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Systematic Structure Calculation in the Z = 50 Region Tin Isotopes*

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The energy spectra, the transition probability (BE2) and the spectroscopic factors are analysed by means of the semimicroscopicmodelapplied to the tin isotopes. In this model the 2^+ vibrations energy of even nuclei are treated by the BCS and RPA method, and the odd tin spectra are obtained by coupling a quasi particle to the 2^+ vibration. A schematic interaction is used: Dirac's delta + quadrupole-quadrupole.

Analisa-se um modelo semi-microscópico aplicado aos isótopos de estanho estudando o espectro, a probabilidade de transição BE2 e os fatores espectroscópicos. Nesse modelo a excitação coletiva 2^+ dos núcleos pares é tratada pelo método BCS e RPA e o espectro dos núcleos ímpares é obtido acoplando-se uma quase partícula a essa vibração 2^+. Usa-se uma interação esquemática: delta de Dirac + quadrupolo-quadrupolo.

1. INTRODUCTION

The odd tin isotopes were firstly calculated in a systematic way by Kisslinger and Sorensen[1]. They employed a pairing plus quadrupole - quadrupole force as the effective interaction hetween the nucleons.

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For the odd tin isotopes there are many calculations[2-8] also that include realistic forces. In this region we have already sufficiently experimental data which enable us to appreciate better the quasi-particle vibration coupling scheme as that used by Kisslinger and Sorensen.

In order to study the experimental tin isotopes systematics[9-16] and microscopic models of intermediate coupling we make a simple microscopic calculation using as the effective interaction the delta plus quadrupole-quadrupole force because it is founded that its reduced matrix elements are similar in the general trend to the realistic matrix elements[17].

Such model calculation is effected in the usual scheme[18] of BCS (Bardeen Cooper and Schrieffer) and RPA (Randon Phase Approximation) to treat the vibrational 2^+ mode of correspbnding even tin isotopes. After this a quasi-particle is coupled to one and to two 2^+ phonon states.

In this paper the notation is essentially that of Baranger[18].

2. FORMALISM

For the system of active nucleons we can write the following hamiltonian

$$H = \Sigma\, \varepsilon_a\, a_a^+\, a_a + \frac{1}{4}\, C\, \langle\alpha\beta|V|\gamma\delta\rangle\, a_\alpha^+\, a_\beta^+\, a_\delta\, a_\gamma \qquad (1)$$

Introducing the Lagrange multiplier λ_a that depends onlyonthe third component of isospin and performing the Bogoliubov-Valatin transformation,

$$a_\alpha^+ = U_\alpha\, c_\alpha^+ + V_\alpha\, S_\alpha\, c_{\bar{\alpha}}$$

$$S_\alpha = (-)^{j_\alpha - m_\alpha}$$

we obtain:

$$H' = H - \Sigma\, \lambda_a\, a_a^+\, a_a$$

$$= \Sigma\, (2j_a + 1) \left[\left(\varepsilon_a - \frac{\mu_a}{2} - \lambda_a\right) V_a^2 - \frac{\Delta_a}{2} U_a V_a \right]$$

$$+ \Sigma \left[(\varepsilon_a - \mu_a - \lambda_a)(U_a^2 - V_a^2) + 2\, \Delta_a U_a V_a \right] c_\alpha^+ c_\alpha$$

$$+ \frac{1}{2} \Sigma \left[(\varepsilon_a - \mu_a - \lambda_a) 2U_a V_a - \Delta_a (U_a^2 - V_a^2) S_\alpha \right] \left[c_\alpha^+ c_{\bar{\alpha}}^+ + c_{\bar{\alpha}} c_\alpha \right]$$

$$+ \frac{1}{4} \Sigma \langle \alpha\beta | V | \gamma\delta \rangle : a_a^+ a_\beta^+ a_\delta a_\gamma . \qquad (2)$$

where: : : indicates the Wick's Normal Product.

$$\mu_a = \sum_b F_{ab}\, \Omega_b\, 2V_b^2 \qquad F_{ab} = \frac{-F_0(aa,bb)}{2\sqrt{\Omega_a\, \Omega_b}} \qquad (3a)$$

$$\Delta_a = \sum_b G_{ab}\, \Omega_b\, U_b\, V_b \qquad G_{ab} = \frac{-G_0(aa,bb)}{2\sqrt{\Omega_a\, \Omega_b}} \qquad (3b)$$

$$\Omega_b = (j_b + \tfrac{1}{2})$$

Imposing the BCS condition on the third term of expression (2), that is, from its elimination, we obtain the following gap equation:

$$\Delta_a = \frac{1}{2} \Sigma\, G_{ab} \frac{\Omega_b}{E_b} \Delta_b \qquad (4a)$$

and

$$E_a = \sqrt{(e_a - \lambda_a)^2 + \Delta_a^2} \qquad (4b)$$

$$e_a = \varepsilon_a - \mu_a \qquad (4c)$$

$$\left\{ \begin{matrix} U_a^2 \\ V_a^2 \end{matrix} \right\} = \frac{1}{2} \left[1 \pm \frac{e_a - \lambda_a}{E_a} \right] \qquad (4d)$$

With the condition of conservation on the average of the neutron number we obtain the following equation:

$$\sum_a \Omega_a \left[1 - \frac{e_a - \lambda_a}{E_a} \right] = \text{number of neutrons}$$

Where the summation is effected by maintaining the z component of isospin.

Discarding the first term of (2), the Hamiltonian can be written in the following form:

$$H = H_1 + H_2 + H_3$$

$$H_1 = \sum E_a c_a^+ c_a$$

$$H_2 = \sum \{ M_2(\alpha\beta\gamma\delta) c_\alpha^+ c_\beta^+ c_\delta c_\gamma + N_2(\alpha\beta\gamma\delta)(c_\alpha^+ c_\beta^+ c_\gamma^+ c_\delta^+ + h.c.) \} \qquad (6)$$

$$H_3 = \sum M_3(\alpha\beta\gamma\delta)(c_\alpha^+ c_\beta^+ c_\gamma^+ c_\delta + h.c.)$$

$\hat{H}_1$ represents the single quasi-particle energy;

$\hat{H}_2$ contributes to particle-hole excitation. It is, then, important to produce fluctuations in the nucleon density and therefore collective vibrational states of even-even nuclei';

$\hat{H}_3$ describes the quasi-particle coupling to particle-hole states, being therefore more important to the odd systems of nucleons.

2.1. The Even-Even Number of Nucleous (RPA)

Representing by $\hat{B}^+_{\lambda JM}$ an operator that creates an excitation with energy ω_{XJ} and angular momentum (JM)

$$\left[\hat{H}_1 + \hat{H}_2\right] \hat{B}^{+}_{\lambda JM}|\phi_0> = \left[\omega_{\lambda J} + E_0\right] \hat{B}^{+}_{\lambda JM}|\phi_0> \tag{7}$$

where $|\phi_0>$ is the ground state with energy E_0 and

$$\hat{B}_{\lambda JM}|\phi_0> = 0$$

we have

$$\left[\hat{B}_{\lambda JM}, \hat{H}_1 + \hat{H}_2\right] = \omega_{\lambda J}\, \hat{B}_{\lambda JM} \tag{8}$$

In order to solvethe above equation, we takc for $B_{\lambda JM}$ the following form:

$$\hat{B}_{\lambda JM} = \sum_{a \geq b} \{X^{A}_{abJ}\, A_{abJM} - (-)^{J-M}\, Y^{\lambda}_{abJ}\, A^{+}_{ab\bar{J}\bar{M}}\} \tag{9}$$

with

$$A^{+}_{abJM} = \frac{1}{\sqrt{1 + \delta_{ab}}} \Sigma(j_a\, m_a\, j_b\, m_b | JM)\, c^{+}_{\alpha}\, c^{+}_{\beta} \tag{10}$$

Introducing (9) in (8) and discarding terms like $c^{+}c$ or terms containing four c operators we obtain a linear equation on $(X^{A}_{abJ}, Y^{A}_{abJ})$:

$$\sum_{q} (A \pm B)_{pq}\, (X^{\lambda}_{qJ} \pm Y^{\lambda}_{qJ}) = \omega_{\lambda J}(X^{\lambda}_{pJ} \mp Y^{\lambda}_{pJ}) \tag{11}$$

where:

$$(A \pm B)_{pq} \equiv E_p\, \delta_{pq} + V^{(\pm)}_p\, F^{(\pm)}_{pq}\, V^{(\pm)}_q + U^{(\mp)}\, G_{pq}\, U^{(\mp)}$$

$$E_p \equiv E_a + E_b$$

$$U^{(\pm)}_p \equiv (u_a u_b \pm v_a v_b)/\sqrt{1 + \delta_{ab}}$$

$$V^{(\pm)}_p \equiv (u_a v_b \pm u_b v_a)/\sqrt{1 + \delta_{ab}}$$

$$G_{pq} \equiv G_J(ab,\ cd)$$

$$F^{(\pm)}_{pq.} \equiv F_J(ab,\ cd) \mp (-)^{j_a + j_b + J} F_J(ba,\ cd)$$

$$p \equiv (a \gtrless b)$$

which F_J arid G_J are defined as

$$\langle\alpha\beta|V|\gamma\delta\rangle = \sum_{LM} G_L(ab,cd)\,(j_a m_a j_b m_b|LM)\,(j_c m_c j_d m_d|LM)$$

$$= \sum_{JM} F_J(ac,db)\,(j_a m_a j_c \bar{m}_c|JM)\,(j_d m_d j_b \bar{m}_b|JM)$$

The RPA is equivalent to neglect terms countaining $c^{+}c$ in comutator of A^{+}_{pJM} and A_{qJM}. This approximation is valid, when the total number of quasi-particles in the ground state is smaller than the total number of quasi-particle states.

Then:

$$[B_{\lambda JM},\ B^{+}_{\lambda' J' M}] = \delta_{\lambda\lambda'}\ \delta_{JJ'}\ \delta_{MM'}$$

$$[B^{+}_{\lambda JM},\ B^{+}_{\lambda' J' M}] = [B_{\lambda JM},\ B_{\lambda' J' M}] = 0 \qquad (12)$$

Therefore we have a system of independent bosons. For high excitation with many bosons we cannot obtain reliable results because in these calculations we have neglected some configuration mixing that must be included for high excitations.

Using the comutation relation(12)

$$H_1 + H_2 \tilde{=} E_0 + \sum \omega_{\lambda J}\ B^{+}_{\lambda JM}\ B_{\lambda JM} \qquad (13)$$

2.2. Odd System of Nucleons

The calculation for the odd system will be made applying the intermediate coupling, i.e., the last particle is taken independently of particle-hole mode.

In the case of odd number of neutrons (odd tin isotopes) we include $\hat{H}$ interaction rewriting it in the form

$$\hat{H}_3 \cong \Sigma\, L^{\lambda}_{abJ}\, C^{+}_{abJM} \left[\bar{B}_{\lambda JM} + (-)^{J-M} B^{+}_{\lambda J\bar{M}} \right] \tag{14}$$

where:

$$C^{+}_{abJM} = \Sigma\, (j_a m_a j_b \bar{m}_b | JM) S_\beta c^{+}_\alpha c^{+}_\beta \tag{15}$$

and

$$L^{\lambda}_{abJ} = (u_a u_b - v_a v_b) F^{\lambda}_{abJ} - \frac{1}{2} (u_a v_b + v_a u_b)\, G^{\lambda}_{abJ} \tag{16}$$

where

$$F^{\lambda}_{abJ} = \frac{1}{2} \sum_{c \geq d} F^{(+)}_J (ab, cd)\, v^{+}_{cd} (X^{\lambda}_{cdJ} + Y^{\lambda}_{cdJ})$$

$$G^{\lambda}_{abJ} = \sum_{c \geq d} G_J (ab, cd)\, \bar{U}_{cd} (X^{\lambda}_{cdJ} + Y^{\lambda}_{cdJ})$$

Then the Hamiltonian can be written approximatelly, without a constant term in the following form:

$$H = \Sigma\, E_\alpha c^{+}_\alpha c_\alpha + \Sigma\, \omega_{\lambda J} B^{+}_{\lambda JM} B_{\lambda JM} + \Sigma\, L^{\lambda}_{abJ} C^{+}_{abJM} \left[\bar{B}_{\lambda JM} + (-)^{J-M} B^{+}_{\lambda J\bar{M}} \right] \tag{17}$$

The quasi-particle couples more strongly to collective modes, i.e., L_{abJ} is strong for collective mode, and we can neglect the other L_{abJ} which are

not collective, The solution of the hamiltonian taking only one collective L_{abJ} can be diagonalized in the space generated by the following states:

$$
\begin{aligned}
|\psi_\alpha> &= c^+_\alpha |\phi_0> \\
|\psi^b_\alpha> &= [c^+_b B^+_J]_\alpha |\phi_0> \\
|\psi^{cI}_\alpha> &= \frac{1}{\sqrt{2}} [c^+_c [B^+_J B^+_J]_I]_\alpha |\phi_0>
\end{aligned}
\tag{18}
$$

where:

$$
\begin{aligned}
& I = 0, 2, 4, \ldots, 2J \\
& [c^+_b B^+_J]_\alpha = \Sigma (j_b m_b JM | j_a m_a) c^+_\beta B^+_{JM} \\
& B_{JM}|\phi_0> = 0 \qquad c_\alpha |\phi_0> = 0
\end{aligned}
$$

Then the energy matrix elements in the two phonon approximation, are:

$$
\begin{aligned}
<\phi_\alpha|H|\phi_\alpha> &= E_a \\
<\phi_\alpha|\hat{H}|\phi^b_\alpha> &= \sqrt{\frac{2J+1}{2j_a+1}}\, L_{abJ} \\
<\phi^b_\alpha|\hat{H}|\phi^{b'}_\alpha> &= (E_b + \omega_J)\ \delta_{bb'} \\
<\phi^b_\alpha|\hat{H}|\phi^{cI}_\alpha> &= \sqrt{2(2J+1)(2I+1)}\ W(j_a j_c JJ;\ I j_b) L_{bcJ} \\
<\phi^{cI}_\alpha|\hat{H}|\phi^{c'I'}_\alpha> &= (E_c + 2\omega_J)\ \delta_{cc'}\ \delta_{II'}
\end{aligned}
\tag{19}
$$

All the other matrix elements are zero.

Finally the eigenvalue equation $H|\phi_\alpha> = w_a|\phi^{cI}_a>$ can be solved assuming the following expansion

$$|\phi_\alpha> = A \quad + \sum_b A \quad |\phi_\alpha^b> + \sum_{cI} A_{ac}^I |\phi_\alpha^{cI}> \qquad (20)$$

and the matrix elements given in (19).

3. A MODEL CALCULATION

In this section we present a simple interaction model, for the active neutrons outside a double magic nuclei.

In the Kisslinger and Sorensen's interaction model the reduced matrix elements for particle-hole and particle-particle interaction are:

$$\begin{aligned} F_J(ab,cd) &= -\chi\, Q_{ab}\, Q_{cd}\, \delta_{J2} \\ G_J(ab,cd) &= -G\, \sqrt{(2j_a+1)(2j_b+1)}\, \delta_{ab}\, \delta_{cd}\, \delta_{J0} \end{aligned} \qquad (21)$$

where,

$$Qab = (a||\, r^2 Y_2\, ||\, b)/\sqrt{5} \qquad (22)$$

In the present work we take:

$$\begin{aligned} F_J(ab,cd) &= -\chi\, Q_{ab}\, Q_{cd}\, \delta_{J2} + \bar{F}_0^{\delta}(ab,cd)\, \delta_{J0} \\ G_J(ab,cd) &= G_0^{\delta}(ab,cd)\, \delta_{J0} \end{aligned} \qquad (23)$$

Where F_0^{δ} and C_0^{6} are reduced matrix elements calculated by the Dirac's delta interaction.

The present model atempts to simulate the pairing and the correction[13] on the single particle energy through the Dirac's delta interaction.

Including the model matrix elements (23) in the RPA equations (11) we have:

$$\frac{1}{2\chi} = S(\omega)$$
$$S(\omega) = \sum_q \frac{E_q (V_q^+ Q_q)^2}{E_q^2 - \omega^2} \tag{24}$$

and the reduced transition probability

$$B_{2\to 0}(E2) = \left| \sum_p V_p^+ e_{\text{eff}} Q_p (X_{p2} + Y_{p2}) \right|^2$$

In this model

$$B_{2\to 0}(E2) = \frac{1}{2} \left[\frac{e_{\text{eff}}}{\chi} \right]^2 \frac{1}{(\frac{\partial S}{\partial \omega})} \tag{25}$$

For the particle-vibration coupling matrix elements (16) we obtain

$$L_{ab} = (U_a U_b - V_a V_b) \frac{Q_{ab}}{\sqrt{2(\frac{\partial S}{\partial \omega})}} \tag{26}$$

As expected, when 2^+ is collective, from (25) $\partial S/\partial\omega$ is small, and the particle-vibretion coupling is strong as can be seen in the formula (26). The derivative is small just for RPA collective solution (24).

4. PARAME'TERS FOR EVEN TIN ISOTOPES

The BCS and RPA equations were solved making a restriction of single particle states for one major shell, that is, considering only states corresponding to the single particle levels:

$$2d_{5/2},\ 1g_{7/2},\ 3s_{1/2},\ 1h_{11/2} \text{ and } 2d_{3/2}$$

The coupling constant of the quadrupole-quadrupole interaction was fixed by the value of the Kisslinger and Sorensen so that in our notation it is:

$$\chi = \left[\frac{0.827}{A}\ \text{MeV}\right] \frac{4\pi}{b^4} \tag{27}$$

where

$$b^2 = \hbar/m\omega_{osc}$$

$$\hbar\omega_{osc} = 41.\ A^{-1/3}\ \text{MeV}$$

The single particle energy ε_a taken from the level $5/2^+$ and the coupling constant V_0 of Dirac's delta, were determined by adjusting the excitation energy of the first theoretical collective state 2^+ to the experimental one for the even isotopes from Sn^{108} to Sn^{126}, through the χ^2 search. The values so determined are (see fig.1)

$$\frac{V_0}{4\pi} = 22.13\ \text{MeV fm}^3$$

$$\begin{aligned} \varepsilon_{11/2^-} &= 2.48\ \text{MeV} & \varepsilon_{7/2^+} &= 0.48\ \text{MeV} \\ \varepsilon_{3/2^+} &= 2.33\ \text{MeV} & \varepsilon_{5/2^+} &= 0.00\ \text{MeV} \\ \varepsilon_{1/2^+} &= 1.48\ \text{MeV} \end{aligned} \tag{28}$$

The adjusted value of V_0 results in Dirac's delta matrix elements that are approximately twice the matrix elements corresponding to realistic pótential as presented in the figure 2. This fact can be interpreted as a renormalization of interaction of active neutrons.

The above comparison of reduced matrix elements of delta interaction with realistic ones presents a similar general trend. On the other hand Kuo and Brown[17] found that the calculated effective force includes in addition to the pairing matrix elenients also strong quadrupole force. It seems to us that the effective force can be simulated by a delta plus lower order of multipole interaction. Following the above procedure the calculated BCS parameters are presented in the tables (1-3).

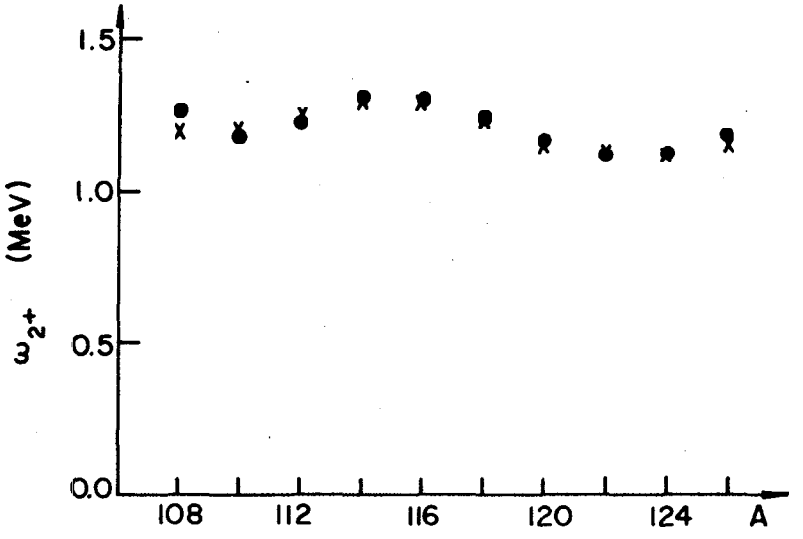

Fig.1 - The lowest 2^+ excitation energy of even tin isotopes in function of mass number A. The calculated value is indicated by •, and the experimental value by x (ref.9-16).

The maxima in the lowest 2^+ excitation energy in the isotopes Sn^{114} and Sn^{116} (Fig.1) correspond to the filling of first sub-shell ($5/2^+$ and $7/2^+$) and of second sub-shell ($1/2^+$) respectively[19] (see Fig.3) which increases the occupation probability fast with the neutron number around Sn^{116} (Fig. 4). This can be understood by the reasonable contribution from

$$\frac{F_0(aa,\ bb)}{\sqrt{(2j_a+1)(2j_b+1)}}$$

for $a=b=1/2$, that makes the correction to the $3s$ 1/2 single particleenergy very appreciable, as can be seen in the figure 2.

The reductzd transition probability $B_{0 \to 2^+}(E2)$ was calculated, because only the fit of 2^+ level is not a condition that model works well.

The figure 5 presents the plot of $B(E2;A)/B(E2;A=112)$ in terms of mass number A. This ratio is taken in order to discard the effective charge

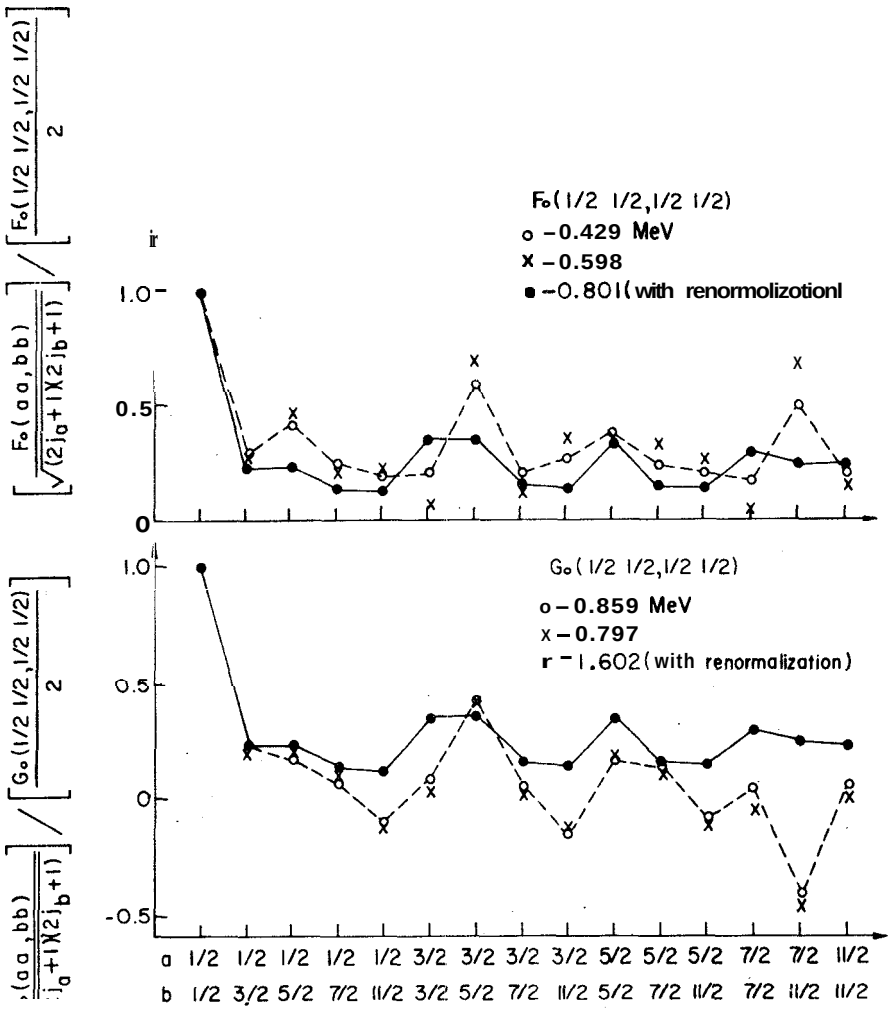

Fig.2 - The interaction reduced matrix elements in the tin isotopes region

Dirac's delta (renormalized)

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	109	111	113	115	117	119	121	123	125	127
$\frac{1}{2}^+$	1.013	1.045	1.067	1.084	1.074	1.042	0.996	0.933	0.850	0.738
$\frac{3}{2}^+$	1.052	1.041	1.012	0.995	1.004	1.013	0.998	0.995	0.881	0.774
$\frac{5}{2}^+$	1.052	1.041	1.012	0.995	1.004	1.013	0.998	0.955	0.881	0.774
$\frac{7}{2}^+$	1.047	1.060	1.036	1.014	1.028	1.054	1.062	1.040	0.983	0.882
$\frac{11}{2}^-$	0.952	0.964	0.947	0.932	0.950	0.979	0.990	0.972	0.920	0.826

Table 1 - A - BCS correlation parameter calculated for the odd tin isotopes without inclusion of blocking effect (Eq. 4a).

	109	111	113	115	117	119	121	123	125	127
$\frac{1}{2}^+$	0.674	0.500	0.302	0.054	-0.200	-0.402	-0.565	-0.706	-0.833	-0.952
$\frac{3}{2}^+$	1.318	1.140	0.970	0.800	0.628	0.460	0.300	0.150	0.009	-0.125
$\frac{5}{2}^+$	-1.012	-1.189	-1.360	-1.530	-1.702	-1.870	-2.030	-2.180	-2.301	-2.455
$\frac{7}{2}^+$	-0.357	-0.554	-0.747	-0.925	-1.089	-1.252	-1.418	-1.585	-1.752	-1.921
$\frac{11}{2}^-$	1.725	1.550	1.378	1.217	1.065	0.911	0.755	0.596	0.437	0.277

Table 2 - $(\varepsilon_a - \nu_a)$ Corrected single particle energy for the odd tin isotopes (Eq.4c).

	109	111	113	115	117	119	121	123	125	127
λ_a	-0.465	-0.354	-0.208	-0.027	0.149	0.286	0.388	0.466	0.529	0.582

Table 3 - λ_a - Fermi energy for the odd tin isotopes without the blocking effect.

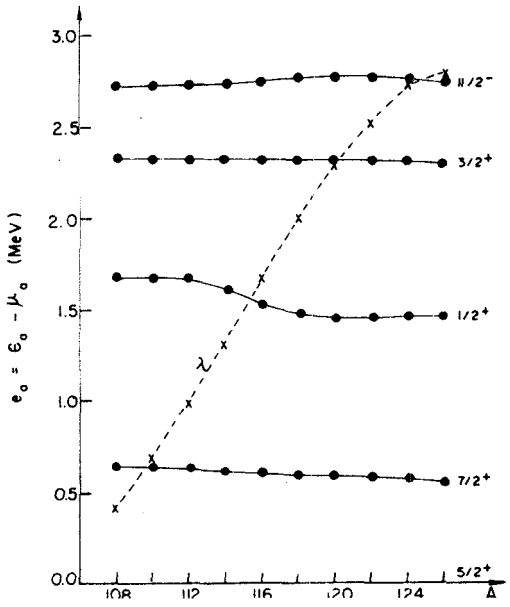
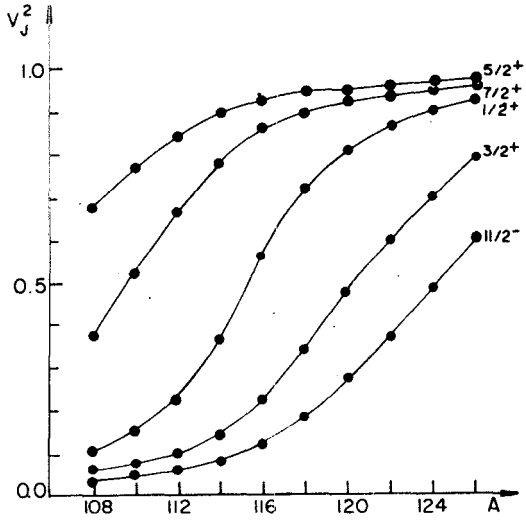


Fig.3 - The single particle energy with self consistent correction μ and chemicai potential λ in function of mass number A.

Fig.4 - The occupation probabilities v_j^2 in function of mass number.



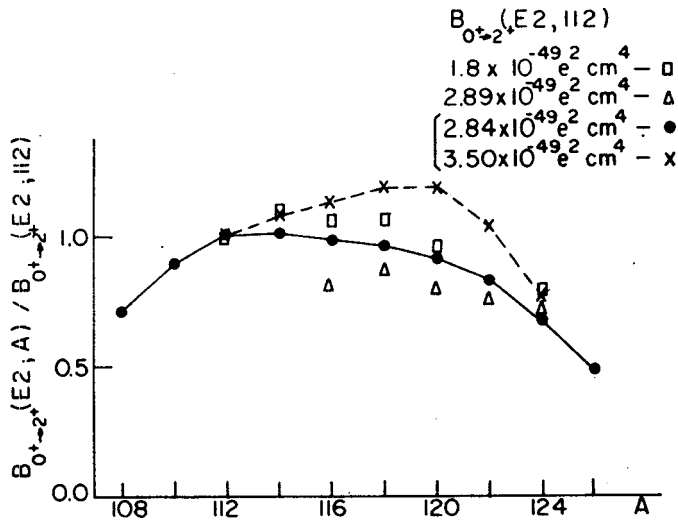


Fig.5 - The reduced transition probabilities in units of $B^{+}_{0\to 2^+}(E2, A-112)$ in function of mass number A. The experimental values[22,23] are indicated by $\Delta^{(22)}$ and $\square^{(23)}$; the theoretical values: the present model by o and x for Kisslinger and Sorensen[1].

factor. One can see that the general trend of the present work seems better than of Kisslinger and Sorensen.

5. ODD TIN ISOTOPES ENERGY LEVELS AND SPECTROSCOPIC FACTOR SYSTEMATICS

Conserving the same pararneters of even-even tin isotopes, that is, the neutron single particle energy and the interaction coupling constant V_0 and χ, the energy levels and spectroscopic factors for odd tin isotopes were determined in quasi-particle plus one and two phonon coupling scheme.

The spectroscopic factor gives the informations about the single particle structure of the nuclear wave function. The following expressions were taken for the theoretical spectroscopic factor[20]

a) For stripping reactions on the even target nucleus.

$$S_{j_a} = u^2_a, A^2_a$$

b) For pick-up reactions on the even target nucleus.

$$S_{j_a} = V_a^2, A_a^2(2j_a + 1)$$

Where V_a, and U_a, are occupation and non occupation probabilities of the target nucleis and A_a is the single-quasi particle amplitude (see 20).

The above expressions refer to the cases in that one describes the 'odd system in terms of one quasi-particle coupled to collective modes of the corresponding even system.

Nevertheless, there are considerable incertainty inthe experimental spectroscopic factor (about 30%). These incertainties are due to the hypothesis involved in the reaction mechanism.

Therefore we make a comparison, mainly about the systematic evolution of the spectroscopic factors in function of the atomic mass.

The calculation were restricted to five single particle levels corresponding to the major shell 50-82 and the hamiltonian were diagonalized in a space that contains $1QP$, IQP + 1 phonon 2^+; $1QP$ + 2 phonon 2^+.

The experimental energy levels of odd tin isotopes and the theoretical ones are presented in the figure 6 and 7. The general agreement of the present calculation with experience is very good if we take in account that all the parameters are fixed for the even tin isotopes and no parameters were adjusted for the odd isotopes.

In the figures (8a-9e) are presented the spectroscopic factors for odd tin isotopes from (d,p) and (p,d) reactions on the even tin isotopes.

There are compared:

1. The experimental spectroscopic factors, mainly based in E.U. Baranger's review[9] and on the recent experimental data.[10-16]

2. Two theoretical calculation: the first is the Kisslinger and Sorensen

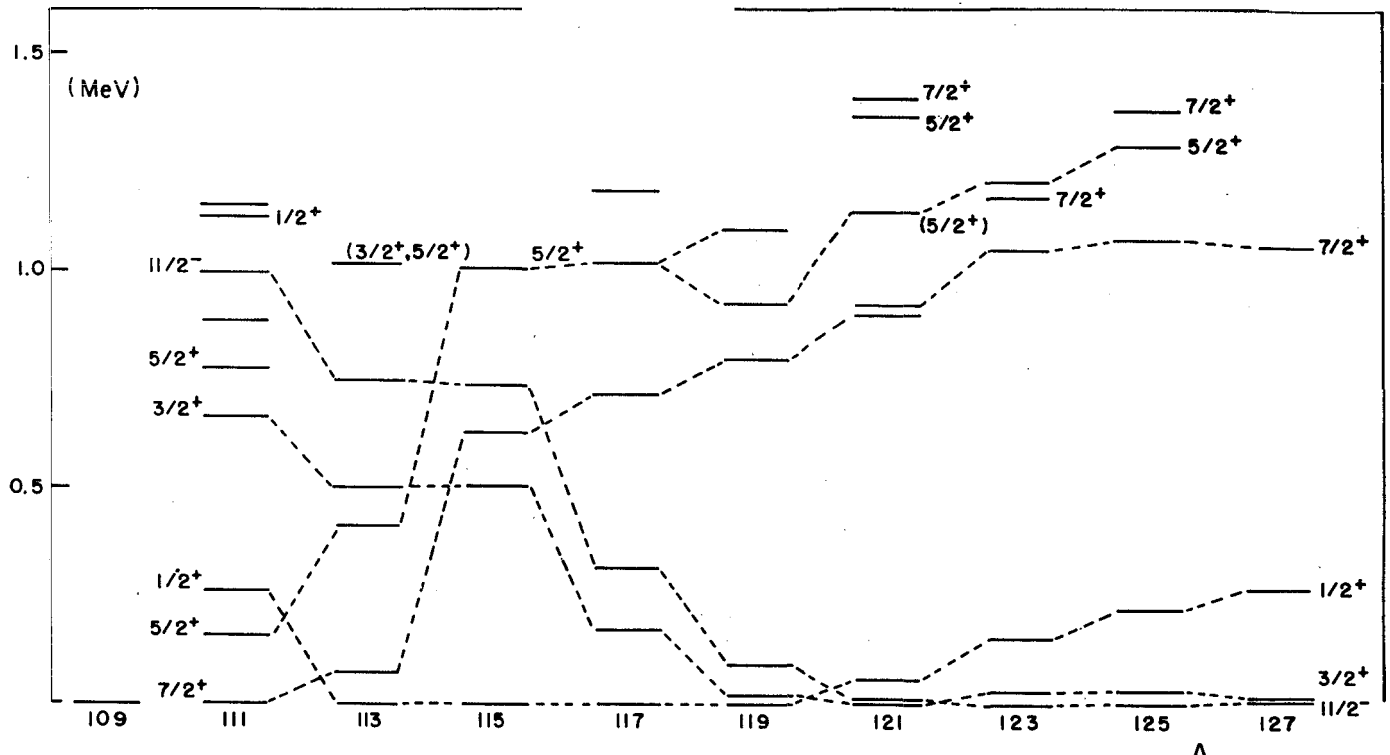


Fig.6 - Experimental odd tin isotopes energy levels (ref. 9-16).

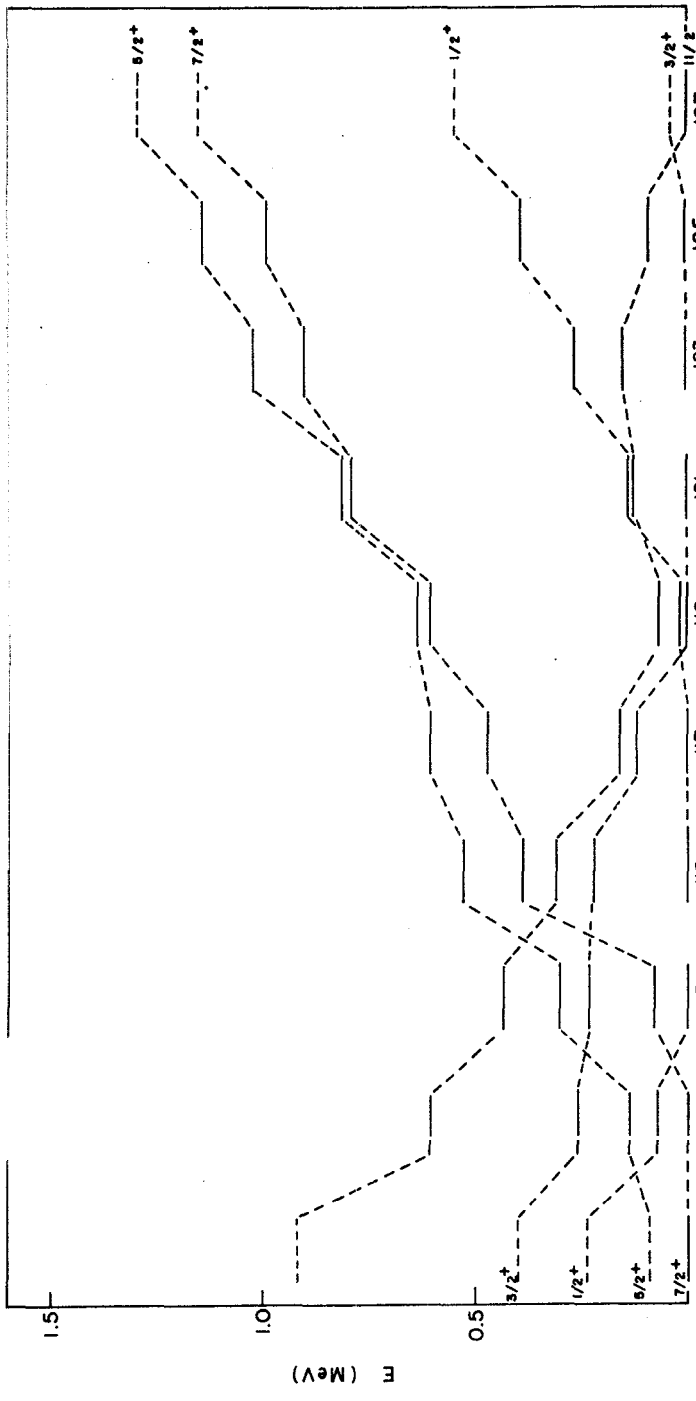
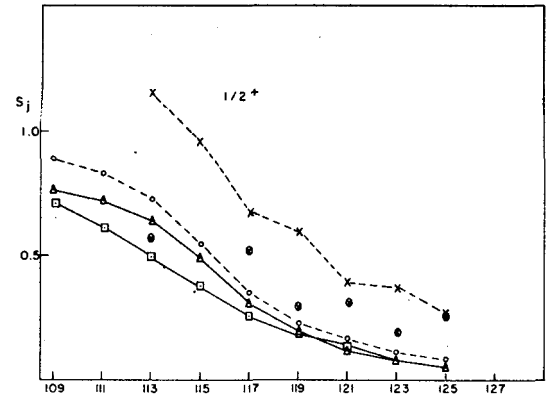
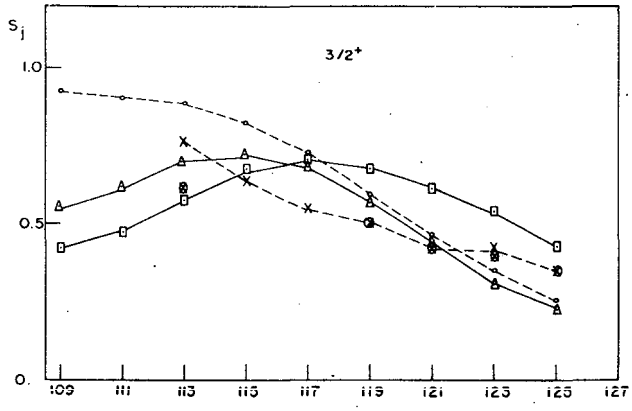
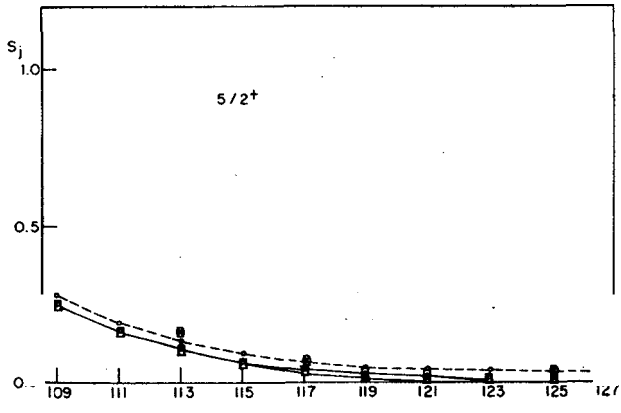
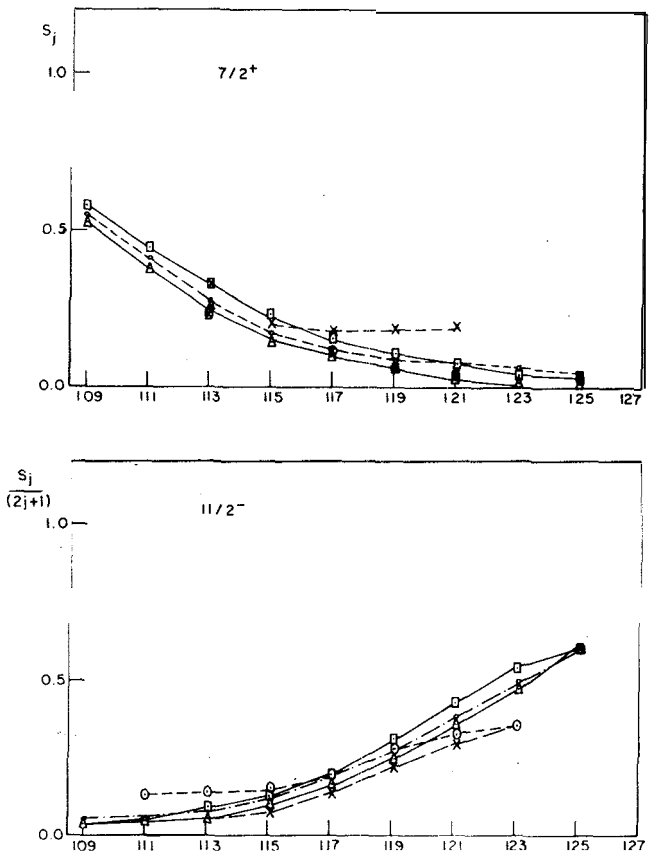


Fig.7 - Calculated energy levels of tin isotopes.


S_j
1.0
0.5
1/2+
109
111
113
115
117
119
121
123
125
127


S_j
1.0
0.5
0.
3/2+
109
111
113
115
117
119
121
123
125
127


S_j
1.0
0.5
0.
5/2+
109
111
113
115
117
119
121
123
125

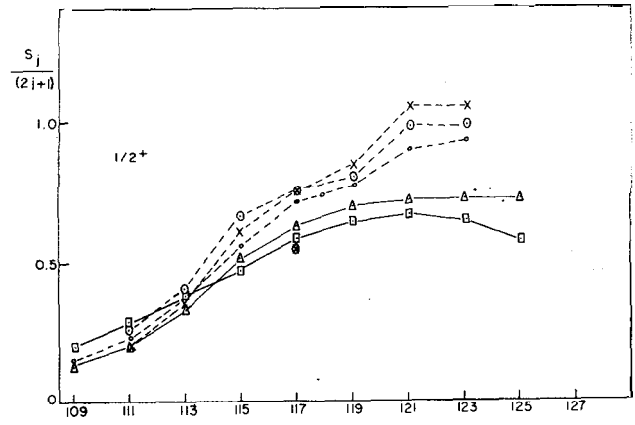


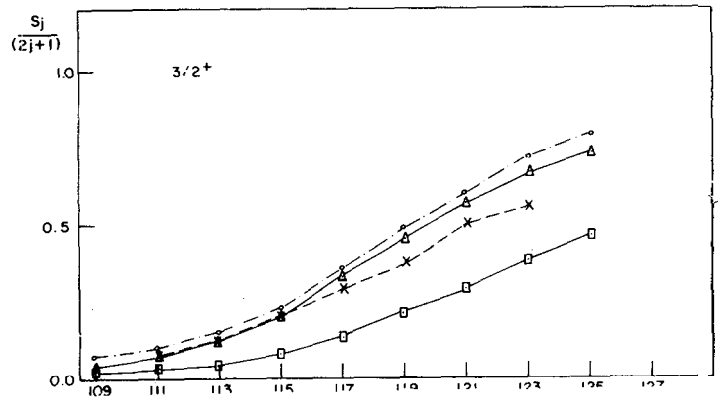
Figs.8a-8d - The theoretical and exper mental S_{dp} spectroscopic factors for the low-lying excitation energy of $1/2^+$, $3/2^+$, $5/2^+$, $7/2^+$ and $11/2^-$ states in each odd tin isotopes (ref. 9-16). x Experimental systematics (ref. 9); ⊗ More recent data (ref. 10-16); o u?: (Non occupation probabilities; □ Pairing plus guadrupole-quadrupole interaction; A Present work.

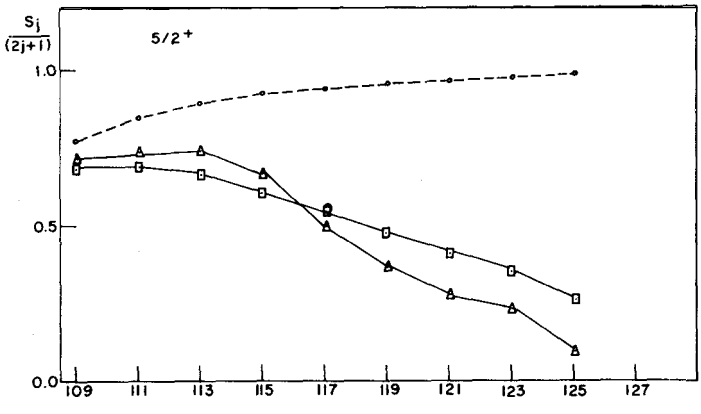
model with pairing plus Q–Q interaction and the second is the present calculation.

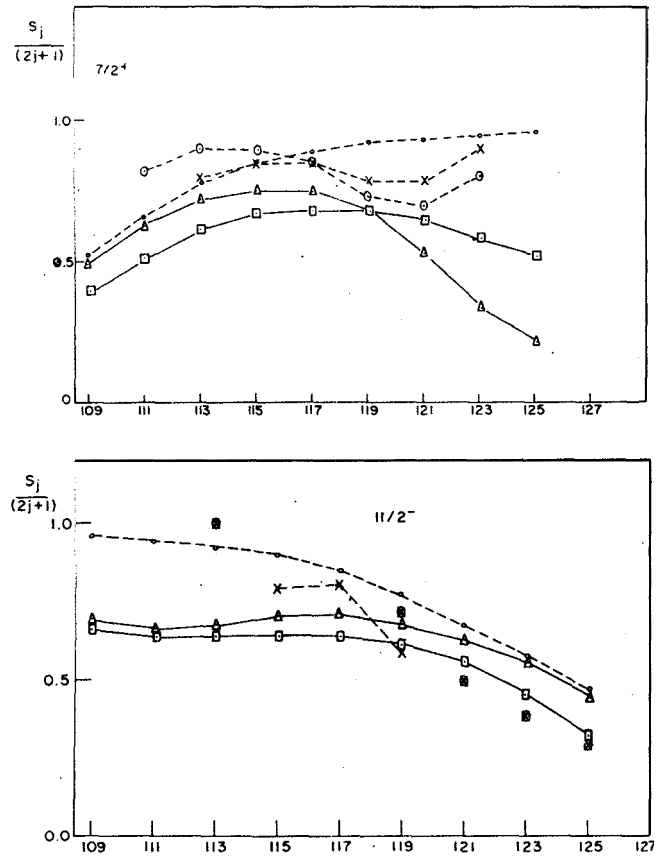
3. The non occupation and occupation probabilities are from the present interaction model.

In the Kisslinger and Sorensen's model here utilized we calculated the coupling constants by fitting to experimental first 2^+ level. The single particle energy and the pairing force intensity G are from reference 1.


$\frac{S_j}{(2j+1)}$
1.0
0.5
0
1/2+
109
111
113
115
117
119
121
123
125
127


$\frac{S_j}{(2j+1)}$
1.0
0.5
0.0
3/2+
109
111
113
115
117
119
121
123
125
127


$\frac{S_j}{(2j+1)}$
1.0
0.5
0.0
5/2+
109
111
113
115
117
119
121
123
125
127



Figs.9a-9d - The calculated and the experimental S_{pd} spectroscopic factors for low lying excitation energy of $1/2^+$, $3/2^+$, $5/2^+$, $7/2^+$ and $11/2^-$ states in each odd tin isotopes (ref. 9-16); x Experimental systematics (ref.9); ● More recent data (ref. 10-16); o v^2 (occupation probabilities); □ Pairing plus quadrupole-quadrupole interaction; Δ Present work.

The analysis of the spectroscopic factors and the amplitudes of quasi-particle states (figures 8-9):

1. The one quasi-particle approximation (BCS) is still suficient to give reasonable description for the low lying $1/2^+$ energy level. The inclusion in the calculation of only one 2^+ ptionon results in some small contribution of $3/2^+ \otimes 2^+$ in the wave functions. It is not necessary the inclusion of two phonons.

2. The low lying $3/2^+$ level has some contributions of $1/2^+ \otimes 2^+$, $3/2^+ \otimes 2^+$ and $7/2^+ \otimes 2^+$ for low tin mass number, and smaller contribution for higher mass number.

3. The low lying $5/2^+$ has contributions mainly of $5/2^+ \otimes 2^+$ for lower mass number and big contribution of $5/2^+ \otimes 2^+$ and small contributions of $3/2^+ \otimes (2^+ \otimes 2^+)_{2^+}$ and $7/2^+ \otimes (2^+ \otimes 2^+)_{4^+}$ for higher mass.

4. The low lying $7/2^+$ has some contribution of $7/2^+ \otimes 2^+$ for lower tin mass number and big contribution of $3/2^+ \otimes 2^+$ and small contributions of $7/2^+ \otimes 2^+$ and $3/2^+ \otimes (2^+ \otimes 2^+)_{4^+}$ for higher mass number.

5. Finally the $11/2^-$ low lying energy level has appreciable contributions of $11/2^- \otimes 2^+$ for low tin mass number.

6. CONCLUDING REMARKS

From the present work we conclude that it is possible to give a reasonable description of nuclei in the $Z=50$ region using a very simple schematic force and a set of single particle energies that were obtained by making theoretical levels 2^+ to fit fhe experimental ones. It is better to emphasize that the utilized single particle energies are the same for both even and odd tin isotopes and that they are kept for all the isotopes.

The present calculation improves the results of Kisslinger and Sorensen for the energy levels and for spectroscopic factors.

The delta interaction matrix elements did not differ qualitatively from the realistic interaction. The calculation of F_0 allowed us to make a discussion about the effect due to a filled sub-shell.

The experimental energy levels systematics of the odd isotopes (v. figure 6) is reasonably well established for the low lying levels (bellow 1 MeV of excitation energy) excepting the $A=115$ isotope. We think that a higher resolution experiment must be made on this nucleus.

The sequence of the low lying $5/2^+$ level is not yet well established. Our calculation does not predict the abrupt variation presented by it from A=113 to A=115 isotopes.

The experimental ground state from a A=123 to A=127 tin isotopes is $11/2^-$ but the present calculation does not reproduce it in a satisfactory way. This could be attributed to the fact that we have not included the quasi-particle coupling to the pairing vibrations (0^+).

An estimation of the pairing effect based in the work of Cusson and Hara[21] shows that it is not able to produce sufficient effects in so that the $11/2^-$ level gets as down as the ground state in A=113 to A=127 isotopes. Nevertheless a more detailed calculation, to be published, had to be realized.

The systematic calculation of spectroscopic factors showed that the low-lying $1/2^+$, $3/2^+$, $11/2^-$ energy levels have contributions of $1QP + 1$ phonon 2^+ and the low-lying $5/2^+$, $7/2^+$ energy levels have contributions of still 2 phonons 2^+. This study shows us that the analysis of spectroscopic factors through the occupation probabilities is not always sufficient, for this may lead to mistakes in the atributions of energy levels as, for example, in the case of $5/2^+$ and $7/2^+$ for A - 123-127.

In the present work only the odd tin isotopes were treated. The antimonium and indium isotopes calculations will appear soon.

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REFERENCES

1. L.S. Kisslinger and R.A. Sorensen, Rev.Mod.Phys.35, 853 (1963).
2. S. Yoshida, Nucl. Phys. 38, 380 (1962).
3. T.T.S. Kuo, E.U. Baranger and M. Baranger, Nucl.Phys. 79, 513 (1966).

4. D.M. Clernent and E.U. Baranger, Nucl. Phys. *A120*, 25 (1969).
5. R. Alzetta and J. Sawicki, Phys. Rev. 173, 1185 (1968).
6. H.C. Lee and K. Hara, Phys. Rev. C3, 325 (1971).
7. N.D. Takacsy and S.Das Gupta, Phys. Rev. *C13*, 399 (1976).
8. W.F. Van Gunsteren and K. Allaart, unpublished.
9. E.U. Baranger, *Advances in Nucl. Phys.* vol.4 edited by M. Baranger and E. Vogt.
10. T. Borello, E.Frota-Pessoa, C.Q.Orsini, O.Dietzsch and E. W. Hamburger, Rev. Bras. Fis.2, 157 (1972).
11. T. Borello-Lewin, C.Q. Orsini, O. Dietzsch and E.W. Hamburger, Nucl. Phys. *A249*, 284 (1975).
12. M.J. Bechara and O. Dietzsch, Phys. Rev.*C12*, 90 (1975).
13. C.R. Binghamand D.L. Hillis, Phys. Rev. *C8*, 729 (1973).
14. P.L. Carson; L.C. McIntyre Jr., Nucl. Phys. *A198*, 289 (1972).
15. R.F. Casten, E.R. Flynn, O. Hausen, T.J. Mulligan, Nucl. Phys. *A180*, 49 (1972).
16. -8. Fogelberg, L.E. De Geer, K. Franson, M.Af Ugglas Z. Physik *A276*, 381 (1976).
17. T.T.S. Kuo and G.E. Brown, Nucl. Phys. 85, 40 (1966).
18. M. Baranger, Phys. Rev. 120, 226 (1960).
19. R. Arvieu, E. Baranger, M. Veneroni, M. Baranger et V. Gillet. Phys. Lett. 4, 119 (1963).
20. S. Yoshida, Phys. Rev. 123, 2122 (1961).
21. R.Y. Cusson and K. Hara; Zeitschrift für Physik, 209, 428 (1968).
22. P.H. Stelson et *al.*, Phys. Rev. *170*, 1172 (1968).
23. D.G. Alkhazov et *al.*, JETP 6, 1036 (1958).