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An Analytical Method for Deterrnining the Thickness of Targets Used for Nuclear Reaction Studies

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A method consisting of a series iintegration of the thin target cross section formula, which is expressed in a form that allows an estimation of the target thickness, ΔE, has been developed. A knowledge of the absolute differential cross sections at a given angle is necessary for at least one group of the emergent reaction particles emitted in the reaction being studied. However, these cross sections need be known only for an energy interval below the incident particle energy, which is slightly larger than the energy loss expected to be incurred in the target. The uncertainty in the estimated target surface density depends mainly on the uncertainties in the cross sections and on the accuracy of a series approximation to the form of these cross sections.

Foi desenvolvido um método que consiste de uma integração em série da fórmula da secção de choque, a qual é expressa numa forma que permite estimar a espessura ΔE do alvo. É necessário o conhecimento das secções de choque diferenciais absolutas em um dado ângulo para no mínimo um grupo de partículas de reação emitidas na reação estudada. Essas secções de choque, contudo, precisam ser conhecidas somente para um intervalo de energia abaixo da energia da partícula incidente. Este intervalo é ligeiramente maior do que a perda de energia da partícula no alvo. A in-

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certeza na densidade superficial estimada do alvo, depende das incertezas nas secções de choque e das aproximações feitas no desenvolvimento em série das secções de choque.

1. INTRODUCTION

Targets are considered to be of intermediate thickness when the energy loss of the incident particle in the target is of the order of the energy resolution of the detector. For the time of flight facility used this represents thicknesses of 100 keV to 1 MeV. This energy loss in the target depends on the stopping power of the target material for the incident particles and hence depends on the incident energy. However, once the energy loss is determined for a given beam energy, it may easily be converted into the energy independent target description more commonly used: surface density or number of target nuclei per cm^2.

The use of such targets for nuclear reaction studies, especially those involving neutrons as reaction products, has the advantage of an appreciable increase in the counting rate or yield of the detected particle. Even though limitations on target thickness are imposed by the inherent structure in the differential cross section curves, intermediate thick targets ai-e convenient for making more rapid surveys of the cross sections. In addition such targets are perfectly appropriate for studying a large variety of nuclear reactions for which the differential cross sections are smoothly varying over large energy ranges.

The difficulty in using this type of target is in determining its surface density with a required accuracy. It is possible, of course, to measure the energy of the beam inducing the nuclear reaction after passage through the target, thus obtaining the energy loss which, with the knowledge of the incident energy, serves to determine the surface density. This however, usually calls for more complicated experimental procedures which are often times impractical.

For this reason, the purpose of this paper is to present an analytical method for determining the thickness of such targets. The method requires an a *priori* knowledge of the absolute differential cross sectionsat a given angle, for at least one group of the emergent reactionparticles being detected, for an energy interval below the incident particle energy which is slightly larger than the energy loss which is expected to be incurred by the target.

For each new target, a spectrum at this incident particle energy and scattering angle are taken and from the yield and known cross sections, the energy loss may be calculated. Checks may be repeated intermittently during the data taking.

2. THE ANALYTICAL METHOD

Consider the "thin target formula" which is valid for an infinitesimal layer of target material:

$$dy = \frac{d\sigma}{d\Omega} \cdot N \cdot n \cdot \Delta\Omega \cdot \varepsilon \qquad (1)$$

where dy is the yield per beam integration of the emergent group of detected particles for which the differential cross sections are known, $d\sigma/d\Omega$ (E) $\equiv \sigma(E)$ is the known differential cross sections at agiven angle, N is the surface density of the target in units of nuclei per cm^2, n is the number of incident particles reaching the target per beam integration, AR is the solid angle subtended by the detector and ε is the overall detector efficiency, which for neutrons is a function of neutron energy.

Expressing the density of the target material in g/cm^3 as $\rho(x)$ where the parameter x corresponds to the target: thickness it is possible to express N as:

$$N = \frac{\rho(x)\,dxAP}{M} \qquad (2)$$

where A is Avogadro's number, M is the molecular weight (grams) of the target material and P is the relative isotopic purity of the target, Substitution of equation (2) in (1) gives

$$dy = \sigma(E)\ n\ \frac{\rho(x) AP\ dx}{M}\ \Delta\Omega\varepsilon \tag{3}$$

which by differentiating with respect to energy and multiplying bothsides by dE gives:

$$\frac{dy}{dE}\ dE = \sigma(E)\ \frac{n\rho(x)\ AP}{M\ dE/dx}\ \Delta\Omega\varepsilon\ dE \tag{4}$$

where the quantity (dE/dx)(E), defined as $S(E)$, is the stopping power of the target material for the incident beam of particles inducing the nuclear reaction.

Rearrangement of equation (4) gives

$$dE = \frac{(dy/dE) dE\ M\ S(E)}{\sigma(E) n\rho AP\Delta\Omega\varepsilon(E)} \tag{5}$$

By substituting the expression for dy/dE in equation *(4)* into equation (5) and integrating, the quantity $\int dE \equiv \Delta E$ is obtained which represents the energy loss which the incident beam suffers in traversing the target. The quantity $\int(dy/dE)dE$ represents under the peak identified with the group of emergent particles whose differential cross sections are given by $\sigma(E)$.

3. DETAILS OF THE INTEGRATION

To integrate equation (5) it is necessary to know the dependence of the target stopping power, detector efficiency and cross sections on energy. These of course depend on the particular reaction being used and for this reason the integration will be demonstrated for a particular case. Our choice is the unresolved $(n_0 + n_1)$ neutron groups from the reaction $^7Li(p,n)\,^7Be$ detected using time of flight techniques and a Nuclear Enterprise 213 liquid scintillator of 5 inch diameter and 1 inch length. The incident proton energy is 10.00 MeV.

It can be seem from the stopping power curves[1] for Li, the reaction kinematics and the neutron detection efficiency[2] of the above scintillator that for protons with energies between 9 MeV and 10 MeV, a linear approximation for both the variables $S(E)$ and $\varepsilon(E)$ is sufficient, i.e.

$$S(E) = S_0 \left[1 + \left(\frac{dS}{dE}\right)_0 \frac{(E-E_0)}{S_0} \right] \qquad (6)$$

$$\varepsilon(E) = \varepsilon_0 \left[1 + \left(\frac{d\varepsilon}{dE}\right)_0 \frac{(E-E_0)}{S_0} \right] \qquad (7)$$

where E_0 in this case is the proton bombarding energy of 10.00 MeV. E is a value of proton energy at some point within the target. From the cross section data of 6orchers[3] for the n_0 and n_1 groups between 9.0 MeV and 10 MeV, it is apparent that a quadratic dependence of o (E) on energy would be a very good approximation. Thus,

$$\sigma(E) = \sigma_0 \left[1 + \left(\frac{d\sigma}{dE}\right)_0 \frac{(E-E_0)}{\sigma_0} + \frac{1}{2} \frac{d^2\sigma}{dE^2} \frac{(E-E_0)^2}{\sigma_0} \right] \qquad (8)$$

It is interesting at this point to check the sign of thequantity $(E-E_0)$. Since for the $^7Li(p,n)\,^7Be$ reaction at 0^0, $d\sigma/dE$ is negative in the energy range of interest (i.e. the cross sections decrease monotonically with energy) then the quantity $(d\sigma/dE)_0\,(E-E_0/\sigma_0)$ is always positive since E is always less than E_0. Thus the differential cross section o increases with decreasing energy E, as it should.

Substituting the analytic forms (6), (7) and (8) in equation (5) and calling

$$\frac{S_0 M}{\sigma_0 n\rho AP\ \Delta\Omega\varepsilon_0} \equiv \frac{BS_0}{\varepsilon_0\sigma_0}$$

where

$$B = \frac{1.075\ M}{n\rho A\Delta\Omega}$$

since the isotopic purity of metalic Lithium is .93 ^7Li, gives

$$dE = \frac{BS_0}{\varepsilon_0\sigma_0} \frac{\frac{dy}{dE}\, dE \left[1 + \left(\frac{dS}{dE}\right)_0 \frac{(E-E_0)}{S_0}\right]}{\left[1 + \left(\frac{d\sigma}{dE}\right)_0 \frac{(E-E_0)}{\sigma_0} + \frac{d^2\sigma}{dE^2} \frac{(E-E_0)^2}{2\sigma_0}\right] \left[1 + \left(\frac{d\varepsilon}{dE}\right)_0 \frac{(E-E_0)}{\varepsilon_0}\right]}$$

Using tables of stopping power[1] and efficiencies[2] evaluation of the terms

$$\left[1 + \left(\frac{dS}{dE}\right)_0 \frac{(E-E_0)}{S_0}\right] \quad \text{and} \quad \left[1 + \qquad (E-E_0)\right]$$

for E_0 = 10.0 MeV and E = 9.0 MeV shows that they differ at most by a few percent. Thus for these calculations they may be considered to cancel.

Denoting

$$\frac{1}{\sigma_0} \left(\frac{d\sigma}{dE}\right)_0 \equiv \ell \tag{9}$$

and

$$\frac{1}{2\sigma_0} \left(\frac{d^2\sigma}{dE^2}\right)_0 \equiv m \tag{10}$$

then

$$dE = \frac{BS_0}{\varepsilon_0\sigma_0} \frac{\frac{dy}{dE}\, dE}{\left[1+\ell(E-E_0)+m(E-E_0)^2\right]}$$

and expansion of the denominator term to third order approximation gives:

$$dE = \frac{BS_0}{\varepsilon_0\sigma_0} \frac{dy}{dE}\, dE \left[1 - \ell(E-E_0)+(R^2-m)(E-E_0)^2+(2\ell m-\ell^3)(E-E_0)^3\right] \tag{11}$$

which upon integration becomes:

$$\Delta E = \frac{BS_0}{\varepsilon_0\sigma_0}\int \frac{dy}{dE}\,dE - \frac{BS_0}{\varepsilon_0\sigma_0}\int \ell\,\frac{dy}{dE}\,(E-E_0)\,dE + \frac{BS_0}{\varepsilon_0\sigma_0}\int (\ell^2-m)\,\frac{dy}{dE}\,(E-E_0)^2 dE +$$

$$+ \frac{BS_0}{\varepsilon_0\sigma_0}\int \frac{dy}{dE}\,(2\ell m-\ell^3)\,(E-E_0)^3 dE \qquad (12)$$

where it should be remembered that $\int(dy/dE)dE$ = area of the n_0+n_1 unresolved groups in the neutron time of flight spectrum.

Employing the same considerations on S, ε and σ, as done in the above paragraphs, to equation (4) gives:

$$\frac{dy}{dE} = \frac{\varepsilon_0\sigma_0}{BS_0}\left[1 + \frac{\left(\frac{d\sigma}{dE}\right)_0 (E-E_0)}{\sigma_0} + \frac{\left(\frac{d^2\sigma}{dE^2}\right)_0 (E-E_0)^2}{2\sigma_0}\right]$$

or

$$\frac{dy}{dE} = \frac{\varepsilon_0\sigma_0}{BS_0}\left[1 + \ell(E-E_0) + m(E-E_0)^2\right] \qquad (13)$$

Substitution of this into equation (12) followed by simplification and integration between E_0 and E where E_0 is the incident proton bombarding energy and E is the proton energy after passge through the target, results, to the third order in ΔE, in:

$$\Delta E + \frac{\ell(\Delta E)^2}{2} + \frac{m(\Delta E)^3}{3} = (\Delta E)_0 \qquad (14)$$

where $(\Delta E)_0 \equiv (BS_0/\varepsilon_0\sigma_0)$ × Area of peak, is negative since the quantity so defined above is intrinsically negative. This equation in ΔE, the energy loss incurred in the intermediate thick target, will be referred to as the energy loss equation.

4. DISCUSSION OF THE ENERGY LOSS EQUATION

Recalling that ℓ and m are related to the first and second derivatives of the differential cross section with respect to energy, (eq. (9) and eq. (10)) three possibilities arise.

The first is the case where σ = constant, i.e. $\ell = 0$, $m = 0$. For this case $\Delta E = (\Delta E)_0$ represents an upper limit (for the case of cross sections which decrease with energy) on the energy loss expected in the target.

The second case is when σ varies linearly with energy. For this case with $\Delta E \ll 1$ MeV the equation may be well approximated by the following quadratic one

$$\Delta E + \frac{\ell(\Delta E)^2}{2} = (\Delta E)_0 \tag{15}$$

The third case is when σ has a quadratic dependence on energy. In this case for $\Delta E \ll 1$ MeV the equation may be approximated by the cubic equation

$$\Delta E + \frac{\ell(\Delta E)^2}{2} + \frac{m(\Delta E)^3}{3} = (\Delta E)_0 \tag{16}$$

It has been found that the convergence of the equation in going from σ= = constant to $\sigma(E)$ = quadratic function of (E) is quite rapid.

For target "thicknesses" greater than 1 MeV the $(\Delta E)^5$ and $(\Delta E)^6$ terms from Equation (12) may not be negligibie and the solution to the equation becomes much more laborious.

5. CONCLUSION

In concluding we present a concrete example taken from our data. From the time of flight spectrum of neutrons from the reaction $^7Li(p,n)$ ^7Be, at 10.00 MeV proton bombarding energy, the area of the (n_0+n_1) peak is

1.357 x 105 cts per beam integration. The number of particles incident on the target per beam integration was 2.5 x 10^{13} and the solid angle subtended by the detector at 0° was $\Delta\Omega$ = 5.19 x 10^{-3} SR. These conditions give a value of B = 1.796 x 10^{-34} cm^3/SR.

From the reactions kinematics and the detector efficiency curve a value of ε_0 = .128 is expected. The stopping power of ^7Li for 10 MeV protons is 19.65 x 10^3 KeV/cm and from Borcher's data the $(n_0 + n_1)$ differential cross section at 10 MeV is 5.79 x 10^{-27} cm^2/SR. Compounding these, gives a value of

$$(\Delta E)_0 = \frac{BS_0}{\varepsilon_0 \sigma_0} = - .646 \text{ MeV}$$

which is the upper limit on the energy loss incurred in this target.

From the cross section data of Borchers[3] a good fit to the 10 MeV region is given by the following quadratic

$$\sigma = 110.89 - 18.96\, E + .845\, E^2$$

Thus at 10 MeV

$$\ell = - 0.356/\text{MeV}$$

$$m = + 0.146/\text{MeV}^2$$

and the energy loss equation gives

$$AE - .178\,(\Delta E)^2 + .049\,(\Delta E)^3 = -.646 \text{ MeV}$$

Solution of this cubíc equatíon gives one real root and two complex, the real root being

$$\Delta E = - .578 \text{ MeV}$$

which represents the energy loss incurred in the target.

It is interesting to note that had we assumed a linear dependence of cross section on energy, i.e. assumed $m = 0$ then the solution to the quadratic equation (15) in ΔE would give

$$\Delta E = -.584 \text{ MeV}$$

which shows the rapid convergence of the terms mentioned previously.

6. UNCERTAINTIES

The greatast source of uncertainty in the calculation of the energy loss comes from the uncertainties associated with the cross section measurements which in most cases lie between 8-20%.

In the calculation of target surface density the uncertainties in the values of stopping power should also be included although these should be much less than the errors in the reaction cross sections.

Differential cross sections at various other energies at 0^{o} for the unresolved $n_0 + n_1$ groups were calculated from our spectra using the target thickness evaluated by this method. The results were in excellent agreement with those reported by Borchers[3].

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