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The Green's Function of a Relativistic Free-Electron in a Uniform Magnetic Field

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A closed representation is obtained for the Green's function of the Dirac equation of a free-electron in a uniform magnetic field.

Deduz-se uma representação fechada para a função de Green para a equação de Dirac para um eletron livre na presença de um campo magnético uniforme.

1. DERIVATION OF THE GREEN'S FUNCTION

The present note contains a simple derivation of the Green's function for a Dirac particle in a uniform magnetic field.

The Green's function satisfies the followings inhomogeneous Dirac equation

$$(i\gamma_\mu \pi^\mu - \mu)S(x,y) = \delta^4(x-y) \tag{1}$$

where $\pi^\mu = \partial^\mu - e A^\mu$ is the energy-momentum four vector;

$$'A = (0, -1/2\, Hx, , 1/2\, Hx_1, 0)$$

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is the electromagnetic four vector potential; $\underset{\sim}{H}$ is the uniform magnetic field in the z-direction and μ is the electron mass.

We can make the following Anzats

$$S(x,y) = (i\gamma_\mu \pi^\mu + \mu)\ F(x,y) \tag{2}$$

and as A does not depend on x_0 and x_3, we may write $F(x,y)$ as a Fourier transform of a transversal Green's function

$$F(x,y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dp_0 dp_3 \ \exp[ip_0(x_0-y_0) - ip_3(x_3-y_3)] K(\vec{s},\vec{s}',p_0,p_3) \tag{3}$$

which satisfies the following differential equation

$$[p_0^2 - p_3^2 - 2\omega\sigma_3 - \mu^2 - \omega^2 s^2 + \nabla_\perp^2 - 2\omega L_3]\ K(\vec{s},\vec{s}';p_0,\ p_3) = \delta(\vec{s}' - \vec{s}) \tag{4}$$

with $\vec{s} \equiv (x_1, x_2)$ and $\omega = e\,H/2$. Assuming $K(\vec{s},\vec{s}';\ p_0,p_3)$ diagonal on σ_3 we may write $\sigma_3 K = sK$ with $s = \pm 1$.

As in the non-relativistic case (Bellandi, Zimerman-1975) we can firstly seek for $K(\vec{s},\ \vec{s}';\ p_0,\ p_3)$ in the series form

$$K(\vec{s},\vec{s}';p_0,p_3) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} K^m(\vec{s},\vec{s}';p_0,p_3)\ \exp[im(\phi - \phi')] \tag{5}$$

where m are the eigenvalues of L_3; s and ϕ are the polar coordinates in the (x_1,x_2) plane.

The partial Green's function $K^m(s,s';\ p\ ,\ p\ )$ satisfies the following differential equation

$$\left[\frac{d^2}{ds^2} + \frac{1}{s}\ \frac{d}{ds} - \frac{m^2}{s^2} - \omega^2 s^2 + \lambda - 2m\omega\right] K^m(s,s';p_0,p_3) = (ss')^{-1/2}\ \delta(s-s') \tag{6}$$

with $\lambda = p_0^2 - p_3^2 - 2\omega s - \mu^2$. A solution of this equation is (we put ω=1)

$$K^m(s,s'\ p_0,p_3) = \Gamma\left(\frac{1+|m|+m-\lambda/2}{2}\right)(ss')^{-1}\ M_{\frac{\lambda-m}{2},\frac{|m|}{2}}(s<)\ W_{\frac{\lambda-m}{2},\frac{|m|}{2}}(s>)\ . \tag{7}$$

Here $s_<$ and $s_>$ are the lesser and greater of s' and s' respectively and M and W are the Whittaker functions. Replacing the product of the Whittaker function in Eq.7 by an integral representation in the complex plane (Buchholz, 1969) we can perform the summation in Eq. 5 and the final expression for the transversal Green's function is

$$K(\vec{s}, \vec{s}\,'; p_0, p_3) = \frac{1}{2\pi} \Gamma(\frac{1}{2} - \frac{\lambda}{4}) \exp\left[\frac{i}{2} |\vec{s} \times \vec{s}\,'|\right] \frac{1}{|\vec{s} - \vec{s}\,'|} W_{\frac{\lambda}{4}; 0}(|\vec{s} - \vec{s}\,'|^2). \tag{8}$$

Since this function is an analytical function, Eq. 8 is valid for arbitrary values of A. This expression must be interpreted as a 4x4 matrix, and for a defined value of s we must write $1/2(1 + s\,\Sigma^3)K$; $\Sigma^3 = \sigma^{12}$.

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