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On Gravitational Solitons

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A gravitational model with exact general solution (depending on two arbitrary functions) is presented. The model exhibits soliton characteristics.

Um modelo gravitacional com exata solução geral (dependendo de duas funções arbitrárias) é apresentado. O modelo exibe caracteristicas de soliton.

1. INTRODUCTION

Recently considerable effort has been made to find soliton solutions corresponding to (non-linear) evotion equations. Indeed the "exponential growth regime" on the number of papers on this subject is fully justified: solitons may represent objects that have some features of elementary particles[1]. In general relativity, this idea could substitute the hope about gravitons. Unfortunately, up to now, soliton and gravitation are unmixed theories: the difficulties concerning coupled non-linear partial differential equations are formidable[2], even in situations simpler than Einstein's equations.

The purpose of this note is to start closing this gap. We propose a soluble gravitational model which exhibits plane wave solution with some remarkable points: we have achieved a general solution for this model (i. e. a solution depending on arbitrary functions) - able to describe solitons with arbitrary shapes. The negative poirit is: we were able to find only two

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dirnensional waves - as is usual in the literature; and these waves were found under somewhat restrictive symrnetry assumptions. We strongly believe that it will be possible to remove these restrictions very soon.

2. A GRAVITATIONAL MODEL

Let us confine ourselves to the universe with line elernent given by

$$(ds)^2 = \sum_{i,j=1}^{4} \delta_{ij}\, g_i\, dx^i dx^j$$

where $g_i = g_i(z)$, with $z = kx^1 - wx^4$.

Under such conditions we have only four non-trivial field equations, namely

$$R_{11} = 0 \Longrightarrow \frac{1}{2} k^2 \left[\left(\frac{g_2'}{g_2}\right)' + \left(\frac{g_3'}{g_3}\right)' + \left(\frac{g_4'}{g_4}\right)'\right] + \frac{1}{2}\omega^2 \left(\frac{g_1'}{g_4}\right)'$$

$$+ \frac{1}{4}\omega^2 \left(\frac{g_1'}{g_4}\right) \left[\frac{g_2'}{g_2} + \frac{g_3'}{g_3} + \frac{g_4'}{g_4} - \frac{g_1'}{g_1}\right] +$$

$$+ \frac{1}{4} k^2 \left[\left(\frac{g_2'}{g_2}\right)^2 + \left(\frac{g_3'}{g_3}\right)^2 + \left(\frac{g_4'}{g_4}\right)^2 - \frac{g_1'}{g_1}\frac{g_2'}{g_2} - \frac{g_1'}{g_1}\frac{g_3'}{g_3} - \frac{g_1'}{g_1}\frac{g_4'}{g_4}\right] = 0$$

$$R_{44} = 0 \Longrightarrow \frac{1}{2}\omega^2 \left[\left(\frac{g_1'}{g_1}\right)' + \left(\frac{g_2'}{g_2}\right)' + \left(\frac{g_3'}{g_3}\right)'\right] + \frac{1}{2} k^2 \left(\frac{g_4'}{g_1}\right)'$$

$$+ \frac{1}{4} k^2 \left(\frac{g_4'}{g_1}\right) \left[\frac{g_1'}{g_1} + \frac{g_2'}{g_2} + \frac{g_3'}{g_3} - \frac{g_4'}{g_4}\right] +$$

$$+ \frac{1}{4}\omega^2 \left[\left(\frac{g_1'}{g_1}\right)^2 + \left(\frac{g_2'}{g_2}\right)^2 + \left(\frac{g_3'}{g_3}\right)^2 - \frac{g_1'}{g_1}\frac{g_4'}{g_4} - \frac{g_2'}{g_2}\frac{g_4'}{g_4} - \frac{g_3'}{g_3}\frac{g_4'}{g_4}\right] = 0$$

$$R_{22} = 0 \Longrightarrow \frac{1}{4}\left(\frac{g'_1}{g_1} - \frac{g'_2}{g_2} + \frac{g'_3}{g_3} + \frac{g'_4}{g_4}\right)\left(k^2\frac{g'_2}{g_1} + \omega^2\frac{g'_2}{g_4}\right) +$$

$$+ \frac{1}{2}k^2\left(\frac{g'_2}{g_1}\right)' + \frac{1}{2}\omega^2\left(\frac{g'_2}{g_4}\right)' = 0$$

$$R_{33} = 0 \Longrightarrow \frac{1}{4}\left(\frac{g'_1}{g_1} \quad \frac{g'_2}{g_2} \quad \frac{g'_3}{g_3} \quad \frac{g'_4}{g_4}\right)\left(k^2\frac{g'_3}{g_1} + \omega^2\frac{g'_3}{g_4}\right) +$$

$$+ \frac{1}{2}k^2\left(\frac{g'_3}{g_1}\right)' + \frac{1}{2}\omega^2\left(\frac{g'_3}{g_4}\right)' = 0$$

whsre $g' = dg/dz$.

The last two equations are satisfied if

$$\frac{k^2}{g_1} + \frac{\omega^2}{g_4} = 0 \tag{1}$$

which implies

$$\frac{g'_1}{g_1} = \frac{g'_4}{g_4} = v \tag{2}$$

Then the first two equations are reduced to the single one:

$$v = \frac{2(v'_2 + v'_3) + v^2_2 + v^2_3}{2(v_2 + v_3)}$$

where v_i is such that $g_i = \exp \int v_i \, dz$, being v_i arbitrary functions of z.[3] In order to have Minkowski metric, assymptotically, we impose the only restriction upon the functions v_i: they must be bounded at infinity, i.e.

$$\lim_{x^1 \to (\pm)\infty} v_i = M_i^{(\pm)}$$

So we have obtained a class of wave solutions for the field equations - a class depending on two arbitrary functions. Among the elements of this class, we have solitary waves by a convenient choice of the functions v_i, In particular we may select, as solutions, solitons already present in other theories.

3. FINAL REMARKS

The success in finding general soliton solutions for the gravitational field opens some interesting possibilities: following some (yet unknown) prescriptions we may adapt these general solutions to particular solitons (sine--Gordon, KdV, etc...). It may not be just a coincidence that general solutions could be found (till now) only in a theory that claims to be unified (or unifiable).

REFERENCES AND NOTES

1. A good review about soliton theory is Alwyn C. Scott, F. Y. C. Chu and David W. McLaughlin, in Proceedings of the IEEE, 62, 1443 (1973).
2. S. Kubo, M. Namiki and I. Ohba, *Soliton Solutions of Coupled Field Equations and Suggestion of the "Soliton Bag" in Dynamical Particle Model.* (Preprint) University of Waseda, Tokyo, 1977.
3. And then the model is solved, by calculatins g_1 and g_2, taken into account eqs. (1) and (2).