills:

$$f_{,\mu}\sigma_{,\nu} - f_{,\nu}\sigma_{,\mu} = 0 \tag{27}$$

In three dimensions, with the particular choice $\vec{n} = \vec{r}/r$, $\sigma \equiv \sigma_r$, we can write (27) in the form:

$$\vec{\nabla} f \wedge \vec{\nabla} \sigma_r = 0 \tag{28}$$

But

$$\vec{\nabla} \sigma_r = \frac{\vec{\sigma}}{r} - \frac{\vec{r}}{r^2} \sigma_r \tag{29}$$

Substituting (29) in (28), multiplying by $\vec{r}$ (dot product), and recalling the linear independence of the Pauli matrices, we arrive at:

$$\vec{r} \wedge \vec{\nabla} f = 0$$

so that the only sclution of (28) is f = constant. We see that when ϕ_μ (cf eq. (21)) derives from a V_0 of the form (23) in which f is not constant, then condition (22) can not be fulfilled and the potentials $A^{(\alpha)}_\mu$ and $A^{(1-\alpha)}_\mu$ can not be connected by any gauge transformation: it may be mentioned that when f = constant, the gauge transformation relating $A^{(\alpha)}_\mu$ and $A^{(1-\alpha)}_U$ is $V = e^{iv\sigma_r}$; tg v = $(1-2\alpha)$tg f.

5. THE CASE OF THREE DIMENSIONS

We have just seen that in three dimensions (and SU(2)) any potential $\vec{A}^{(\alpha)}$ of the form (10) which derives from a group element $U = e^{if\sigma}$ with a non-constant f cannot be connected by a gauge transformation with the potential $\vec{A}^{(1-\alpha)}$ which gives the same field but different current.However, when f is constant, the theorem fails and $\vec{A}^{(\alpha)}$ ceases to be physically different from $\vec{A}^{(1-\alpha)}$.

We shall consider now the case $f = \pi/2$ for which (23) gives $V_0 = i\sigma$, transforming

$$\vec{A}^{(\alpha)} = \alpha\, \sigma \vec{\nabla} \sigma \tag{30}$$

into $A^{(1-\alpha)}$

We shall, in this case, construct another potential (not equivalent to (30)) which gives the same field,

$$\vec{B}^{(\alpha)} = \alpha(1-\alpha)\, \vec{\nabla} \sigma \wedge \vec{\nabla} \sigma = \vec{B}^{(1-\alpha)} \tag{31}$$

[$\vec{B}$ is the dual of F_{ij}]

For that purpose we shall first prove the following lemma:

<u>Lemma</u>

$$\vec{k} = \vec{\nabla} \sigma \wedge \vec{\nabla} \sigma\, \sigma \tag{32}$$

is an ordinary vector (free from Pauli matrices) having zero divergence.

<u>Proof.</u> (recall $\sigma = \sigma_i n_i$)

$$K_i = \varepsilon_{ijk}\, \sigma_{,j}\, \sigma_{,k}\, \sigma$$

$$K_i = \varepsilon_{ijk}\, n_{a,j}\, n_{b,k}\, (\delta_{ab} + i\, \varepsilon_{abc}\, \sigma_c)\sigma$$

The δ_{ab} term does not contribute (as $n_a n_{a,i} = 0$), we are left with

$$K_i = i\, \varepsilon_{ijk}\, n_{a,j}\, n_{b,k}\, \varepsilon_{abc}\, n_d (\delta_{cd} + i\, \varepsilon_{cde}\, \sigma_e) \tag{33}$$

This time the term in ε_{cde} does not contribute (as $\varepsilon_{abc}\, \varepsilon_{cde} = \delta_{ad}\, \delta_{be} - \delta_{ae}\, \delta_{bd}$). Therefore, no σ matrix survives in (33) and the first part of our lemma is proved.

The divergence of (33) is:

$$\partial_i K_i = i\, \varepsilon_{ijk}\, n_{a,j}\, n_{b,k}\, n_{c,i}\, \varepsilon_{abc} = i\, f_{abc}\, \varepsilon_{abc}\ ,$$

where

$$f_{abc} = \varepsilon_{ijk}\, n_{a,j}\, n_{b,k}\, n_{c,i} \tag{34}$$

is a completely antisymmetric tensor of the third rank totally orthogonal to $\vec{n}(n_a f_{abc} = 0)$. The tensor (34) belongs then to a two-dimensional sub-space (orthogonal to $\vec{n}$). But we know that a completely antisymmetric tensor cannot exist when the rank is greater than the number of dimensions. So f_{abc} is identically zero and the lemma is proved.

As an immediate consequence we have

$$\vec{\nabla}\sigma \wedge \vec{\nabla}\sigma = i\, \sigma\, \vec{\nabla} \wedge \vec{a} \tag{35}$$

for some vector field $\vec{a}(\vec{r})$.

We are now in position to prove the following.

Theorem 3

The potentials $\vec{A}^{(\alpha)}$ $(\vec{A}^{(1-\alpha)})$ (given by (30)) and

$$\vec{A}^{(\alpha)} = \frac{1}{2}\, \sigma\, \vec{\nabla}\sigma - i\left(\alpha - \frac{1}{2}\right)^2 \sigma\, \vec{a} \tag{36}$$

($\vec{a}$ satisfying (35)), give exactly the same field (31)

Proof. Let us compute the field due to $\vec{A}^{(\alpha)}$

$$\vec{\nabla} \wedge \vec{A}^{(\alpha)} = \frac{1}{2}\, \vec{\nabla}\sigma \wedge \vec{\nabla}\sigma - i\left(\alpha - \frac{1}{2}\right)^2 (\vec{\nabla}\sigma \wedge \vec{a} + \sigma\, \vec{\nabla} \wedge \vec{a})\ .$$

Or, using (35),

$$\vec{\nabla} \wedge \vec{A}^{(\alpha)} = \left[\frac{1}{2} - \left(\alpha - \frac{1}{2}\right)^2\right] \vec{\nabla}\sigma \wedge \vec{\nabla}\sigma - i\left(\alpha - \frac{1}{2}\right)^2 \vec{\nabla}\sigma \wedge \vec{a} \tag{37}$$

On the other hand, noting that

$$\{\sigma, \vec{\nabla}\sigma\} = \vec{\nabla}\sigma^2 = 0\ , \tag{38}$$

$$\vec{A}^{(\alpha)} \wedge \vec{A}^{(\alpha)} = -\frac{1}{4} \vec{\nabla}\sigma \wedge \vec{\nabla}\sigma + i\left(\alpha - \frac{1}{2}\right)^2 \vec{\nabla}\sigma \wedge \vec{a} \qquad (39)$$

When we add (37) and (39) we find:

$$\vec{B}^{(\alpha)} = \left[\frac{1}{4} - \left(\alpha - \frac{1}{2}\right)^2\right] \vec{\nabla}\sigma \wedge \vec{\nabla}\sigma ,$$

which coincides with (31) and proves the theorem.

It is not difficult to compute the currents corresponding to (30) and (36). We only give the final answer:

$$\vec{j}^{(\alpha)} = i\, \alpha(1-\alpha)(1-2\alpha)\vec{\nabla}\sigma \wedge \vec{\nabla} \wedge \vec{a} + i\alpha(1-\alpha)\sigma \vec{\nabla} \wedge \vec{\nabla} \wedge \vec{a} \qquad (40)$$

$$\vec{j}^{(\alpha)} = i\, \alpha(1-\alpha)\, \sigma \vec{\nabla} \wedge \vec{\nabla} \wedge \vec{a} \qquad (41)$$

It is easy to see that, while $\vec{j}^{(1-\alpha)} = \sigma\, \vec{j}^{(\alpha)}\sigma$, no possible gauge transformation relating (30) and (36) can exist; for such a transformation would commute with o (as $\vec{B}$ is invariant), but then is would also commute with $\vec{j}^{(\alpha)}$ leaving it invariant, contradicting (40) - (41).

When $\vec{b} = \vec{\nabla} \wedge \vec{a}$ is curless, $\vec{j}^{(\alpha)} = 0$, while

$$\vec{j}^{(\alpha)} = i\, \alpha(1-\alpha)\ (1-2\alpha)\vec{\nabla}\sigma \wedge \vec{b} = -\vec{j}^{(1-\alpha)}$$

The example of reference (1) belongs to this class with $\vec{b} = \vec{r}/r^3$. Note also that (36) is not an "almost pure gauge"[6].

6. EXAMPLE IN FOUR DIMENSIONS.

Let us take the following group element:

$$U = |X|^{-1}\ (X_0 + i\ \vec{\sigma}.\vec{r}) \qquad X = (X_\mu\ X_\mu)^{1/2} \qquad (42)$$

From it we deduce the potential:

$$A_\mu^{(\alpha)} = \alpha\, U^{-1}\, \partial_\mu\, U = -\,2\,i\,\alpha\, \frac{\sigma_{\mu\nu}\, X_\mu}{X^2} \tag{43}$$

where $\sigma_{ij} = \frac{1}{2}\, \varepsilon_{ijk}\, \sigma_k$ and $\sigma_i - \frac{1}{2}\,\sigma_{i4}$ $(\sigma_{\mu\nu} = -\,\sigma_{\nu\mu})$.

The field corresponding to (43) is:

$$F_{\mu\nu}^{(\alpha)} = \frac{4i\alpha\ (1-\alpha)}{X^4}\ (X_\mu\ \sigma_{\nu\rho}\ X_\rho - X_\nu\ \sigma_{\mu\rho}\ X_\rho + X^2\ \sigma_{\mu\nu}), \tag{44}$$

and the current:

$$j_\mu^{(\alpha)} = 8i\alpha(1-\alpha)\ (1-2\alpha)\ \frac{\sigma_{\mu\rho}\ X_\rho}{x^4} = -\ j_\mu^{(1-\alpha)} \tag{45}$$

We see that $j_\mu^{(1/2)} = 0$ and eq. (17) is in force. Then, two equal and opposite currents give raise to the same field (without being physically equivalent).

In this case it is possible to show directly that a gauge transformation relating $A^{(\alpha)}$ and $A^{(1-\alpha)}$ cannot exist. In fact, such a transformation V should commute with $F_{\mu\nu}^{(\alpha)}$ (eq. (44)) and anticommute with $j_\mu^{(\alpha)}$ (eq. (45)). The last condition is easily shown to imply the anticommutativity of V with all three Pauli matrices.

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