Revista Brasileira de Física, Vol. 8, Nº 3, 1978

The Muon Magnetic Moment in a Model of Lepton Structure

J. LEITE LOPES and J. MARTINS SIMÕES*
Centre de Recherches Nucléaires, Université Louis Pasteur, Strasbourg, France

Recebido em 9 de Março de 1978

In this note we calculate the magnetic moment form factor for the muon, according to a recently proposed model of lepton structure.

Nesta note, calculamos o fator de forma do momento magnético do muon, de acordo com um modelo recentemente proposto para a estrutura do lepton.

1. INTRODUCTION

In this note, we calculate the muon magnetic moment according to the model of lepton structure which has been recently proposed[1,2,3].

The basic feature of this model is to consider leptons and hadrons on the same levei, since they are both observables. As hadrons seem to be composite of quarks, a quark structure for the leptons (e,μ) with the inclusion of neutral leptons, was suggested[1].

In order to account for the non-observation of such interactions at the known enei-gies, it was then proposed[2] that the neutral leptons are heavy and thet the charged leptons may interact with mesons according to a Lagrangian which has a mixture of scalar and pseudoscalar couplings, of the type:

* with a fellowship of CNPq

$$L_1 = g_1 \{\bar{e}(1+\gamma^5)L_{1\theta}\pi^- + \bar{L}_{1\theta}(1-\gamma^5)e\,\pi^+ + \mu^-(1+\gamma^5)L_{2\theta}K^- + $$
$$+ \bar{L}_{2\theta}(1-\gamma^5)\mu K^+\} \tag{1}$$

where, θ designating a mixing angle

$$L_{1\theta} \equiv L_1 \cos\theta + L_2 \sin\theta$$
$$L_{2\theta} \equiv L_2 \cos\theta - L_1 \sin\theta \tag{2}$$

and

$$m_{L1} \simeq m_{L2}$$

Another possible interaction, of a vector-axial vector character, is (we will discuss it in another paper)

$$L_2 = g_2 \{\bar{e}\,\gamma^\mu(1+\gamma^5)L_{1\theta}\,\rho^-_\mu + \bar{\mu}\,\gamma^\mu(1+\gamma^5)L_{2\theta}\,K^{*-}_\mu + h.c.\} \tag{1'}$$

where g_1 and g_2 are the respective coupling constants.

As the experimental value of the muon magnetic moment is known with increasing accuracy[4], and agrees very well with the theoretical calculations[5], we shall use this result to estimate a lower bound to the mass of the heavy leptons.

We shall consider only the general form for L_1 and add the $\sin\theta$ and $\cos\theta$ factors in the end.

2. THE CONTRIBUTION TO THE MAGNETIC MOMENT FROM A SCALAR-PSEUDOSCALAR INTERACTION

In this section we consider the contribution of the first interaction (1). The notation is taken over from Bjorken-Drell[7].

The graph to calculate is indicated in fig. 1

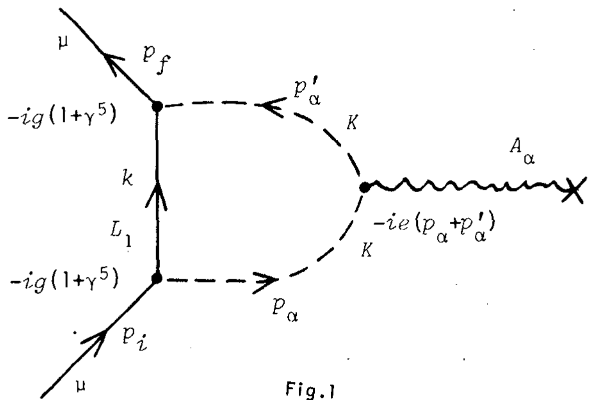

Fig.1

Using the well-known properties of Dirac's matrices

$$(1+\gamma^5)(1-\gamma^5) = 0$$

$$\{\gamma^\mu , \gamma^5\}_+ = 0$$

the S-matrix of this graph is:

$$S = N_f \, e \, g^2 \, 2 \int \frac{d^4k}{(2\pi)^4} \bar{u}(p_f) \left\{ \frac{\not{k}(1-\gamma^5)}{k^2 - M_L^2} \frac{1}{(p_i-k)^2-m^2_K} \frac{1}{(p_f-k)^2-m^2_K} \right\} u(p_i) \cdot$$

$$\cdot \, (p_i^\alpha + p_f^\alpha - 2k^\alpha)\, A_\alpha \qquad (3)$$

where N_f is a normalization factor.

Let us leave aside the γ^5 term which does not contribute to the magnetic moment and define

$$P_\alpha \equiv \frac{p_{i\alpha} + p_{f\alpha}}{2}$$

we then get for the remaining term in the expression (3)

$$S' = N_f \frac{4eg^2}{(2\pi)^4} \; A^{\alpha} \; \bar{u}(p_f) \; I_{\alpha} \; u(p_i) \tag{4}$$

where I_a is the integral

$$I_{\alpha} = \int d^4k \cdot \frac{\not{k} \; (P_{\alpha} - k_{\alpha})}{(k^2 - M_L^2)\,((p_i - k)^2 - m^2{}_K)\,((p_f - k)^2 - m^2{}_K)} \tag{5}$$

Now, the following integral

$$I = \int d^4k \; \frac{(1 \; ; \; k_{\mu} \; ; \; k_{\mu} \; k_{\nu})}{(k^2 - 2p_i k - \Delta_1)\,(k^2 - 2p_f k - \Delta_1)\,(k^2 - \Delta_2)} \tag{6}$$

has been evaluated[6] and gives

$$I = \frac{i\pi^2}{2} \; (A(\Delta) \; ; \; 2P_{\mu} B(\Delta) \; ; \; g_{\mu\nu} C(\Delta) + (p_{i\mu} p_{f\nu} + p_{i\nu} p_{f\mu}) D(\Delta)$$

$$+ (p_{i\mu} p_{i\nu} + p_{f\mu} p_{f\nu}) E(\Delta)) \tag{7}$$

where

$$A(\Delta) = 2 \int_0^1 dx \int_0^1 dy \; \frac{x}{\Delta}$$

$$B(\Delta) = 2 \int_0^1 dx \int_0^1 dy \; \frac{x^2 y}{\Delta}$$

$$C(\Delta) = \frac{1}{i\pi^2} \int_0^1 dx \int_0^1 dy \int d^4k \; \frac{k^2 x}{(k^2 + \Delta)^3}$$

$$D(\Delta) = 2 \int_0^1 dx \int_0^1 dy \; \frac{x^3 y (1 - y)}{\Delta}$$

$$E(\Delta) = 2 \int_0^1 dx \int_0^1 dy \; \frac{x^3 y^2}{\Delta}$$

$$\Delta = q^2 \, x^2 \, y(1 - y) - m^2{}_{\mu} \, x^2 + (\Delta_2 - \Delta_1) x - \Delta_2$$

$$q \equiv p_f - p_i$$

Applying the result (7) to (5)

$$I_\alpha = \frac{i\pi^2}{2}\,(0\ ;\ \gamma_\nu P_\alpha\ (2P^\nu\ B(\Delta))\ ;\ -\ \gamma^\nu [g_{\alpha\nu}C(\Delta)\ +\ (p_{i\alpha}p_{f\nu} + p_{i\nu}p_{f\alpha})D(\Delta)\ +$$

$$+\ (p_{i\alpha}p_{i\nu} + p_{f\alpha}p_{f\nu})E(\Delta)])$$

The $C(\Delta)$ **term** does not contribute to the magnetic moment. It is a divergente to be absorbed in the charge renormalization.

Noting that this integral is between $\bar{u}(p_f)$ and $u(p_i)$ and using Dirac's equation, we can replace each of the terms

$$p_{(i,f)\nu}\ \gamma^\nu\ ;\ P_\nu\ \gamma^\mu\ \text{by}\ m_\mu$$

Thus

$$I_\alpha = i\,\frac{\pi^2}{2}\,(0\ ;\ 2\ m_\mu P_\alpha B(\Delta)\ ;\ -\ 2m_\mu P_\alpha\ D(\Delta)\ -\ 2m_\mu P_\alpha\ E(\Delta))$$

$$= i\ \pi^2\ m_\mu\ 2P_\alpha \int_0^1 dx \int_0^1 dy\ \frac{y(x^2 - x^3)}{\Delta}$$

As the magnetic moment is calculated in the limit $q^2 \to 0$ and using the approximaticm $M_L^2 + m^2_\mu - m_K^2 = M_L^2$ we get

$$\Delta \simeq -\ m^2_\mu\ x^2 + M_L^2\ x - M_L^2 \tag{8}$$

and the terms of **the** S-matrix that we have kept are given from **eq.**(4) by

$$S' = N_f\,\frac{eg^2}{4\pi^2}\ i\ m_\mu\ A_\nu\ P^\nu\ I_F\ \bar{u}(p_f)\ u(p_i) \tag{9}$$

with

$$I_F = \int_0^1 dx \, \frac{x^2 - x^3}{\Delta} \tag{10}$$

Using the Gordon decomposition

$$i\, P_\nu \, \bar{u}(p_f)\, u(p_i) = i\, m_\mu \, \bar{u}(p_f)\, \gamma_\nu \, u(p_i) + \frac{1}{2}\, \bar{u}(p_f)\, \sigma_{\nu\alpha}\, u(p_i)\, q^\alpha$$

and noting that the first term does not contribute to the magnetic moment, we keep only the term

$$S'' = N_f \left| \frac{g^2}{4\pi^2} \, m^2_\mu \, I_F \right| \mu_0 \, \bar{u}(p_f)\, \sigma^{\nu\alpha}\, u(p_i)\, q_\nu \, A_\alpha \tag{11}$$

where

$$\mu_0 \equiv \frac{e}{2m_\mu} \tag{12}$$

3. THE MUON MAGNETIC MOMENT DUE TO A VECKOR-AXIAL VECTOR INTERACTION

In this section we consider the contribution of the second possible interaction (1'). The notation is taken from the paper by Fujikawa et *al.*[8].

The graph to calculate is indicated in Fig.2

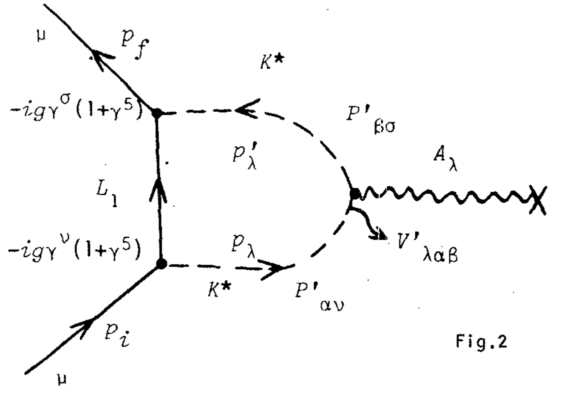

Fig.2

As the spin-one particles present divergences in a more complicated way than thoss with spin zero or 1/2 one must be very careful to separate the finite part from the infinite ones in our integrals.

The usual method for this separation is the ξ-process of Lee and Yang, used in the R_ξ gauge (see Ref.8).

Another regularization scheme however has been proposed by Bollini and Giambiagi[9]. As in this method, called dimensional regularization, the calculations are more straigthforward, we shall employ it.

We use a n-dimensional generalization of the γ^5 matrix which satisfies $\{\gamma^\mu, \gamma^5\}_+ = 0$ since we note that in the case of the Weinberg-Salammodel (whose charged part Lagrangian has the same structure as our (1')), the calculation of the niuon magnetic moment in the dimensional regularization method[10] agrees with other calculations[11].

With the properties of Dirac's matrices extended to the n-dimensional space[9] the S-matrix has the form

$$S = -N_f\, eg^2\; 2\int \frac{d^n k}{(2\pi)^n}\, \bar{u}(p_f)\{ \frac{\gamma^\sigma \not{k}}{k^2-M_L^2}\, \gamma^\nu (1+\gamma^5)\}\, u(p_i) P_{\beta\sigma} V^{\lambda\alpha\beta}\, P_{\alpha\nu}\, A_\lambda (p_f-p_i) \tag{13}$$

where[8]

$$P_{\beta\sigma} = \frac{-g_{\beta\sigma} + \dfrac{(p_f-k)_\beta (p_f-k)_\sigma}{m^2_{K*}}}{(p_f-k)^2 - m^2_{K*}}$$

$$P_{\alpha\nu} = \frac{-g_{\alpha\nu} + \dfrac{(p_i-k)_\alpha (p_i-k)_\nu}{m^2_{K*}}}{(p_i-k)^2 - m^2_{K*}} \tag{14}$$

$$V_{\lambda\alpha\beta} = g_{\alpha\beta}(p_i+p_f-2k)_\lambda - g_{\alpha\lambda}(2p_i-p_f-k)_\beta - g_{\beta\lambda}(2p_f-p_i-k)_\alpha$$

The γ^5 term does not contribute to the magnetic **moment** so we **leave it** aside. The rcmaining term of (13) is

$$S_1 = -N_f \frac{2eg^2}{(2\pi)^n} A^\lambda \bar{u}(p_f)\ I_\lambda\ u(p_i) \tag{15}$$

where

$$I_\lambda = \int d^n k\ \{ \gamma^\sigma \not{k} \gamma^\nu \} \frac{1}{k^2-M_L^2} P_{\beta\sigma} P_{\alpha\nu} V_\lambda{}^{\alpha\beta} \tag{16}$$

If we use the following results

$$\bar{u}(p_f)\not{p}_f = \bar{u}(p_f) m_\mu \qquad p_i{}^2 = p_f{}^2 = m^2{}_\mu$$

$$\not{p}_i\ u(p_i) = m_\mu\ u(p_i) \qquad q \equiv p_f - p_i$$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} \qquad q^2 \to 0$$

$$\gamma^\mu \not{a}\ \gamma_\mu = (2-n)\not{a} \qquad \not{a}\not{b} = 2a.b - \not{b}\not{a}$$

where $\eta^{\mu\nu}$ is the generalization of $g^{\mu\nu}$ in a n-**dimensional space, and neglect** the pure y^A terms, then we are left with

$$I'_\lambda = \frac{2}{m^2{}_{K^*}} \int d^n k\ \frac{(P_\lambda-k_\lambda)((2-n)m^2{}_{K^*}\not{k} + 2m_\mu k^2-(m_\mu+k^2)\not{k})}{(k^2-M_L^2)((p_i-k)^2-m^2{}_{K^*})((p_f-k)^2-m^2{}_{K^*})} \tag{17}$$

As in section 2, we introduce the same **Feynman** parameters. The **denominator** in (17) becomes $[(k-M)^2+\Delta]^3$ where

$$M \equiv -q\,x\,y + p_f\,x$$

We now shift the k origin and use

$$\int d^n k \frac{k_\mu}{(k^2 + \Delta)^3} = 0 \qquad \int d^n k \frac{k_\mu k_\nu k_\sigma}{(k^2 + \Delta)^3} = 0$$

$$\int d^n k \frac{k_\mu k_\nu}{(k^2 + \Delta)^3} = \frac{\eta_{\mu\nu}}{n} \int d^n k \frac{k^2}{(k^2 + \Delta)^3}$$

$$\int d^n k \frac{k_\mu k_\nu k_\lambda k_\sigma}{(k^2 + \Delta)^3} = \frac{1}{n!} (\eta_{\mu\nu} \eta_{\lambda\sigma} + \eta_{\mu\lambda} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\lambda}) \int d^n k \frac{k^4}{(k^2 + \Delta)^3}$$

The terms containing

$$k_\mu \not{k} \rightarrow \frac{\eta_{\mu\nu}}{n} \gamma^\nu k^2 = \frac{k^2}{n} \gamma_\mu$$

$$k_\mu k^2 \not{k} \rightarrow \frac{k^4}{n!} \gamma^\mu$$

are absorbed in the charge renorrnalization and

$$2\, k.\, M \not{k} \rightarrow \frac{2\eta_{\alpha\beta}}{4} M^\alpha \gamma^\beta k^2 = \frac{1}{2} \not{M} k^2$$

The numerator in (17) becomes

$$\frac{P_\lambda}{m^2_{K^*}} \left[2(2 - n)m^2_{K^*} \not{M} + 4m_\mu M^2 - 2(m^2_\mu + M^2) \not{M} + (4m_\mu - 3 \not{M}). k^2 \right] -$$

$$- \frac{M_\lambda}{m^2_{K^*}} \left[2(2 - n)m^2_{K^*} \not{M} + 4m_\mu M^2 - 2(m^2_\mu + M^2) \not{M} + (6m_\mu - 4 \not{M})k^2\right] \quad (18)$$

we consider first the k^2 terrn in (18)

$$I''_\lambda = \frac{1}{m^2_{K*}} \int_0^1 2x\, dx \int_0^1 dy \int d^n k \frac{\{P_\lambda(4m_\mu - 3\not{M}) - M_\lambda(6m_\mu - 4\not{M})\} \quad k^2}{(k^2 + \Delta)^3}$$

changing variables we have

$$M_\lambda \rightarrow 2xy\, P_\lambda$$

$$\not{M} \rightarrow 2xy\, m_\mu$$

$$M_\lambda \not{M} \rightarrow 2x^2\, P_\lambda\, m_\mu y$$

and integrating over y

$$I''_\lambda = \frac{m_\mu}{m^2_{K*}} P_\lambda \int_0^1 dx \int d^n k \frac{k^2(4x - 9x^2 + 4x^3)}{(k^2 + \Delta)^3}$$

the k integral can be evaluated

$$\int_{-\infty}^{+\infty} \frac{k^2}{(k^2+\Delta)^3} d^n k = \frac{2i\, \Pi^{n/2}}{\Gamma(n/2)} \int_0^\infty dk \frac{k^{n+1}}{(k^2+\Delta)^3} \equiv \frac{2i\, \Pi^{n/2}}{\Gamma(n/2)} \frac{1}{2} \int_0^\infty \frac{p^{n/2}}{(p+\Delta)^3} dp =$$

$$= \frac{i\, \Pi^{n/2}}{\Gamma(n/2)} \Delta^{(n/2 - 2)} \frac{\Gamma(n/2 + 1)\, \Gamma(2 - n/2)}{\Gamma(3)}$$

and

$$I''_\lambda = \frac{2m_\mu}{m^2_{K*}} P_\lambda \int_0^1 dx(4x - 9x^2 + 4x^3)(i\Pi^{n/2} \frac{n}{2} \frac{1}{\Gamma(3)} \Delta^{n/2 - 2}\, \Gamma(2 - \frac{\nu}{2}))$$

as

$$\Gamma(2 - \frac{n}{2}) = \frac{\Gamma(3 - \frac{n}{2})}{2 - \frac{n}{2}}$$

$$\Delta^{\frac{n}{2} - 2} \equiv e^{(\frac{n}{2} - 2) \ln \Delta} = 1 + (\frac{n}{2} - 2)\ln \Delta + \frac{1}{2!}(\frac{n}{2} - 2)^2 \ln \Delta + \dots\dots$$

we have

$$\Gamma(2-\frac{n}{2})\ \Delta^{\frac{n}{2}-2} = \Gamma(3-\frac{n}{2})\left[\frac{1}{2-\frac{n}{2}} - \ln\Delta + \frac{1}{2!}(\frac{n}{2}-2)\ \ln^2\Delta + \ldots\right]$$

As

$$\int_0^1 dx\ (4x - 9x^2 + 4x^3) = 0\ ,$$

the first term in the series vanishes.

Taking then the limit $n \to 4$, we have only the second term

$$I''_\lambda = -\,i\ \Pi^2\ \frac{m_\mu}{m^2_{K^*}}\ 2\ P_\lambda \int_0^1 dx\ (4x - qx^2 + 4x^3)\ \ln\Delta \qquad (19)$$

The remaining part of (18) is

$$I'''_\lambda = \int_0^1 2x\ dx \int d^n k\ \frac{P_\lambda(2(2-n)\,m_\mu(x-x^2)) + P_\lambda \frac{m^3_\mu}{m^2_{K^*}}(6x^2-2x-6x^3+2x^4)}{(k^2+\Delta)^3}$$

We take as leading term the first one in the numerator, and use the integral

$$\int d^n k\ \frac{1}{(k^2+\Delta)^3} = \frac{i\ \Pi^{n/2}}{\Gamma(3)}\ \Delta^{\frac{n}{2}-3}\ \Gamma(3-\frac{n}{2}) \underset{n\to 4}{=} \frac{i\Pi^2}{2\Delta}$$

$$I'''_\lambda = -\,i\ 4m_\mu\ P_\lambda\ \Pi^2 \int_0^1 dx\ \frac{(x^2-x^3)}{\Delta} = -\,i\ 4\Pi^2\ m_\mu\ P_\lambda\ I_F \qquad (20)$$

As $I''_\lambda << I'''_\lambda$, we take in consideration only I'''_λ

From (15) the S-matrix is

$$S = N_f \frac{2eg^2}{(2\pi)^4} i \Pi^2 m_\mu . 4 P_\lambda I_F A^\lambda \bar{u}(p_f) u(p_i)$$

Aplying the Gordon decomposition $(i P_\lambda \to 1/2\ \sigma_{\lambda\nu} q^\nu)$ the S-matrix becomes

$$S = N_f (g^2 \frac{m^2_\mu}{2\pi^2} I_F) \mu_0 \bar{u}(p_f) \sigma_{\lambda\nu} q^\lambda A^\nu u(p_i) \qquad (21)$$

4. COMPARISON WITH EXPERIMENT

We assume that the coupling constants g_1 and g_2 are of order "e" which implies

$$\frac{g^2_{1,2}}{4\pi} \simeq \alpha$$

By comparing the expressions (11) and *(21)* for S with the lowest order Q.E.D.

$$N_f \mu_0 \bar{u}(p_f) \sigma^{\nu\lambda} u(p_i) q_\nu A_\lambda$$

we obtain the correction to μ_0

$$|\delta_{\mu_0}| = \varepsilon_{1,2} \frac{\alpha}{\pi} m^2_\mu I_F = \varepsilon_{1,2} \frac{\alpha}{\pi} \int_0^1 dx \frac{x^2 - x^3}{x^2 + (1-x) \frac{M_L^2}{m^2_\mu}}$$

where $\varepsilon_1 \equiv 1$ refers to the scalar-pseudoscalar interaction (1) and $\varepsilon_2 \equiv 2$ to the vector-axial vector interaction (1').

The final result is

$$|\delta_{\mu_0}| = \varepsilon_{1,2} \frac{\alpha}{\pi} \frac{m^2_\mu}{M_L^2} \left(\frac{1}{3} - \frac{m^2_\mu}{M^2} \ln \frac{M_L^2}{m^2_\mu} \right)$$

with $M_{L1} \simeq M_{L2}$. The inclusion of the sin θ and cos θ factors does not change this result.

The agreement between theory and experiment[4,5] implies $|\delta_{\mu_0}| < 2 \times 10^{-8}$, which gives a bound for M_L of $M_L > 20$ GeV for the pseudo scalar interaction and $M_L > 28$ GeV for the pseudo vector interaction.

One of the authors (J.M.S.) would like to thank N. Fleury, Ch.Ragiadakos, J.M. Brucker and J. Husser for several discussions.

REFERENCES

1. J. Leite Lopes, Rev. Bras. Fis. 5, 37 (1975).
2. J. Leite Lopes and N. Fleury, Lett. Nuov. Cim. 19, 7 (1977).
3. S. Barshay and J. Leite Lopes, Phys. Lett. *68B*, 174 (1977).
4. J. Bailey *et al.*, Phys. Lett. *67B*, 225 (1977).
5. J. Calmet et *al.*, Rev. Mod. Phys. 49, 21 (1972).
6. B.D. Fried, Phys. Rev. 88, 1142 (1952).
7. Bjorken and Drell, *Relativistic Quantum Fields*, Mc Graw-Hill (1965).
8. K. Fujikawa, B.W. Lee, A.I. Sanda, Phys. Rev. D6, 2923 (1972).
9. C.G. Bollini and J.J. Giambiagi, Nuov. Cim. *12B*, 20 (1972).
10. W.A. Bardeen et *al.*, Nucl. Phys. *46B*, 319 (1972).
11. S.J. Brodsky and J.D. Sullivan, Phys. Rev. *156*, 1644 (1967).