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On the Initial Conditions for a Universe with Variable G and c

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Initial conditions are discussed for a closed universe with matter injection process and with time varying G and c. it is shown that the usual interpretation for the cosmic microwave background radiation can be accepted for this model universe, provided a certain initial condition is imposed on the Pryce-Hoyle field. The time varying c is responsible for two red-shift laws one for wave-length and the other for the frequency. If radiation temperature obeys the red-shift law for frequency, the primordial Planck spectrum can be reproduced along cosmic expansion.

Condições iniciais são analisadas para um universo fechado com processo de injeção de matéria G e c funções do tempo. Mostra-se que a interpretação usual da radiação cósmica do corpo negro pode ser aceita para esse modelo cosmológico desde que certa condição inicial seja imposta para o campo de Pryce-Hoyle. O c variável é responsável por duas leis de "red-shift", uma para comprimento de onda e outra para frequência. Se a temperatura de radiação obedece á lei do "red-shift" para frequência, o espectro Planck primordial pode ser reproduzido ao longo da expansão cósmica.

1. EQUATIONS OF THE MODEL UNIVERSE

The equations of the present cosmological model are of a type involving positive curvature and matter injection process[1] which are the following:

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$$R_{00} - \frac{1}{2} g_{00} R + g_{00} \Lambda = - \kappa T_{00}$$

$$R_{11} - \frac{1}{2} g_{11} R + g_{11} \Lambda = - \kappa T_{11} \tag{1}$$

where,

$$T_{\mu\nu} = (\rho + \frac{P}{c^2}) u_\mu u_\nu - g_{\mu\nu} \frac{P}{c^2} - f(\lambda_\mu \lambda_\nu - \frac{1}{2} g_{\mu\nu} \lambda_\alpha \lambda^\alpha)$$

and,

$$\lambda^\mu_{;\mu} = f^{-1} n(x^0) \tag{2}$$

$n(x^0)$ is the number of injected particles per unit proper volume.

An additional relation is introduced which represents Mach's principle:

$$MG/c^2 R = \alpha\pi/4 \ , \tag{3}$$

being a a numerical constant.

This set of four equations determine uniquely the functions $\rho(x^0)$, $\rho(x^0)$, $\lambda'(x^0)$ and $\rho(x^0)$.

Introducing into this model certain ideas developed by Narlikar[2], the following conclusions were inferred[3]:

$$G \propto R^{-1}(t)$$

$$c^2 \propto G(t) \tag{4}$$

which means that Einstein's κ is effectively constant, and the differential equations derived from the combination of (1), (2) and (3) are not substantially modified. We must bear in mind that, according to (4) we have:

$$dx^0 = c(t) dt \tag{5}$$

2. MODIFIED RED-SHIFT LAW

For this cosmological model, the red-shift law presents the form:

$$(1+\tilde{z}) = R_p/R = [c(t_p)\ dt_p]/[c(t)\ dt] = \tilde{\lambda}_p/\tilde{\lambda} \ , \tag{6}$$

where the index "p" refers to the present epoch. The measured red-shift z is:

$$(1+z) = \tilde{\lambda}_p/\lambda \ . \tag{7}$$

For, λ is the non-shifted wave-length at the present epoch which is different from $\tilde{\lambda}$ which was emitted in the past when $c(t)$ was higher.

According to the Balmer equation:

$$\nu/c \propto (\varepsilon_0 c)^{-1} \ . \tag{8}$$

Bearing in mind that[3,4],

$$\varepsilon_0(t)c(t) = \text{const.} \ , \tag{9}$$

the following red-shift law for frequency is deduced, different from law (6):

$$\tilde{\nu}/\tilde{\nu}_p = (R_p/R)^{3/2} \ . \tag{10}$$

It can be shown that:

$$(1+\tilde{z}) = (1+z)^2 \ , \tag{11}$$

$$c(t) = c_p(1+z) \ . \tag{12}$$

3. INITIAL CONDITIONS COMPATIBLE WITH THE USUAL INTERPRETATION FOR THE COSMIC BLACK BODY MICROWAVE RADIATION

According to the usual interpretation, the present thermal spectrum of the cosmic background radiation with 2,7 ^{O}K temperature, is a fossil image projected from the primordial fireball, the initial stage of the universe as proposed by Gamow[5]. In this fireball stage, according to Alpher, Bethe and Gamow[6], matter is considered a neutron fluid at a temperature of 10^{10} ^{O}K. This hypothesis was raised by Gamow[5] in order to develop a theory that could account for the present abundance of the elements, and the start of the building up period is shown to be 20 sec. after zero cosmic time[6]. The density of rnatter at this situation should be 10^{6} times smaller than the density of radiation, which should be therefore equivalent to the density of water[6,7]. This means hence that the expansion of the primordial stage is drived almost completely by radiation pressure.

As shown by Weinberg[8] the Planck spectrum at the primordial phase can be red-shifted without deformation, to the present 2,7 ^{O}K spectrum. We have speculated[1] that the cosmic background radiation in the context of our cosmological model could perhaps be interpreted in different terms, to wit. A universe model with a cold start, drived almost exclusively by the Hoyle repulsive fieid, such that the Planck spectrum could be accounted for through a hypothesis suggested by Weinberg[8] for Hoyle's model universe. It seems to us that the usual interpretation could be nonetheless maintained if we modify the initial conditions for the present cosmological model.

Let us assume that the primordial state of the present model at time zero should be a perfect neutron fluid. This is possible, since for cosmological models with the Pryce-Hoyle field, the initial singularity is avoided. The matter injection process could be therefore viewed as present throughout the total cosmic space: the totallity of the universe then, is a single "white hole", in contradistinction with subsequent stages when quasars and radiogalaxies were formed, which were postulated as being "white holes"[1,9]. In the standard models, the breaking up of the primordial fluid takes place when the Jeans principle of gravitational instability begins to develop[10]. But in the present theoretical framework, the stage when the breaking up starts is not solely determined by the interplay of gravitational attraction and repulsion by conventional pressure (kinetic and radiation). For, there is a repulsion produced by the Pryce-Hoyle field.

Solution of equations (1) and (2) through (3), gives the result:

$$\frac{1}{R}\frac{dR}{dx^0} = \left[\frac{\kappa\rho}{3} - \frac{1}{R^2} - \frac{1}{3R^6}(\Gamma R + \Theta)^2\right]^{1/2} , \tag{13}$$

being $R(x^0)$ the "radius of curvature" of the closed universe obeying a *Robertson-Walker* metric:

$$ds^2 = -\frac{R^2}{\left[1 + \frac{r^2}{4}\right]^2}\left[dr^2 + r^2\sin^2\theta d\phi^2 + r^2 d\theta^2\right] + (dx^0)^2 . \tag{14}$$

$$\rho = mn + \frac{A}{c^5}T^4 + (\gamma-1)^{-1}\frac{nkT}{c^2} , \tag{15}$$

where m is the nucleon mass and n the number density of particles. A, γ and k are constants, and c is considered a function of cosmic time[3]. We see in relation (13) that it is impossible to exist a singularity at the origin of time due to the third term in the radical, which comes from the Pryce-Hoyle field. According to relation (3):

$$\kappa mn = \alpha/R^2 . \tag{16}$$

The conditions proposed by Gamow for the standard cosmological models at 20 sec. cosmic time, could be reproduced in this model at zero cosmic time, on account of being impossible a singularity. In order that the breaking up of the fluid be determined after a lapse of time necessary for fusion and synthesis of the elements, it would be reasonable to assume at $x^0 = 0$, the following initial condition:

$$\lambda'(0) = 0 \ . \tag{17}$$

since,

$$\lambda'(x^0) = \text{const.}\ R^{-3}\ (\Gamma R + \Theta)\ , \tag{18}$$

it follows:

$$\Gamma R(0) = -\Theta\ . \tag{19}$$

This initial condition is also advantageous on other grounds, to wit: for high values of R, Θ may be neglected[3] and not Γ, so that Γ could be chosen such as to avoid incompatibility of the varying G with data obtained from geology and palaeontology.

Relations (4) were deduced from matter injection conditions prevailing during cosmic expansion, after formation of quasars and radiogalaxies, so, after the breaking up of the primordial fireball fluid. Thus, during the fireball phase it is probably not possible to know how G and c vary with time, Nevertheless it could be possible to assume G and c aproximately constant, because the duration of the fireball stage is extremely small in comparison with the age of the universe. Calculationsofthe cosmic age are not impaired by this assumption. Besides that, knowledge of the functions G and c are not essential for dealing with the equations at the primordial stage, since that which is important is κ (a constant) and not G, and "time" x^0 can be used instead of t.

Of course relation (13) presupposes:

$$\Lambda = 0\ . \tag{20}$$

In figure 1, the behaviour of functions λ' and R.

From equations (1), (2) and (3) the following expression for the cosmic stress comes forth:

$$\kappa p/c^2 = -\frac{\kappa\rho}{3} - 2(R''/R) + \frac{2}{3}\kappa f(\lambda')^2\ , \tag{21}$$

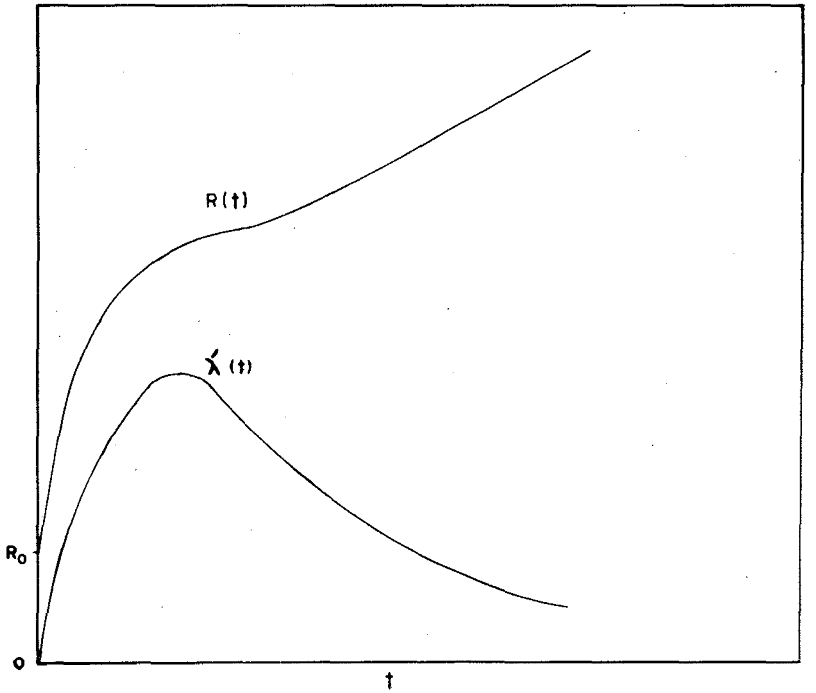

Fig.1 - Theoretical behaviour of the scaling function R and the matter injection λ'. For this cosmological model it is impossible $R = 0$.

where

$$\kappa f(\lambda')^2 = R^{-6}(\Gamma R + \Theta)^2 \ . \qquad (22)$$

We are going to assume that radiation temperature obeys a red-shift law identical to (10), and this is essential in order that the Planck spectrum be reproduced along cosmic expansion, what will be seen afterwards. Thus:

$$T = T_p \ (R_p/R)^{3/2} \ . \qquad (23)$$

Combining relation (23) with (22), (21), (15) and (13), and as well neglecting the contribution of the kinetic term in (15), it can be inferred the following expression:

$$\kappa p = B^{-1}\ \{-\ \frac{\alpha}{3} R^{-3} + \frac{1}{6}\ \Xi\ R^{-9/2} + \frac{2}{3} R^{-6}\ (\Gamma R + \Theta)\Gamma\}\ . \qquad (24)$$

Constant B comes from the relation:

$$c^2 = (BR)^{-1}\ . \qquad (25)$$

The first term in (24) arrives from the inertial contribution to the pressure plus relation (3), and we see that it gives a negative contribution to the cosmic pressure. This term predorninates after a certain epoch of the cosmic expansion, such that the overall pressure is negative. This curious result comes about from the initial choice of a positive curvature for the present cosmological model. We have pointed out before[1,9] along with McCrea's reasoning[11], that negative stresses are possible in general relativity, and this transcends the usual analogy between Newtonian fluid dynamics and relativity. The energy balance equation for the preçent universe model is of the type[8]:

$$\frac{d}{dx^0}\ \{[\rho - f(\lambda')^2]R^3\} + \{\frac{p}{c^2} - f(\lambda')^2\}\ \frac{dR^3}{dx^0} = 0\ . \qquad (26)$$

in which the pressure term is negative after the breaking up the primordial fluid.

Equation (24) was deduced, assuming relation (25), that is, it is valid in so far as we deal with cosmic times following the formation of quasars and radiogalaxies. For the initial stage we cannot use relation (25) as we have pointed out before. Therefore, during the beginning of the cosrnic expansion the pressure function should be expressed as follows (see figure 2):

$$\kappa p/c^2 = \frac{1}{3}\ [2\Gamma/R^4 - \alpha/R^2 - \kappa u - \kappa u' R]\ , \qquad (27)$$

being u the radiation preçsure term and the "dash", x^0 differentiation.

Radiation flux should vary with cosrnic expansion according to the differential equation,

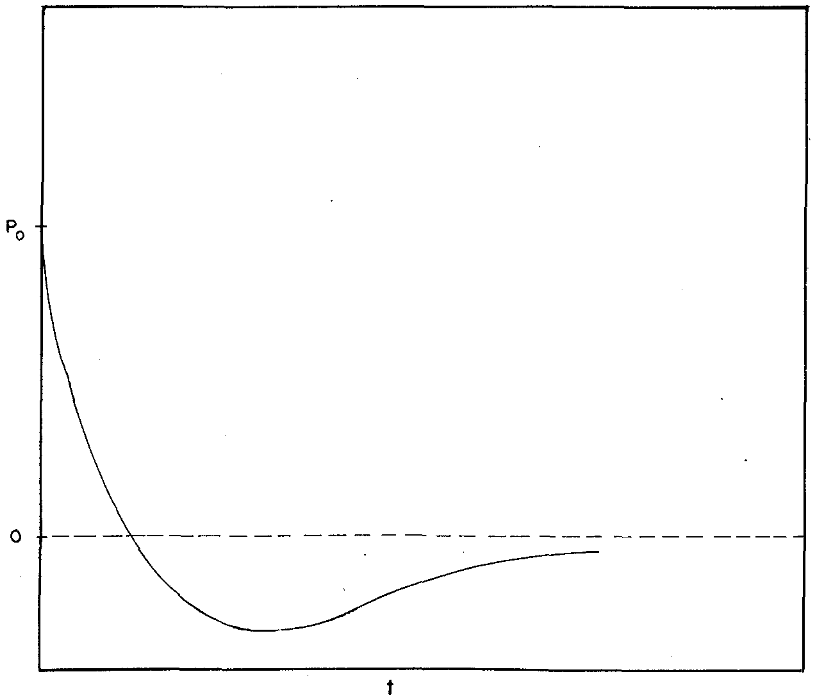

Fig.2 - Theoretical behaviour of the pressure function. The pressure at the origin is not infinite since there is no singularity. After the fireball stage the pressure assumes negative values.

$$dS(\nu(t),t) = - Z(R(t),\nu(t),t)S(\nu(t),t)dt \ , \tag{28}$$

being $Z(\nu,R,t)$ a function responsible for the decrease of the flux due to the expansion, scattering and absorption effects. Integration of (28) between time zero and present time t_p gives:

$$S(\nu_p,t_p) = \frac{2\nu_0}{c_0^2}\left[e^{\frac{h\nu_0}{kT_0}} - 1\right]^{-1} \exp\left[-\int_0^{t_p} Z\, dt\right] \ , \tag{29}$$

where ν_0 and T_0 are values for time zero. If radiation temperature obeys the same red-shift law as the frequency, i.e., relation (23), the Planck spectrum can be reproduced at the present time. So, we obtain:

$$S(\nu_p, t_p) = \frac{2\nu_0}{c_0^2}\left[e^{\frac{h\nu_p}{kT_p}} - 1\right]^{-1} \exp\left[-\int_0^{t_p} Z\, dt\right] . \qquad (30)$$

4. CONCLUSIONS

It is possible to interpret the cosmic microwave black body radiation for our cosmological model, as being originated in an initial fireball stage. It can be seen that the variability of c with time, (and a constant Einstein's κ) is not incompatible with the usual interpretation for this radiation. Of course the duration of the fireball phase necessary for the building up process of the elements, demands adequate numerical values for Γ and 0, and for the initial value R_0 of the "radius of curvature", as well. These parameters are a novelty in cosmology, and this situation should necessarily complicate still the already complex theory of the primordial synthesis of the elements. For, the Hoyle function associated with Γ and 0 represents not only a cosmic repulsion, but also a matter injection process given by equation (2), and this is surely a source of new difficulties in calculations for the theory of the building up of the elements.*

REFERENCES

1. F.M. Gomide, M. Uehara, Prog. Theor. Phys. 53, 1365 (1975).
2. J.V. Narlikar, Nature, 247, 99 (1974).
3. F.M. Gomide, Lett. Nuovo Cimento, 15, 595 (1976).
4. J.N. Bahcall, M. Schmidt, Phys. Rev. Let. *19*, 1294 (1967).
5. G. Gamow, Phys. Rev. 70, 572 (1946).
6. α, β, γ , Phys. Rev. 73, 803 (1948).
7. G. Gamow, Nature, 162, 680 (1948).

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8. S. Weinberg, *Gravitation and Cosmotogy. Principles and Applications of the General Theory of Relativity*. John Wiley and Sons, Inc. (1972).

9. F.M. Gomide, Nuovo Cimento *12B*, 11 (1972).

10. G. Gamow, Phys. Rev. 74, 505 (1948).

11. W.H. McCrea, Proc. Roy. Soc. *206A*, 563 (1951).