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A Study on a New Tabulation of Kaon-Proton Total Cross Sections

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A new tabulation of kaon-proton total cross sections, with a treatment of data and errors,which is particularly suitable in calculating with dispersion relations, is presented.

Apresentam-se novas tabelas de seções de choque para espalhamento kaon--proton, como também um tratamento dos dados e erros que se revela particularmente conveniente quando se calcula com relações de dispersão.

1. INTRODUCTION

Forward dispersion relations are currently used with the purpose of predicting the value of coupling constants and of thereal parts of a scattering amplitude[1]. Their use involves the evaluation of integrais of the type

$$I = \int_a^b h(\omega)\ \sigma(\omega)\, d\omega, \qquad (1.1)$$

where $\sigma(\omega)$ is the total cross section, at the laboratory energy, w, for the process under consideration (or for the crossed one), and $h(\omega)$ is a weight function which may be singular at some energy value. In such a case, I must be evaluated as a principal value integral.

In the evaluation of (1.1), there are two sorts of ambiguity: first of all, the function $\sigma(\omega)$ can be known only at discrete energies ω_i, and each value $\sigma(\omega_i)$ is affected both by a systematic and a statistical error

which must be taken into account if one wants to estimate the uncertainty on I; secondly, the particular form of $h(\omega)$ may enhance these uncertainties, at least in some energy region. From this discussion, it is clear that, in order to evaluate both the value of I and of Al with high accuracy, one must have a good set of input data. In particular, one must get rid of two possible sources of spurious effects: a) for a given experiment some points may be accidentally displaced in comparison with the remaining ones; b) when many experiments have been performed in the same energy region, there may be some systematic discrepancy between them.

The first effect can produce a structure in the real part, due to the principal value integration, and there is no doubt that it is generally unreasonable to interpret a single shifted point as an evidence for a structure in the amplitude. The second effect can have various consequences, according to the method which is followed in the interpolation of the data points, and often makes it hard to estimate significantly the errors on the calculated quantities. Therefore, it is important that the total cross section data be carefully examined before they are used in dispersion relations. In a sense, this is what was currently done already in the sixties, for example in the πN analyses performed by the Karlsruhe group[2], where the experimental points of $\sigma(\omega)$ were replaced by a suitable interpolating function. In that way, however, the estirnate of the statistical error on I is a difficult task. This is only one of the two extreme approaches which can be pursued, the other being that of rejecting any suspicious point.

More recently, this problem has been deeply examined by Ferrari[3], still in the case of πN scattering, For this reaction, the problem of distinguishing statistical from systematic errors is particularly important, even more than for other reactions, like those considered in the present paper, where only occasionally there are two experiments in the same energy region, with comparable statistical errors and exhibiting systematic discrepancies.

In the πN case, Ferrari, besides taking into account the systematic errors on the various sets of points coming from different experiments,

proposed a procedure to smooth the data so that no spurious structures are introduced. His procedure is intermediate between the two above extreme approaches. In fact, he performs a fit to several data points in energy regions determined by inspection of the data. Whenever the best fit curve does not cross the error bar of a given point, the value of this is replaced by that predicted by the fit to the neighbouring points. However, to keep a memory of this substitution, to the new point is attributed an extended error, of the order of the difference between experimental and fitted values.

2. THE Kp TOTAL SECTIONS

In this work, we follow the procedure of Ref.3 for Kp scattering. As we mentioned above, in our case we do not pay much attention to the problem of the systematic errors; this is not a serious limitation because in these reactions they are generally folded into the statistical ones and it would be almost impossible to extract them, and to distinguish the two contributions.

Before discussing our calculations, let us briefly summarize the experimental situat'ion of Kp total cross sections.

The reaction K^-p has an interesting feature; it exhibits a remarkable structure which is also extended below its threshold because of the existente of coupled channels having the same quantum numbers as the K^-p system and a lower mass ($\pi\Lambda$, $\pi\Sigma$, $\pi\pi\Lambda$).

It is unnecessary to exhibit a tabulation of the imaginary part of the amplitude in the unphysical region; however, it might be useful to recall that the procedure to estimate it is to perform an analytic continuation of the amplitude from the low-energy region. The formalism currently used for this purpose was originally proposed by Dalitz and Tuan[4]. It is based on a K-matrix parametrization, for which the earliest approximation was one of a constant scattering length. Already in this form, the solutions exhibit* a s-wave resonance corresponding to the Y_0^* (1405)

* We disregard some old analyses which presented also non-resonant amplitudes as possible solutions[5].

and this feature is preserved in the most recent analyses which parametrize the K-matrix using either an effective-range or a zero-range approximation. Typical representatives of the two alternatives are Refs. 6 and 7, respectively.

The analyses of these works are obviously confined to the very low energy region. At higher energies, we analyze directly the data; whenever it is possible, we only make use of the most recent and accurate data, neglecting previous data which in most cases are much less accurate. For this reason, they are not very important in the fits even when they fail to follow the trend of the most recent experiments. The list of the experiments which we considered in our fits up to 3.3 GeV is given is Refs. 8-13; in the tabulations, we shall quote the experimental source for each point so that one can identify the data we neglected, for example by comparison with compilations of KN data[14]. In the region between 3.3 GeV and 6 GeV, only very old data are available, Refs. 12, 15, 16, and their errors are larger than those at lower and higher energies.

The situation for K^+p is quite different in the sense that one does not observe such a complex structure. In fact, in this channel there is only some structure in the 1 GeV region, for which conjectures were made that it could be interpreted as an evidence for the existence of exotic resonances.

We have selected the experimental points for K^+p scattering according to the same criteria as for K^-p. The experiments from which we extracted the data are those of Refs. 9-13 and Refs. 17-24. Also, for this reactions the data in the region from 3.3 GeV to 6.0 GeV are rather scarce and old[16-25].

At higher energies, there is an improvement in the status of our knowledge of $K^{\pm}p$ total cross sections. The region from 6 to 20 GeV was studied about ten years ago at Brookhaven[26], and later the knowledge was extended to higher energies using the Serpukhov[27] and Fermilab[28] accelerators. At these energies it is more convenient to give the values for the combinations

$$\sigma^{(-)} = \sigma(K^-p) - \sigma(K^+p) ,$$
$$\sigma^{(+)} = \sigma(K^-p) + \sigma(K^+p) , \qquad (2.1)$$

and this for various reasons: first of all, the theoretical analysis of $\sigma^{(\pm)}$ in terms of a Regge pole model is much simpler because these combinations separate the contributions of poles of different parity. This allows one to perform separate and significant fits, each with a smaller number of parameters, instead of performing a unique fit to all data. Moreover, it is convenient to write the dispersion integrals in terms of $\sigma^{(-)}$ and $\sigma^{(+)}$; this makes the estimate of the errors more convenient than when using $\sigma(K^-p)$ and $\sigma(K^+p)$, because in the latter case one should consider the correlation of the errors coming from the fact that the same pole term contributes to both cross sections.

3. PARAMETRIZATION AND CALCULATIONS

In the previous Section, we described the experimental situation of $K^{\pm}p$ total cross sections. As we wrote there, a first selection of the data was done by excluding those data bearing a statistical error much larger than that of the most recent experiments.

Another problem of choice of data occurs when, in a certain energy range, there exist several points from different experiments. If in such a case the points did not show any systematic shift, they were all considered in the fit. If instead it appeared that there was some systematic difference between the points from different experiments, we selected one set of data, on the basis of the statistical accuracy and of the matching with the data in the nearby energy regions. We were particularly careful in handling this problem; in our fitting procedure the matching was insured by performing fits with the last few points of each experiment together with the first few points of the next one, and constraining the continuity with the parametrizations of both regions.

Continuity constraints were imposed by requiring that, at the energy ω, separating the two sets which were fitted, both a and $d\sigma/d\omega$ have the

same value for the two parametrizations. This was done not only for the connection between different experiments but also when the structure of the cross section suggested to break up the interval covered by one experiment into subintervals.

Let us now present our calculations. It is convenient to discuss them following the same partition of the data as in Section 2.

For energy values lower than 6 GeV, we parametrized the cross section for each reaction in the form

$$\sigma = a + bT + cT^2 + dT^3 \ , \qquad (3.1)$$

where T is the kaon kinetic energy

The parameters a,b,c,d were determined in each interval by fitting the corresponding data with the continuity constraints.

We shall make only a few comments on these fits, referring the reader to the tabulation of our i-esults.

The energy from which we start to use the parametrization (3.1), for the reaction K^-p, is that of the lowest energy point of Ref.8, i.e. ω = 551 MeV. Below, we use the K-matrix parametrizations of Refs. 6 and 7 which are in good agreement for the shape of the curve, with only a slight discrepancy in magnitude. We then impose the continuity between the two regions, and use the points from Refs. 8 and 9. The rich structure of the reaction and the form of the parametrization forced us to break up the energy range into many intervals in order not to destroy that structure by our smoothing procedure.

One could think that by choosing a different number of points in the fit, for a particular region, one could obtain different best fit curves. This is not so because the continuity constraints, although unable to determine uniquely how many points are to be taken for a given fit, are sufficient to make the results independent of such a choice.

The situation is definitely simpler for K^+p; in this case, it is possible to select few regions, each with a relatively high number of points. Taking advantage of the wellknown fact that only *s*- and p-waves are present at the lowest energies, i.e. below ω = 857 MeV, we used there only the first three terms of the expansion[3].

For both reactions, the region between 3.3 GeV and 6 GeV is much less well known. There, the data do not allow to perform significant fits, and for this reason we treated that region in a different way; namely, we used there an interpolation from the two contiguous regions, exploiting the usual constraints, but giving no importance to the actual value of the experimental data. This procedure is not as arbitrary as it could appear, if one remembers that the purpose of our work is to determine a set of values of total cross sections which is sufficiently smooth and with a reasonable estimate of the errors. Now, all the data in the region are very old and have larger errors than the points from more recent experiments; therefore, since we used those errors as an estimate of the uncertainty on our data in the region, and since the old data are not in disagreement with the calculated values, we may use our values with confidence. A further argument in favour of this procedure is that it is known that the data of the region below 3.3 GeV, and those of the region above 6 GeV, join quite smoothly[29].

Above 6 GeV, we have data up to energies as high as 280 GeV. In this interval, the Regge theory states that the combinations $\sigma^{(-)}$ and $\sigma^{(+)}$ have contributions from poles with different quantum numbers; in particular, one has

$$\begin{aligned} \sigma^{(-)} &= F_\omega + F_\rho \,, \\ \sigma^{(+)} &= F_p + F_{A_2} \,, \end{aligned} \tag{3.2}$$

where by F_x we denote, apart from kinematical factors, the amplitude corresponding to the exchange of a pole having the quantum numbers of the particle x.

The use of Kp data, only, is not sufficient to determine the parameters of all the poles; this would require the simultaneous use of Kn data

because the contributions from ρ and A, have opposite signs. Therefore, we assumed that $F_\omega + F_P$ could be approximated by one effective pole, and that the contributions to $\sigma^{(+)}$ come from the Pomeranchuk pole and from a secondary pole which may be considered as an effective pole averaging the p' and A_2 contributions.

The fits were performed assuming a Regge pole expansion; for the even amplitude we also added a rising logarithmic term, whereas for the odd one we considered both a pure Regge pole model and one with cuts in the complex plane. Typical values of the parameters which can be used for the Regge expansion are: β_ρ = 17.80 mb, α_ρ = 0.465 for the crossing-odd combination, parametrized as $\sigma^{(-)} = \beta_\rho\, \omega^{\alpha_\rho - 1}$, and β_P = 14.78 nb, $\beta_{p'}$ = 40.46 mb, $a_{p'}$ = 0.571, c = 4.334 mb, γ = 0.967 for the crossing-even one, parametrized as $\sigma^{(+)} = \beta_p + \beta_{p'}\omega^{\alpha_{p'} - 1} + c(\log \omega)^\gamma$; for both parametrizations, ω is given in GeV.

Our results are displayed in Tables I-VI, where for each point we give the incident kaon momentum in the laboratory system, the calculated values of a and Δσ, the experimental values of the same quantities, and the experimental source.

In Tables 5 and 6, we exhibit values of parameters which fit best the total cross sections for the various energy intervals. These parameters do not have a particular physical significance, since other sets of values can give fits of comparable quality. We report them in order to allow the reader to calculate cross sections, at other energy intervals, in a dispersive integral.

CONCLUSIONS

To conclude this work, it may be interesting to give a brief account of possible applications of Kp dispersion relations which have some interest in the present status of the matter.

The first application which can be thought of is to calculate again the NYK coupling constants, and the Kp real parts. The interest for this is

TABLE 1

TOTAL CROSS SECTIONS FOR K-P BETWEEN 0.245 AND 6. GEV/C

PLAB (GEV/C)	SIGMA- CALC. (MB)	ERROR CALC. (MB)	SIGMA- EXP. (MB)	ERROR EXP. (MB)	REFERENCE
.245	101.330	14.816	113.800	8.000	MAST/75 [8]
.255	97.086	5.300	98.000	5.300	MAST/75
.265	92.697	4.400	94.000	4.400	MAST/75
.275	88.294	9.397	96.700	4.200	MAST/75
.285	84.025	9.416	75.100	3.000	MAST/75
.295	80.059	3.500	82.500	3.500	MAST/75
.305	76.591	3.200	78.600	3.500	MAST/75
.315	73.834	5.037	70.900	3.200	MAST/75
.325	72.030	2.800	76.000	3.100	MAST/75
.335	71.441	2.800	71.500	2.800	MAST/75
.345	70.827	2.900	71.300	2.900	MAST/75
.355	68.924	2.300	68.800	2.300	MAST/75
.365	70.016	2.000	70.000	2.000	MAST/75
.375	77.905	4.425	81.800	2.100	MAST/75
.385	88.947	1.700	88.300	1.700	MAST/75
.395	91.395	1.900	91.600	1.900	MAST/75
.405	80.524	4.277	76.600	1.700	MAST/75
.415	70.879	2.000	70.700	2.000	MAST/75
.425	63.640	4.015	66.700	2.600	MAST/75
.435	58.651	4.503	54.900	2.500	MAST/75
.445	55.332	3.000	55.700	3.000	MAST/75
.455	53.142	4.200	57.200	4.200	MAST/75
.475	49.801	.963	48.960	.470	BOWEN/70 [9]
.506	45.088	.450	44.940	.450	BOWEN/70
.536	41.667	.561	42.080	.380	BOWEN/70
.566	39.230	.340	39.290	.340	BOWEN/70
.596	37.595	.330	37.760	.330	BOWEN/70
.627	36.508	.280	36.410	.280	BOWEN/70
.657	35.788	.330	35.530	.330	BOWEN/70
.686	35.244	.370	35.290	.370	BOWEN/70
.717	35.309	.350	35.640	.350	BOWEN/70
.736	35.958	.350	35.640	.350	BUGG/68 [10]
.767	38.457	.340	38.570	.340	BUGG/68
.794	40.852	.260	40.960	.260	BUGG/68
.819	40.958	.300	40.670	.300	BUGG/68
.839	40.417	.658	39.860	.350	BUGG/68
.860	40.651	.560	41.130	.290	BUGG/68
.881	41.545	.855	42.360	.260	BUGG/68
.899	42.710	.737	43.410	.230	BUGG/68
.936	45.766	.415	45.460	.280	BUGG/68
.965	48.250	.956	47.330	.260	BUGG/68
.991	50.088	.395	49.790	.260	BUGG/68
1.020	51.226	.614	51.700	.390	BUGG/68
1.043	51.099	1.133	52.170	.370	BUGG/68
1.070	49.387	.360	49.300	.360	BUGG/68
1.093	46.724	.652	46.130	.270	BUGG/68
1.119	43.463	.260	43.230	.260	BUGG/68
1.144	40.672	.916	39.800	.280	BUGG/68
1.169	38.478	.260	38.670	.260	BUGG/68
1.194	36.668	.355	36.920	.250	BUGG/68
1.219	35.073	.250	35.210	.250	BUGG/68
1.244	33.584	.220	33.390	.220	BUGG/68
1.270	32.281	.293	32.520	.170	COOL/70 [11]
1.274	32.129	.230	32.110	.230	BUGG/68
1.283	31.831	.675	32.500	.090	COOL/70
1.306	31.316	.220	31.170	.220	BUGG/68
1.320	31.152	.140	31.030	.140	COOL/70
1.325	31.117	.554	30.600	.200	BUGG/68
1.336	31.079	.080	31.120	.080	COOL/70
1.350	31.094	.900	31.200	.900	COOK/61 [12]
1.370	31.212	.110	31.270	.110	COOL/70
1.375	31.255	.385	30.900	.150	BUGG/68
1.384	31.338	.261	31.090	.080	COOL/70
1.406	31.562	.170	31.630	.170	BUGG/68
1.434	31.793	.080	31.830	.080	COOL/70
1.442	31.912	.374	32.280	.070	BUGG/68
1.448	32.012	.831	32.840	.080	COOL/70
1.509	33.295	.192	33.470	.080	COOL/70
1.549	33.882	.155	34.000	.100	COOL/70
1.583	34.150	.080	34.170	.080	COOL/70
1.637	34.176	.133	34.070	.080	COOL/70
1.687	33.816	.070	33.840	.070	COOL/70
1.735	33.177	.070	33.190	.070	COOL/70
1.785	32.261	.070	32.260	.070	COOL/70
1.835	31.258	.281	31.530	.070	COOL/70
1.840	31.181	.070	31.190	.070	COOL/70
1.879	30.740	.130	30.850	.070	COOL/70
1.885	30.685	.143	30.810	.070	COOL/70
1.929	30.393	.120	30.490	.070	COOL/70
1.985	30.215	.060	30.170	.060	COOL/70

TABLE 1 (CTD)

PLAB (GEV/C)	SIGMA- CALC. (MB)	ERROR CALC. (MB)	SIGMA- EXP. (MB)	ERKOR EXP. (MB)	HEFERENCE
2.035	30.116	.140	29.990	.060	COOL/70
2.080	29.972	.102	29.890	.060	COOL/70
2.124	29.694	.131	29.810	.060	COOL/70
2.164	29.805	.060	29.790	.060	COOL/70
2.206	29.813	.060	29.870	.060	COOL/70
2.240	29.797	.060	29.790	.060	COOL/70
2.250	29.786	.150	29.790	.060	COOL/70
2.301	29.674	.060	29.630	.060	COOL/70
2.350	29.445	.050	29.440	.050	ABRAMS/70 [13]
2.351	29.439	.150	29.380	.150	COOL/70
2.392	29.164	.117	29.070	.070	COOL/70
2.400	29.107	.050	29.100	.050	ABRAMS/70
2.423	28.946	.070	28.880	.070	COOL/70
2.450	28.776	.118	28.870	.070	ABRAMS/70
2.467	28.677	.203	28.500	.110	BUGG/68
2.500	28.504	.060	28.460	.060	ABRAMS/70
2.550	28.297	.060	28.300	.060	ABRAMS/70
2.600	28.129	.107	28.210	.070	ABRAMS/70
2.650	27.977	.141	28.100	.070	ABRAMS/70
2.700	27.841	.192	28.020	.070	ABRAMS/70
2.750	27.716	.134	27.830	.070	ABRAMS/70
2.800	27.601	.136	27.720	.070	ABRAMS/70
2.850	27.494	.244	27.730	.060	ABRAMS/70
2.900	27.393	.197	27.580	.060	ABRAMS/70
2.950	27.295	.224	27.510	.060	ABRAMS/70
3.000	27.198	.230	27.420	.060	ABRAMS/70
3.050	27.101	.116	27.200	.060	ABRAMS/70
3.100	27.000	.060	27.000	.060	ABRAMS/70
3.150	26.894	.105	26.980	.060	ABRAMS/70
3.200	26.781	.060	26.780	.060	ABRAMS/70
3.250	26.658	.070	26.700	.070	ABRAMS/70
3.300	26.523	.216	26.730	.060	ABRAMS/70
3.500	25.925	1.411	27.150	.700	DIDDENS/63 [15]
3.710	25.569	1.557	26.900	.700	DIDDENS/63
3.920	25.185	1.071	25.950	.750	DIDDENS/63
3.980	25.108	.700	25.400	.700	COOK/61
4.000	25.083	3.650	28.200	1.900	BAKER/63 [16]
4.130	24.941	.600	25.140	.600	DIDDENS/63
4.340	24.762	1.259	25.840	.650	DIDDENS/63
4.760	24.549	.700	24.740	.700	DIDDENS/63
5.180	24.438	.700	24.030	.700	DIDDENS/63
5.500	24.355	.800	24.300	.800	BAKER/63

TABLE 2

TOTAL CROSS SECTIONS FOR K+P BETWEEN 0.145 AND 6. GEV/C

PLAB (GEV/C)	SIGMA+ CALC. (MB)	ERROR CALC. (MB)	SIGMA+ EXP. (MB)	ERROR EXP. (MB)	REFERENCE	
.145	11.497	.800	11.800	.800	CAMERON/74	[17]
.175	11.622	.500	12.100	.500	CAMERON/74	
.205	11.758	.300	11.600	.300	CAMERON/74	
.235	11.902	.300	11.800	.300	CAMERON/74	
.265	12.049	.540	11.600	.300	CAMERON/74	
.295	12.194	.300	12.300	.300	CAMERON/74	
.325	12.332	1.370	11.100	.600	CAMERON/74	
.355	12.458	1.170	11.400	.500	CAMERON/74	
.366	12.500	.855	13.280	.350	BOWEN/70	
.385	12.567	1.361	11.400	.700	CAMERON/74	
.405	12.629	.972	13.550	.310	BOWEN/70	
.417	12.661	.710	12.280	.710	CARROLL/73	[18]
.432	12.695	.500	12.600	.500	ADAMS/73	[19]
.440	12.711	.230	12.970	.230	BOWEN/70	
.444	12.718	.250	12.690	.250	CARROLL/73	
.475	12.756	.705	13.440	.170	BOWEN/70	
.479	12.758	.500	12.900	.500	ADAMS/73	
.482	12.760	.450	12.930	.450	CARROLL/73	
.500	12.762	.200	12.600	.200	CAMERON/74	
.505	12.760	.160	12.580	.160	BOWEN/70	
.506	12.759	.326	12.990	.230	BOWEN/70	
.520	12.749	.130	12.200	.130	GOLDHABER/62	[20]
.525	12.743	.500	12.300	.500	ADAMS/73	
.536	12.727	.427	12.350	.200	BOWEN/70	
.560	12.675	.252	12.440	.090	CARROLL/73	
.566	12.658	.436	13.050	.190	BOWEN/70	
.569	12.648	.900	12.700	.900	BOWEN/73	[21]
.592	12.565	2.182	14.360	1.240	BUGG/68	
.593	12.560	.950	13.050	.950	BOWEN/73	
.596	12.547	.437	12.940	.190	BOWEN/70	
.611	12.476	.180	12.320	.090	CARROLL/73	
.618	12.440	.650	12.650	.650	BOWEN/73	
.620	12.429	.490	12.910	.490	BUGG/68	
.627	12.389	.140	12.330	.140	BOWEN/70	
.643	12.288	.850	12.500	.850	BOWEN/73	
.644	12.281	.768	12.880	.480	BUGG/68	
.657	12.190	.200	12.200	.200	BOWEN/70	
.666	12.121	.090	12.210	.090	CARROLL/73	
.668	12.105	1.039	13.100	.300	BOWEN/73	
.686	11.993	.620	11.400	.180	BOWEN/70	
.698	11.952	.750	12.600	.750	BOWEN/73	
.713	11.938	.871	11.140	.350	BUGG/68	
.717	11.942	.840	11.140	.250	BOWEN/70	
.719	11.945	.315	12.250	.080	CARROLL/73	
.727	11.964	.629	12.450	.400	BOWEN/73	
.757	12.151	.640	12.650	.400	BOWEN/73	
.768	12.265	.286	12.540	.080	BUGG/68	
.770	12.276	.949	13.000	.949	COOL/70	
.786	12.502	.300	12.800	.300	BOWEN/73	
.795	12.629	.171	12.760	.110	CARROLL/73	
.817	12.939	.090	13.200	.300	BOWEN/73	
.820	12.981	.134	13.080	.090	CARROLL/73	
.823	13.024	.250	12.970	.250	BUGG/68	
.846	13.351	.300	13.450	.300	BOWEN/73	
.847	13.365	.159	13.250	.110	CARROLL/73	
.864	13.610	.466	13.210	.240	BUGG/68	
.872	13.726	.268	13.470	.080	CARROLL/73	
.891	14.006	.540	14.390	.380	COOL/70	
.897	14.096	.110	14.050	.110	CARROLL/73	
.900	14.141	1.254	13.100	.700	GIACOMELLI/70	[22]
.904	14.201	.190	14.230	.190	BUGG/68	
.916	14.385	.300	14.200	.300	BOWEN/73	
.924	14.509	.114	14.590	.080	CARROLL/73	
.938	14.730	.210	14.590	.210	BUGG/68	
.942	14.794	.832	15.570	.300	COOL/70	
.951	14.940	.110	14.860	.110	CARROLL/73	
.969	15.239	.449	15.630	.220	BLAND/68	[23]
.970	15.256	.671	15.400	.671	COOK /61	
.977	15.376	.165	15.520	.080	CARROLL/73	
.985	15.515	.769	16.200	.350	BOWEN/73	
.992	15.638	.428	15.970	.270	COOL/70	
1.020	16.153	.300	16.100	.300	BOWEN/73	
1.021	16.172	.100	16.110	.100	CARROLL/73	
1.029	16.326	.658	15.690	.170	BUGG/68	
1.043	16.603	.826	17.390	.250	COOL/70	
1.055	16.849	.300	16.950	.300	BOWEN/73	

TABLE 2 (CID)

PLAB (GEV/C)	SIGMA+ CALC. (MB)	ERROR CALC. (MB)	SIGMA+ EXP. (MB)	ERROR EXP. (MB)	REFERENCE
1.060	16.953	.880	16.400	.880	GIACOMELLI/70
1.066	17.081	.090	17.030	.090	CARROLL/73
1.084	17.261	.279	17.040	.170	BUGG/68
1.090	17.345	.300	17.600	.300	BOWEN/73
1.094	17.399	.375	17.120	.250	COOL/70
1.125	17.767	.300	17.550	.300	BOWEN/73
1.130	17.819	1.050	18.080	1.050	GIACOMELLI/70
1.140	17.914	.170	18.020	.170	BUGG/68
1.144	17.950	.150	18.090	.150	COOL/70
1.160	18.080	.350	17.950	.350	BOWEN/73
1.170	18.150	.671	18.100	.671	COOK/61
1.189	18.261	.263	18.060	.170	BUGG/68
1.194	18.285	.221	18.470	.120	COOL/61
1.210	18.350	.900	18.580	.900	GIACOMELLI/70
1.238	18.419	.353	18.110	.170	BUGG/68
1.245	18.428	.120	18.540	.120	COOL/70
1.250	18.432	2.508	20.710	1.050	GIACOMELLI/70
1.293	18.403	.150	18.440	.150	BUGG/68
1.295	18.399	.236	18.610	.110	COOL/70
1.300	18.389	.949	17.900	.949	COOK/61
1.320	18.350	1.344	19.330	.920	GIACOMELLI/70
1.345	18.303	.170	18.440	.100	COOL/70
1.347	18.299	.160	18.270	.160	BUGG/68
1.360	18.240	.840	18.640	.840	GIACOMELLI/70
1.395	18.214	.100	18.270	.100	COOL/70
1.408	18.192	.262	17.970	.140	BUGG/68
1.440	18.140	.762	18.100	.762	COOK/61
1.445	18.132	.100	18.040	.100	COOL/70
1.468	18.097	.160	17.940	.160	BUGG/68
1.480	18.079	.100	18.040	.100	GIACOMELLI/70
1.495	18.057	.156	17.930	.090	COOL/70
1.550	17.982	.350	17.700	.208	ABRAMS/70
1.563	17.965	.336	17.660	.140	BUGG/68
1.596	17.925	.197	17.750	.090	COOL/70
1.600	17.920	.292	17.710	.203	ABRAMS/70
1.646	17.869	.060	17.860	.060	COOL/70
1.654	17.860	.900	18.300	.900	BARROWES/59 [24]
1.690	17.824	.671	17.500	.671	COOK/61
1.696	17.818	.060	17.850	.060	COOL/70
1.700	17.814	.195	17.730	.195	ABRAMS/70
1.746	17.772	.060	17.800	.060	COOL/70
1.750	17.769	.192	17.830	.192	ABRAMS/70
1.796	17.732	.091	17.800	.060	COOL/70
1.800	17.728	.325	17.980	.206	ABRAMS/70
1.850	17.693	.191	17.770	.191	ABRAMS/70
1.877	17.675	2.317	15.500	.800	BARROWES/59
1.896	17.664	.158	17.810	.060	COOL/70
1.900	17.661	.191	17.790	.191	ABRAMS/70
1.945	17.636	.277	17.410	.160	BUGG/68
1.950	17.634	.191	17.750	.191	ABRAMS/70
1.970	17.624	.880	16.900	.500	COOK/61
1.996	17.611	.080	17.600	.080	COOL/70
2.000	17.609	.186	17.630	.186	ABRAMS/70
2.050	17.589	.191	17.720	.191	ABRAMS/70
2.096	17.572	.080	17.510	.080	COOL/70
2.100	17.570	.189	17.560	.189	ABRAMS/70
2.150	17.555	.189	17.570	.189	ABRAMS/70
2.196	17.542	.080	17.540	.080	COOL/70
2.200	17.541	.189	17.600	.189	ABRAMS/70
2.260	17.526	.589	17.100	.589	COOK/61
2.300	17.518	.184	17.440	.184	ABRAMS/70
2.350	17.508	.180	17.520	.180	ABRAMS/70
2.396	17.499	.080	17.550	.080	COOL/70
2.400	17.498	.186	17.560	.186	ABRAMS/70
2.450	17.489	.179	17.480	.179	ABRAMS/70
2.473	17.485	.264	17.250	.120	ABRAMS/70
2.500	17.479	.182	17.490	.182	ABRAMS/70
2.550	17.469	.179	17.440	.179	ABRAMS/70
2.600	17.458	.180	17.500	.180	ABRAMS/70
2.650	17.445	.179	17.470	.179	ABRAMS/70
2.700	17.430	.179	17.410	.179	ABRAMS/70
2.750	17.415	.179	17.410	.179	ABRAMS/70
2.800	17.393	.179	17.400	.179	ABRAMS/70
2.830	17.380	.896	16.700	.583	COOK/61
2.850	17.371	.178	17.300	.178	ABRAMS/70
2.900	17.345	.176	17.340	.176	ABRAMS/70
2.950	17.314	.178	17.300	.178	ABRAMS/70
3.000	17.280	.176	17.190	.176	ABRAMS/70
3.050	17.241	.176	17.140	.176	ABRAMS/70
3.100	17.197	.178	17.080	.178	ABRAMS/70
3.150	17.147	.176	17.150	.176	ABRAMS/70
3.200	17.092	.174	17.130	.174	ABRAMS/70
3.250	17.030	.174	17.130	.174	ABRAMS/70
3.300	16.961	.250	17.140	.174	ABRAMS/70
3.460	16.723	1.992	15.000	1.000	VOVENKO/62 [25]
3.700	16.547	1.000	17.500	1.000	VOVENKO/62
4.000	16.474	1.200	17.600	1.200	BAKER/63
4.350	16.545	1.614	18.000	.700	VOVENKO/62
4.750	16.758	2.830	19.500	.700	VOVENKO/62

TABLE 3

CROSSING-ODD COMBINATION BETYEEN 6. AHD 240. GEV/C

PLAB (GEV/C)	SIGMA(-) CALC. (MB)	ERROR CALC. (MB)	SIGMA(-) EXP. (MB)	ERROR EXP. (MB)	REFERENCE
6.000	6.818	.316	7.000	.316	GALBRAITH /65 [26]
10.000	5.195	.224	5.200	.224	GALBRAITH/65
12.000	4.713	.470	4.300	.224	GALBRAITH/65
14.000	4.340	.329	4.100	.224	GALBRAITH/65
15.000	4.184	.239	4.290	.224	DENISOV/73-71 [27]
16.000	4.042	.412	4.300	.412	GALBRAITH/65
18.000	3.795	4.601	5.300	4.601	GALBRAITH/65
20.000	3.558	.180	3.520	.180	DENISOV/73-GALBRAITH/65
25.000	3.184	.156	3.040	.156	DENISOV/73-71
30.000	2.889	.279	3.120	.156	DENISOV/73-71
35.000	2.660	.150	2.610	.150	DENISOV/73-71
40.000	2.477	.177	2.540	.177	DENISOV/73-71
45.000	2.326	.402	2.690	.170	DENISOV/73-71
50.000	2.198	.128	2.240	.128	CARROLL/76 [28]
55.000	2.089	.319	2.320	.220	DENISOV/73-71
70.000	1.837	.094	1.860	.094	CARROLL/76
100.000	1.518	.092	1.570	.092	CARROLL/76
120.000	1.377	.092	1.450	.092	CARROLL/76
150.000	1.222	.090	1.240	.090	CARROLL/76
170.000	1.143	.177	1.010	.117	CARROLL/76
200.000	1.048	.214	.860	.102	CARROLL/76
240.000	.950	.252	.740	.230	CARROLL/76

TABLE 4

CROSSING-EVEN COMBINATION BETWEEN 6. AND 240. GEV/C

PLAB (GEV/C)	SIGMA(+) CALC. (MB)	ERROR CALC. (MB)	SIGMA(+) EXP. (MB)	ERROR EXP. (MB)	REFERENCE
6.000	41.145	.316	41.000	.316	GALBRAITH/65
10.000	39.555	.332	39.800	.224	GALBRAITH/65
12.000	39.165	.347	38.900	.224	GALBRAITH/65
14.000	38.901	.224	38.900	.224	GALBRAITH/65
15.000	38.801	.239	38.910	.239	DENISOV/73-GALBRAITH/65
18.000	38.587	4.601	39.500	4.600	GALBRAITH/65
20.000	38.496	.180	38.520	.180	GALBRAITH/65
25.000	38.375	.156	38.400	.156	DENISOV/73-71
30.000	38.344	.267	38.560	.156	DENISOV/73-71
35.000	38.362	.150	38.250	.150	DENISOV/73-71
40.000	38.406	.292	38.640	.177	DENISOV/73-71
45.000	38.471	.170	38.450	.170	DENISOV/73-71
50.000	38.544	.224	38.360	.128	CARROLL/76
55.000	38.623	.220	38.660	.220	DENISOV/73-71
70.000	38.874	.094	38.900	.094	CARROLL/76
100.000	39.369	.092	39.330	.092	CARROLL/76
120.000	39.673	.092	39.730	.092	CARROLL/76
150.000	40.086	.160	39.960	.099	CARROLL/76
170.000	40.337	.117	40.290	.117	CARROLL/76
200.000	40.681	.103	40.660	.103	CARROLL/76
240.000	41.090	.252	41.300	.139	CARROLL/76

T_{sup} (FM-1)	A (FM+2)	B (FM+3)	C (FM+4)	D (FM+5)	T_{sup} (FM - 1)
.26054+00	.10862+02	.26068+02	-.14173+03	.14971+03	.52823+00
.52823+00	-.50360+03	.27491+04	-.49204+04	.29278+04	.63521+00
.63521+00	.10628+04	-.50672+04	.80428+04	-.42205+04	.71589+00
.71589+00	.21860+03	-.67831+03	.72330+03	-.25882+03	.95763+00
.95763+00	.18282+02	-.25304+02	.14820+02	-.29590+01	.16551+01
.16551+01	-.17438+02	.39480+02	-.24347+02	.49340+01	.20977+01
.20977+01	.25631+03	-.36317+03	.17292+03	-.27258+02	.23296+01
.23296+01	.87570+02	-.90817+02	.32383+02	-.37675+01	.34968+01
.34968+01	.99851+02	-.77866+02	.21963+02	-.21340+01	.36585+01
.36585+01	.14024+03	-.95142+02	.22355+02	-.17753+01	.42729+01
.42729+01	.12280+03	-.72132+02	.14451+02	-.96218+00	.50653+01
.50653+01	.10825+02	-.28130+01	.17328+00	.16366-01	.54149+01
.54149+01	-.23812+02	.11488+02	-.15650+01	.67797-01	.69610+01
.69610+01	.70548+02	-.25342+02	.31747+01	-.13277+00	.85487+01
.85487+01	.34133+02	-.84017+00	-.10625+01	.85908-01	.88830+01
.88830+01	.31413+02	-.98656+01	.11399+01	-.43866-01	.97953+01
.97953+01	.78229+01	-.71355+00	.87110-02	.13402-02	.10661+02
.10661+02	.81204+01	-.11868+01	.89644-01	-.23329-02	.14408+02
.14408+02	.51122+01	-.32821+00	.13641-01	-.19161-03	.28008+02

Table 5. Best fit parameters for K-P.

T_{sup} (FM - 1)	A (FM+2)	B (FM+3)	C (FM+4)	D (FM+5)	T_{sup} (FM - 1)
.00000+00	.11194+01	.30267+00	-.14602+00	.00000+00	.17050+01
.17050+01	.31530+01	-.20827+01	.55346+00	.00000+00	.21791+01
.21791+01	.16698-01	.98228+00	-.27815+00	.39269-01	.32931+01
.32931+01	-.38512+01	.32939+01	-.61210+00	.35819-01	.45129+01
.45129+01	.22126+01	-.13156+00	.12796-01	-.42703-03	.14408+02
.14408+02	.36598+01	-.28654+00	.13194-01	-.19363-03	.28008+02

Table 6. Best fit parameters for K+P.

not only the well known relation of the problem with SU(3), and the still more basic assumption of analyticity, but there is also another point. These calculations can be performed using several different methods[29,30]; although they are equivalent (in the sense that if the analyticity assumption is correct they must lead to the same result), it may not be so because of the different weights given to different data affected by a statistical error. Thus, the consistency of the various calculations is a way of testing the mutual consistency of the data on the total cross sections, and on the real parts, and the clarification of this problem requires the use of a good set of input data.

A consistency test of this type is, e.g., provided by the socalled Haber-Schaim method[31]. This method has been used only rarely for KN dispersion relations[32] in the past, and has been reconsidered by us very recently in a work[33] where we make use of the total cross sections given in the present paper. Moreover, in the same framework, it might be interesting to reconsider some less conventional ideas and techniques, like for instance the use of inverse dispersion relations[34], and compare the corresponding results obtained with those provided by the conventional methods.

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