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A Two-Component Model for High Energy Collisions

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We study a model for high-energy collisions in which two distinct and independent mechanisms are present: a) pionization, depicted as an excitation of the meson field induced by a classical source representing the incident particles in interaction, and b) fragmentation, described as a two-stage process consisting in the incident- particle excitation and its subsequent decay. We show that the model exhibits an excellent agreement with all the ISR data on $d\sigma_{el}/dt$, σ_t, σ_{inel} and σ_{el}, and its consistency with $\omega\, d\sigma/d\vec{k}$, $<n>$ and the fragmentation cross section is discussed.

Estuda-se um modelo, para colisões a altas energias, em que atuam dois mecanismos distintos e independentes, a saber, (i) a pionização, descrita como excitação do campo mesônico induzida por uma fonte clãssica que representa as partículas incidentes em interação, e b) a fragmentação, entendida como um processo, em dois estágios, qce consiste na excitação de uma das partículas incidentes seguida por seu decaimento. Mostra-se que o modelo exibe excelente acordo com os dados de ISR para $d\sigma_{el}/dt$, σ_t, σ_{inel} e σ_{el}, e discute-se sua consistência com $\omega\, d\sigma/d\vec{k}$, $<n>$ e a seção de choque de fragmentação.

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1. INTRODUCTION

Nowadays, we have a reasonably large variety of experimental data concerning high-energy collision of hadrons and, as a consequence, some common features of these interactions have been put in evidence. The aim of the present article is to discuss a particular model based on the idea of particle production by a quantized field with a time-dependent classical source, and also fragmentation of the incident particles. A combination of these two mechanisms, which will be defined later, shows a possibility of accomodating all the main characteristics of high-energy reactions into a single scheme.

The experimental data suggest that, in high-energy collisions of hadrons, the finite extension of these particles plays an important role. This is most clearly seen in the diffraction pattern of $d\sigma_{el}/dt$, although the constancy of $\langle k_\perp \rangle$ and the approximately linear increase of $\langle k_\parallel \rangle$ with s, for the produced particles in these reactions, may also be interpreted as due to the finite and nearly constant transversal range of the interaction and to its flatter and flatter longitudinal dimension.

Eikonal models, formulated in terms of an impact parameter, constitute quite appropriate devices to implement the above idea, besides presenting another interesting feature which is the (s-channel) unitarity.

It also becomes clear[1] that, if we are interested in describing collisions at energies equal or higher than those available at ISR, a mechanism which produces increasing cross-sections must be included. That is, a purely geometrical model is not satisfactory for our purpose.

The idea of making an analogy between the multiparticle production and bremsstrahlung has been proposed a long time ago by Heisenberg[2] and, since then, many authors have developed this idea and studied several aspects of the reactions (Refs.3-5). One can conveniently formulate the problem by considering some classical extended current, which becomes effective only during a short time of interaction between the incident particles, and radiating some quantized field. This type of model predicts a series of qualitatively interesting results in the asymptotic limit when the energy (or equivalently s) goes to infinity. We could mention, for instance (see Sec.2)

a) $\sigma_t, \sigma_{el}, \sigma_{inel} \underset{s\to\infty}{\sim} \log(\log s)$, b) $\langle n\rangle \underset{s\to\infty}{\sim} \dfrac{\log s}{\log(\log s)}$,

c) constancy of $\langle k_\perp\rangle$, d) scaling: $\langle k_\parallel\rangle \underset{s\to\infty}{\sim} \sqrt{s}$, and

e) shrinking of $d\sigma_{el}/dt$.

All these properties (except the exact functional form which is not well established) are really verified at high energies, so that this model may be looked at as a possible explanation of the origin of the energy dependence of the observable quantities.

Serious difficulties arise, however, if one tries to fit the experimental data quantitatively. Although several approximations are involved in this model, such as neglecting spins, isospins and recoils of the incident particles, we feel that the discrepancies are too large. We express these difficulties as follows[6]:

A) If one fixes the parameters by fitting $d\sigma_{el}/dt$, the prediction for $\omega\, d\sigma/d\vec{k}$ becomes extremely low (by a factor of 10 - 20);

B) With the same parameters, the average multiplicity predicted by the model is too small and also increases very slowly (this last behavior is due to a rapid - more than expected - increase of σ_{inel}).

Moreover, other important features of high-energy collisions do not appear in this model, namely:

C) The model does not predict reactions of the class $a+b\to a+$anything, with a small missing mass, which appear as a narrow peak near $x=1$ in the inclusive cross section;

D) The multiplicity distribution, σ_n, is always peaked at $n=1$, contrary to the existing data.

At this point, it seems to us natural to try a model which includes explicitly the possibility of the incident-particle fragmentation, which will give a component with an approximately constant cross section, as well as the possibility of producing particles via the mechanism mentioned above. Of course, this kind of mdel retains all the attractive asymptotical properties a) - e) mentioned above, while at smaller s values the inclusion of a constant component wi'll allow σ_t, σ_{inel} and σ_{el} to increase much more slowly, and where fragmentation particles will dominate both $\omega\, d\sigma/d\vec{k}$ and $\langle n\rangle$. It is true that in doing so, we are just including as another input the property mentioned in C), without trying to obtain it from some principle. However, we can avoid, in this way, the difficulties A), B) and D), getting also a much more satisfactory agreement of the other results with the data.

A similar attempt has been done by Henyey and Sukhatme[5], based on the idea of diffractive dissociation, in a close analogy to the work by Good and Walker[7]. In this way, they have been able to achieve a considerable improvement over the simple classical source mdel, but still were unable to find a particular parametrization which resembles the important features of the data.

Here, instead, a two-component mdel is discussed in which the incident particles can either be excited with a subsequent decay (we ca11 this component fragmentation) or radiate a field in analogy with bremsstrahlung (let us cal1 this component pionization as usual, although the produced particles are not necessarily pions), or *both* of *them.*

Two-component models have been discussed by severa1 authors in connection with the multiplicity distribution and correlation among the produced particles[8]. Here, we focus our attention mstly on σ_t, σ_{inel}, σ_{el}, $d\sigma_{el}/dt$, σ_{dif} and $\omega\, d\sigma/d\vec{k}$, and by choosing a specific (eikonal) formalism, satisfying unitarity, discuss its implications on these observables.

In what fol lows, we first describe, for the sake of completeness, a simple "classical-source" model as mentioned above, showing the properties a) through e). This model is improved in Sec.3, including also fragmentation. The main output of the model which can be immediately derived will be shown in Sec.4, where a quantitative fit to the experimental data is tried. A diçcussion showing the possibility of getting the experimentally measured $\omega\, d\sigma/d\vec{k}$ as well as $\langle n\rangle$ is also presented there.

Finally, the main conclusions of the present work are summarized in the last Section, where a discussion concerning other observable quantities is also given.

2. PIONIZATION: CLASSICAL SOURCE MODEL

Let us initially describe a simple model for particle production in which the multiply-produced particles are described by a field satisfying the equation

$$(\Box + \mu^2)\phi(x) = J(x,\vec{b}) \ , \tag{2.1}$$

where $J(x,\vec{b})$ is a classical source, defined appropriately in terms of the incident particles. Here, as in all the following discussion, the spins, isotopic spins and charges both of the incident and emitted particles are neglected for simplicity. In writing the equation above, no-recoil approximation is implicitly assumed and, as the notation already indicates, an impact-parameter representation is used for the nucleons (let us consider p-p collisions to fix the ideas). The model we are describing in this Section is essentially the same discussed in Ref.4, except the way in which is formuiated.

As is well known[9], Eq. (2.1) is readily solved and the corresponding S-matrix is given by

$$S_\pi(\vec{b}) = \exp\left[i\int \frac{d\vec{k}}{\sqrt{2\omega}}\, j(k,\vec{b})\; a^\dagger_{in}(k)\right] \exp\left[i\int \frac{d\vec{k}}{\sqrt{\omega}}\, j^*(k,\vec{b})\; a_{in}(k)\right] \times$$

$$\times \exp\left[-\frac{1}{2}\int \frac{d\vec{k}}{2\omega}\, |\, j\,(k,\vec{b})\,|^2\right] \ , \tag{2.2}$$

where

$$J(x,\vec{b}) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \int \frac{d\vec{k}}{(2\pi)^{3/2}}\, j(k,\vec{b})\, e^{-ikx} \ . \tag{2.3}$$

Let us define

$$\chi_\pi(\vec{b}) = \frac{1}{2}\int \frac{d\vec{k}}{2\omega} \, | j\,(k,\vec{b})|^2 \ . \qquad (2.4)$$

Although not explicitly indicated, the source $J(x,\vec{b})$ may depend on the incident energy, as we actually assume, and so do $j(k,\vec{b})$ and $\chi_\pi(\vec{b})$. With this notation, the matrix element for transition from an incident $|\,pp\rangle$ state to a final $|pp, k_1,\ldots,k_n\rangle$ state, where $k_1,\ldots,k_n$ refer to momenta of the produced pions, reads

$$\langle pp,\, k_1,\ldots,k_n|S|pp\rangle \simeq e^{-\chi_\pi(\vec{b})} \prod_{\alpha=1}^{n} \frac{ij(k_\alpha,\vec{b})}{\sqrt{2\omega_\alpha}} \ , \qquad (2.5)$$

where the recoil of the protons has been neglected.

The corresponding cross section (except the elastic one) is written as

$$\sigma_n = \frac{1}{n!}\int d\vec{b}\; e^{-2\chi_\pi(\vec{b})} \left[2\,\chi_\pi(\vec{b})\right]^n . \qquad (2.6)$$

Inclusive cross sections $\omega\, d\sigma/d\vec{k}$ may be evaluated in a similar way by squaring Eq.(2.5), integrating it over all the momentum variables except one, and summing the integrals over n with an appropriate statistical factot. The result is

$$\omega \frac{d\sigma}{d\vec{k}} = \frac{1}{2}\int d\vec{b}\; |j(k,\vec{b})|^2 \ . \qquad (2.7)$$

For the elastic channel, we have to subtract 1 from the S-matrix element in Eq.(2.5), which gives the amplitude

$$F(s,t) = \frac{i}{2\pi}\int d\vec{b}\;\; e^{i\vec{b}.\vec{\kappa}} \left[1 - e^{-\chi_\pi(\vec{b})}\right] , \qquad (2.8)$$

in terms of which the differential cross section becomes

$$\frac{d\sigma_{el}}{dt} = \pi \, |F(s,t)|^2 \, . \tag{2.9}$$

Here, as usual, we wrote $t = -\vec{\kappa}^2$, and the integration is on a plane perpendicular to the incident direction.

From Eqs.(2.8) and (2.9), it follows by t-integration that

$$\sigma_{el}(s) = \int d\vec{b} \, \left| 1 - e^{-\chi_\pi(\vec{b})} \right|^2 . \tag{2.10}$$

On the other hand, the optical theorem gives

$$\sigma_t(s) = 2 \int d\vec{b} \, \left[1 - e^{-\chi_\pi(\vec{b})}\right] , \tag{2.11}$$

and by subtracting Eq.(2.10) from (2.11), it follows that

$$\sigma_{inel}(s) = \int d\vec{b} \, \left[1 - e^{-2\chi_\pi(\vec{b})}\right] . \tag{2.12}$$

The last equation can alternatively be derived summing a_n, given by Eq. (2.6), which shows a consistency with the unitarity requirements. The average multiplicity of pions may be calculated by using Eq.(2.6):

$$\langle n \rangle = \sum_{n=1}^{\infty} n \, \frac{\sigma_n}{\sigma_{inel}} = \frac{1}{\sigma_{inel}} \int d\vec{b} \, \left[2 \, \chi_\pi(\vec{b})\right] . \tag{2.13}$$

Now we come to the question of how to choose the source function $J(x,\vec{b})$ in Eq. (2.1). Following the bremsstrahlung idea, the final particles (neglecting the interaction among them) are emitted by the incident protons but only during their mutual interaction. One may write a certain hadronic-charge distribution $\rho_i(x)$ for each proton, which is assumed to be spherically symmetric in its own system, $\rho(r)$. Looked at from an arbitrary system moving along the collision axis (see Fig.1), such a distribution will show itself contracted and in motion with a velocity $\vec{\beta}_i$:

$$\rho_i(x) = \gamma_i \, \rho\left[\gamma_i(\vec{z} - \vec{\beta}_i t) + (\vec{r}_\perp \mp \frac{\vec{b}}{2})\right] . \qquad (2.14)$$

However, as far as particle emission is concerned, this charge distribution wi11 become effective only for a short time interval during which the collision takes plece, and if the impact parameter $\vec{b}$ is such that the particles can actually interact. We try to take this finite-range effect into account by writing

$$J(x,\vec{b}) = g|\vec{\beta}_1 - \vec{\beta}_2| \; \rho_1(x) \; \rho_2(x) \;, \qquad (2.15)$$

where g is a constant proportional to the intensity of the interaction. The factor $|\vec{\beta}_1 - \vec{\beta}_2|$ has been introduced in order to make $J(x,\vec{b})$ a scaiar with respect to Lorentz transformations along the z-axis. A similar source has been used by other authors (Refs.4,5).

Introducing the Fourier transform of the hadronic-charge density $\tilde{\rho}(u^2)$,

$$\rho_i(x) = \gamma_i \int \frac{d^3u}{(2\pi)^{3/2}} \tilde{\rho}(u^2) \; \exp\{i\vec{u}.[\vec{r}_\perp \mp \frac{\vec{b}}{2} + \gamma_i(\vec{z} - \vec{\beta}_i t)]\} \quad (2.16)$$

and calculating $j(k,\vec{b})$ from Eq. (2.3), we have, in the C.M. system,

$$j(k,\vec{b}) = \frac{1}{(2\pi)^{3/2}} \int d^4x \; e^{ikx} \; J(x,\vec{b})$$

$$= \frac{g}{\sqrt{2\pi}} \int d^2u_\perp \;\; \tilde{\rho}(u^2) \; \tilde{\rho}(u'^2) \; \exp\{-i\vec{b} \cdot [\vec{u}_\perp - (k_\perp/2)]\} \;, \qquad (2.17)$$

where

$$\vec{u} \equiv \left[\frac{1}{2\gamma}(k_\parallel + \frac{k_0}{\beta}), \; \vec{u}_\perp\right] , \quad \vec{u}' \equiv \left[\frac{1}{2\gamma}(k_\parallel - \frac{k_0}{\beta}), \; -\vec{u}_\perp + \vec{k}_\perp\right].$$

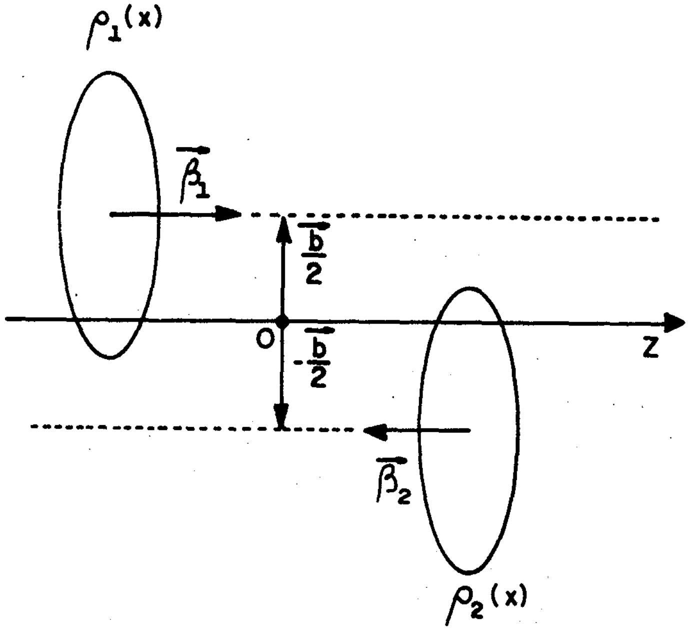

Fig.1. Definition of the coordinate system.

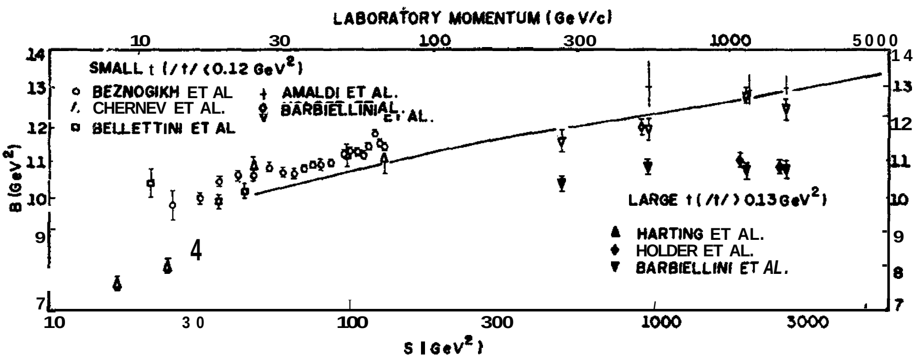

Fig.4. The slope parameter $B(s)$, at $t = 0$, predicted by our model. The experimntal data have been taken from Ref.15 where references to all the data are given.

The equation above shows that the $k_\perp$-dependence of $j(k,\vec{b})$ is more or less independent of the incident energy, while the $k_\parallel$ dependence appears through the variable $k_\parallel/\gamma$. This property, together with Eqs.(2.4) and (2.7-2.13), guarantees all the asymptotic behaviors a) - e) mentioned in the Introduction. Let us illustrate this point by assuming a simple parametrization for the hadronic-charge density, viz.

$$\tilde{\rho}(u^2) = e^{-\alpha u^2} . \tag{2.18}$$

Substituting this expression into Eq.(2.17), we get for $s \to \infty$

$$j(k,\vec{b}) = \frac{g}{2\alpha} \sqrt{\pi/2} \exp \{ - \frac{4\alpha m^2}{s} k_\parallel^2 - \frac{\alpha \vec{k}_\perp^2}{2} - \frac{\vec{b}^2}{8\alpha} \} \tag{2.19}$$

where m is the proton mass. The insertion of this into Eq. (2.7) gives

$$\omega \frac{d\sigma}{d\vec{k}} = \frac{g^2\pi^2}{4\alpha} \exp \{ - \frac{8\alpha m^2}{s} k_\parallel^2 - \alpha \vec{k}_\perp^2 \} , \tag{2.20}$$

which shows the constancy of $\langle k_\perp \rangle$ and the scaling of $\omega\, d\sigma/d\vec{k}$.

Eq. (2.19) also implies that $\chi_\pi \sim \log s$ as $s \to \infty$. This can be immediately seen by noting that the energy dependence of χ_π arises from the $k_\parallel$ integration of Eq. (2.4) which is roughly $\sim \int dk_\parallel / k_\parallel$, the upper limit being determined by the cutoff factor $|j(k,\vec{b})|^2$ which expands proportionally to $\sqrt{s}$.

Detailed integration of Eq. (2.4) gives

$$\chi_\pi(\vec{b}) \underset{s \to \infty}{\simeq} A(\log s - B) e^{-\frac{b^2}{4\alpha}} , \tag{2.21}$$

with

$$A = \frac{\pi^2 g^2}{32 a^3} \quad \text{and} \quad B = \gamma + \log(2\alpha m^2 \mu_\perp^2) , \tag{2.22}$$

where $\mu_\perp^2 = \langle \mu^2 + k_\perp^2 \rangle$; γ, in Eq. (2.22), denotes Euler's constant.

Substituting Eq. (2.21) into (2.11), we have

$$\sigma_t(s) \approx 2 \int d\vec{b} \left[1 - e^{-\chi_\pi(\vec{b})}\right] = -2 \sum_{n=1}^{\infty} \frac{1}{n!} \int d\vec{b} \left[-\chi_\pi(\vec{b})\right]^n$$

$$\simeq -2 \sum_{n=1}^{\infty} \frac{\left[-A\ (\log s - B)\right]^n}{n!} \int d\vec{b}\ e^{-\frac{nb^2}{4\alpha}}$$

$$\approx -8\alpha\pi \sum_{n=1}^{\infty} \frac{\left[-A\ (\log s - B)\right]^n}{n(n!)} ,$$

and, therefore,

$$\sigma_t(s) \simeq 8\alpha\pi \{\gamma + \log[A(\log s - B)] - \text{Ei}\ [-A(\log s - B)]\} , \qquad (2.23)$$

where Ei stands for the exponential integral.

The leading term of this equation is

$$\sigma_t(s) \underset{s \to \infty}{\longrightarrow} 8\alpha\pi \log(\log s) . \qquad (2.23a)$$

Analogously, the substitution of Eq.(2.21) into (2.10) and (2.12) gives

$$\sigma_{el}(s) \simeq 4\alpha\pi \{(\gamma - \log 2) + \log [A(\log s - B)]$$

$$- 2\ \text{Ei}\ [-A(\log s - B)] + \text{Ei}\ [-2A(\log s - B)]\} , \qquad (2.24)$$

and

$$\sigma_{inel}(s) \simeq 4\alpha\pi \{\gamma + \log[2A(\log s - B)] - \text{Ei}[-2A(\log s - B)]\} , \qquad (2.25)$$

whose leading terms are

$$\sigma_{el}(s) \xrightarrow[s \to \infty]{} 4\alpha\pi \log(\log s) , \qquad (2.24a)$$

and

$$\sigma_{inel}(s) \xrightarrow[s \to \infty]{} 4\alpha\pi \log(\log s) . \qquad (2.25a)$$

The average multiplicity $<n>$ is obtained from Eq.(2.13), by using Eqs. (2.21) and (2.25):

$$<n> = \frac{2}{\sigma_{inel}} \int d\vec{b}\; \chi(\vec{b}) = \frac{8\pi\alpha A\left[\log s - B\right]}{\sigma_{inel}} \xrightarrow[s \to \infty]{} \frac{2A \log s}{\log(\log s)} . \qquad (2.26)$$

Finally, the shrinking of $d\sigma_{el}/dt$ is evident from Eqs. (2.8) and (2.9), for $\chi_\pi(\vec{b})$ is now an increasing function of s.

3. THE TWO COMPONENT MODEL

In the preceding Sectiori, a simple classical source model has been described, which exhibits several interesting asymptotic properties. However, as already mentioned in Introduction ((A) -(D)), it meets serious difficulties as soon as a quantitative fit is attempted[6]. Here, we present a possible impruvement to that model, which maintains all of its desirable features, while avoiding the aforementioned difficulties.

Let us suppose that the high-energy collisions between two particles occur through either of the two following mechanisms, or both of them (let us consider pp collisions as in the preceding Section):

i) Excitation of one or both of the incident particles, which decay subsequently. We call this process fragmentation and assume it "independent" of the energy. ii) Particles (or clusters) may be irradiated like in bremsstrahlung. We call this component pionization, and it will exhibit an energy dependence.

These two mechanisms are assumed to be independent one from the other, and this is expressed by writing the S-matrix in the product form

$$S = S_1 \, S_2 \, S_\pi \quad , \tag{3.1}$$

where S_1, S_2 and S_π comute. Here, S_i corresponds to the fragmentation of the i-th incident particle, S_π to pionization, the impact parameter of the incident particles being kept fixed.

Physically, independence of the two mechanisms means first that pionization causes negligible changes on the incident particles so that fragmentation, which we are assuming independent of the incident energy in the sense of "limiting fragmentation hypothesis" (Ref.10), occurs as if no particle has been emitted. Secondly, it means that during the short time of interaction, the hadronic-charge distribution remains constant, even when there is some excitation of the incident particles and, consequently, the source responsible for the pionization stays the same regardless of whether the incident particles suffer fragmentation or not. Also, no-recoil approximation is implicit in using fixed impact parameters for the protons.

The unitarity of the S-matrix and the above assumption require that each factor S_i and S_π be unitary. This condition leads, for instance, to the following useful sum rule which will be employed later:

$$\langle p|S_i^\dagger S_i|p\rangle = \sum_f |\langle f|S_i|p\rangle|^2 = 1 \qquad (i = 1,2) \ , \tag{3.2}$$

where $|p\rangle$ represents the incident proton state and the sum goes overall possible final states, including the elastic channel.

The pionization is described in the way already discussed in the preceding Section, where the source function $J(x,\vec{b})$, in Eq.(2.1), is now assumed to be independent of the proton fragmentation, in accordance with the discussion above. Using the same notation as in Sec. 2, the matrix element for a transition from an incident two-proton state $|pp\rangle$ to a final state $|f_1,f_2;k_1,\ldots,k_n\rangle$, where f_1,f_2 refer to the incident proton fragmentation, and $k_1, \ldots, k_n$ denote the momenta of the pionization particles, reads

$$\langle f_1, f_2; k_1, \ldots, k_n | S | pp \rangle$$

$$\simeq \langle f_1 | S_1 | p \rangle \langle f_2 | S_2 | p \rangle \, e^{-\chi_\pi(\vec{b})} \prod_{\alpha=1}^{n} \frac{i j(k_\alpha, \vec{b})}{\sqrt{2\omega_\alpha}} \tag{3.3}$$

The corresponding cross section is written

$$\sigma_n(f_1, f_2) = \frac{1}{n!} \int d\vec{b} \, e^{-2\chi_\pi(\vec{b})} \, (2\chi_\pi(\vec{b}))^n \, |\langle f_1 | S_1 | p \rangle \langle f_2 | S_2 | p \rangle|^2 . \tag{3.4}$$

If we look at the production of n pionization particles, without worrying about what happens with the incident protons, we have to sum Eq.(3.4) over f_1 and f_2, recovering, on account of Eq.(3.2), the familiar expression already given as Eq.(2.6). In an entirely similar way, contributions to inclusive cross-section coming from pionization is shown to be given by Eq.(2.7).

The elastic amplitude, Eq.(2.8) is modified to

$$F(s,t) = \frac{i}{2\pi} \int d\vec{b} \, e^{i\vec{b}.\vec{\kappa}} \left[1 - e^{-\chi_\pi} \langle p | S_1 | p \rangle \langle p | S_2 | p \rangle \right] , \tag{3.5}$$

where an additional factor representing the beam attenuation due to fragmentation does appear now. Analogous change in Eqs. (2.10) -(2.12) gives

$$\sigma_{el}(s) = \int d\vec{b} \, |1 - e^{-\chi_\pi} \langle p | S_1 | p \rangle \langle p | S_2 | p \rangle|^2 , \tag{3.6}$$

$$\sigma_t(s) = 2 \int d\vec{b} \left[1 - e^{-\chi_\pi} |\langle p | S_1 | p \rangle \langle p | S_2 | p \rangle|^2 \right] , \tag{3.7}$$

and

$$\sigma_{inel}(s) = \int d\vec{b} \left[1 - e^{-2\chi_\pi} |\langle p | S_1 | p \rangle \langle p | S_2 | p \rangle|^2 \right] . \tag{3.8}$$

It is also easily seen that Eq. (2.13) corresponds now to the contribution from pionization to the average pion multiplicity, provided σ_{inel} is appropriately reinterpreted as given by Eq. (3.8).

Let us now consider a pure fragmentation, i.e., an inelastic process in which no pionization particle is emitted. We have for single fragmentation, by Fourier-transforming Eq.(3.3) with $n=0$ and taking one of the f_i equal to p,

$$G(f_1;s,t) \simeq -\frac{i}{2\pi}\int d\vec{b}\, e^{i\vec{b}.\vec{K}}\, e^{-\chi_\pi} <f_1|S_1|p> <p|S_2|p> \qquad (3.9)$$

and

$$\frac{d\sigma(f_1)}{dt} = \pi\ |G(f_1;s,t)|^2\ , \qquad (3.10)$$

with a similar result for $d\sigma(f_2)/dt$. Here, and in the following equations, we neglect the small shift in the t-threshold.

In a corresponding way, the double-fragmentation amplitude reads

$$G(f_1,f_2;s,t) \simeq -\frac{i}{2\pi}\int d\vec{b}\, e^{i\vec{b}.\vec{K}}\, e^{-\chi_\pi} <f_1|S_1|p> <f_2|S_2|p>\ , \qquad (3.11)$$

and the cross section

$$\frac{d}{dt}\sigma(f_1,f_2) = \pi\ |G(f_1,f_2;s,t)|^2 . \qquad (3.12)$$

The integration of Eqs.(3.10) and (3.12) over t gives us

$$\sigma(f_1) \simeq \int d\vec{b}\, e^{-2\chi_\pi}\ |<f_1|S_1|\ > <p|S_2|p>|^2\ , \qquad (3.13)$$

and

$$\sigma(f_1,f_2) \simeq \int d\vec{b}\, e^{-2\chi_\pi}\ |<f_1|S_1|p> <f_2|S_2|p>|^2\ . \qquad (3.14)$$

In actual problems, however, one is often more interested in a sum over several states in the mass range from M to $M + dM$, and not in a transition into a single state specified by f_i. As a matter of fact, what the states f_i are is often not clear in actual problems. Thus, we assume the matrix element $<f_i|S_i|p>$ a function of mass M only (and so are $G(f_i; s, t)$ $G(f_1, f_2; s, t)$) and, by introducing a density of states $n(M)$, write

$$\frac{d^2\sigma}{dt\ dM_i} \simeq \pi\ |G(M_i;s,t)|^2\ n(M_i) \tag{3.10a}$$

and

$$\frac{d^3\sigma}{dt\ dM_1\ dM_2} \simeq \pi\ |G(M_1,M_2;s,t)|^2\ n(M_1)\ n(M_2)\ . \tag{3.12a}$$

The corresponding formulas for Eqs.(3.13) and (3.14) are

$$\frac{d\sigma}{dM_1} \simeq \int d\vec{b}\ e^{-2\chi_\pi} n(M_1)\,|<M_1|S_1|p><p|S_2|p>|^2, \tag{3.13a}$$

and

$$\frac{d^2\sigma}{dM_1 dM_2} \simeq \int d\vec{b}\ e^{-2\chi_\pi}\ n(M_1)\,|<M_1|S_1|p>|^2\ n(M_2)\ \ |<M_2|S_2|p>|^2\ . \tag{3.14a}$$

The total single- and double-fragmentation cross-sections are then, on account of Eq.(3.2),

$$\sigma^1 = \sigma^2 = \int d\vec{b}\ e^{-2\chi_\pi}\ |<p|S_2|p>|^2\ \int dM_1\ n(M_1)\,|<M_1|S_1|p>|^2$$

$$= \int d\vec{b}\ e^{-2\chi_\pi}\ |<p|S_2|p>|^2\ \left[1 - |<p|S_1|p>|^2\right] \tag{3.15}$$

and

$$\sigma^{1,2} = \int d\vec{b}\ e^{-2\chi_\pi}\ \left[1 - |<p|S_2|p>|^2\right]\ \left[1 - |<p|S_1|p>|^2\right]\ . \tag{3.16}$$

In the following, the parametrization

$$|\langle p|S_1|p\rangle|^2 = |\langle p|S_2|p\rangle|^2 = e^{-2\chi_f(\vec{b})} \tag{3.17}$$

will be adopted. A particular choice of χ_f for a quantitative comparison with the experimental data will be considered in the next Section. Here, we argue that χ_f is expected to be approximately proportional to $\langle F^2\rangle$, the two-dimensional Fourier transform of the square of the proton form factor. This follows if we accept the Chou-Yang model[11] as valid for fragmentation. At intermediate energies, bremsstrahlung is still negligible ($p_{lab} \sim 20$ Gev/c), but high enough to allow a geometrical description of collisions, the above assumption may be approximately satisfied, what can be inferred from the excellent results obtained therein[11].

Introducing the above notation, we summarize here some of the formulas derived above, which will be employed in the next Section. That is, Eqs. (3.5)-(3.8), (3.15) and (3.16), may be rewritten as

$$F(s,t) = \frac{i}{2\pi}\int d\vec{b}\, e^{i\vec{b}.\vec{K}}\left[1 - e^{-\chi_\pi - 2\chi_f}\right], \tag{3.18a}$$

$$\sigma_{el}(s) = \int d\vec{b}\, \left|1 - e^{-\chi_\pi - 2\chi_f}\right|^2 , \tag{3.18b}$$

$$\sigma_t(s) = 2\int d\vec{b}\, \left[1 - e^{-\chi_\pi - 2\chi_f}\right] , \tag{3.18c}$$

$$\sigma_{inel}(s) = \int d\vec{b}\, \left[1 - e^{-2(\chi_\pi + 2\chi_f)}\right] , \tag{3.18d}$$

$$\sigma^1 = \sigma^2 = \int d\vec{b}\, e^{-2(\chi_\pi + \chi_f)}\left[1 - e^{-2\chi_f}\right] , \tag{3.18e}$$

$$\sigma^{1,2} = \int d\vec{b}\, e^{-2\chi_\pi}\left[1 - e^{-\chi_f}\right]^2 . \tag{3.18f}$$

In Eqs. (3.18), χ_f is assumed to be constant (for sufficiently high energy), whereas χ_π is given by Eq. (2.21) and increases logarithmically with s.

4. COMPARISON WITH EXPERIMENTS

The results obtained in the preceding Section are now compared with the experimental data.

First, we notice that there exist certain quantities, like those given by Eqs. (3.18), which depend only on χ_f and χ_π. These quantites can easily be compared with the data and this will be done explicitly in the present Section.

Moreover, there are other quantities the computation of which requires a more detailed knowledge of fragmentation, such as the off-diagonal elements of S_i, the state density $n(M)$ and the momentum distribution of the decay products of these states. The cross sections $d^2\sigma/dtdM$, $d^3\sigma/dtdM_1dM_2$, $d\sigma/dM$ and $d^2\sigma/dM_1dM_2$, given by Eqs. (3.10a)-(3.14a), as well as $\omega\, d\sigma/d\vec{k}$, a_n and $<n>$, are such observables. An attempt to calculate $d^2\sigma/dMdt$ has been made in one of our works[••], though restricted to the small-mass region. As expected, the absorption effect, which is embodied through the exponential factor $\exp(-\chi_f)$, gives more peripheral impact-parameter amplitude (profile function) or equivalently shrinking of the forward peak as compared to the elastic channel. In this paper, however, we are not going to rediscuss or improve the results previously obtained, but will simply present a discussion on the possibility of reconciling the data on $\omega\, d\sigma/d\vec{k}$ and $<n>$ with our scheme.

Let us begin by fixing the function χ_f. As discussed in the previous Section, we expected that χ_f be approximately proportional to $<F^2>$, and constant with energy. The purpose of this paper is not to get the "best-fit" of the experimental data, but to verify whether a simple mechanism as discussed here allows one to reproduce *all* the essential features of high-energy collisions. Thus, we are not interested in determining the exact form of χ_f, nor $\rho(x)$ for pionization. We prefer, instead, to parametrize them in a way which allows of an easy computational manipulation. We write

$$\chi_f = Ce^{-\beta b^2} , \quad (4.1)$$

with C and β two adjustable parameters. A comparison with $\langle F^2 \rangle$ gives an estimate of β, and in the following calculation we take

$$\frac{1}{\beta} = 13(\mathrm{GeV}/c)^{-2} . \quad (4.2)$$

This value for β corresponds to an average in the interval $0 < \kappa^2 < 0.8$ GeV^2, and, as a consequence of this parametrization, we do not expect a good agreement of $d\sigma_{el}/dt$ in the large-t region (see Fig.2).

The parameter C in Eq.(4.1) could be evaluated by using, for instance, data on $d\sigma_{el}/dt$ at intermediate energies (p ~ 20 GeV/c), where, according to our description, pionization is not important yet. We, however, choose slightly smaller value for C, considering the overall agreement of the result with the data, namely,

$$C = 0,5 . \quad (4.3)$$

Once χ_f is fixed, it remains only to determine two parameters, for instance g and a, in Eqs. (2.21) and (2.22), in order that a numerical comparison of the quantities given by Eqs.(3.18) with experiments can be carried out. We do this by fitting $d\sigma_{el}/dt$ for $-t<0.4$ $(\mathrm{GeV}/c)^2$, at the highest energy where such data are available. As is known, $d\sigma_{el}/dt$ at these energies, and at such t intervals, shows two distinct components with different slope parameters (see Fig.2). In our description, the wider component corresponds roughly to fragmentation, whereas the narrower one is due to pionization. This interpretation is consistent with the experimental evidence that the large-t part is very little energy dependent, while the small-t peak shrinks continuously with energy (see Fig. 3).

At s = 2809 GeV^2, a good fit is obtained with

$$\alpha = 8.5 \ (\mathrm{GeV}/c)^{-2} , \quad x = A(\log s - B) = 0.290 , \quad (4.4)$$

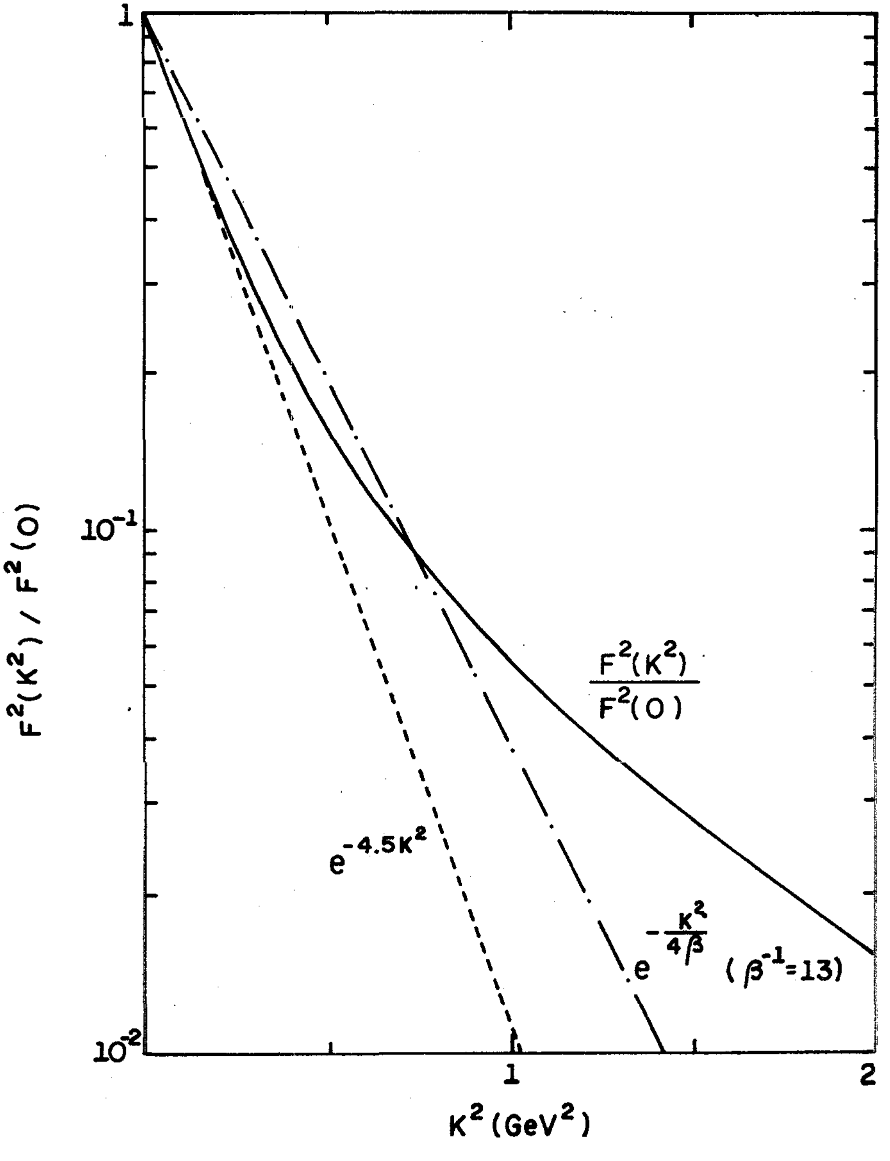

Fig.2. The square of the eletromagnetic form factor as a function of κ^2. The approximate curve used in our calculation is also shown (broken line). The dotted line is the exponential extrapolation of $F^2(\kappa^2)$ at small κ^2.

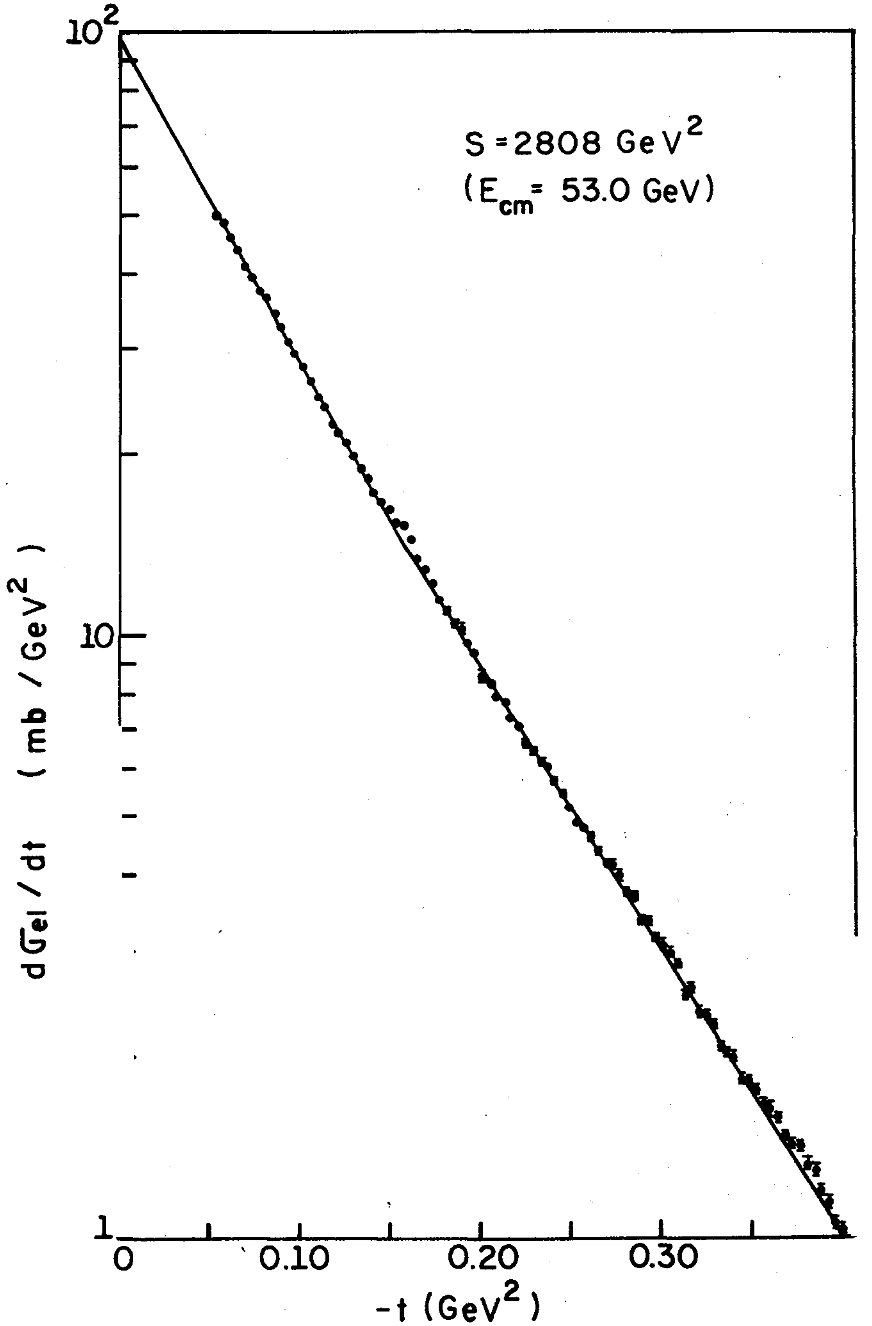

Fig.3. $d\sigma_{el}/dt$ calculated at s = 2808 Gev^2 compared with the experimental data[15].

from which, using Eq.(2.22), we find

$$A = 4.44 \times 10^{-2} \ , \ B = 1.41 \quad . \tag{4.5}$$

Now, we are ready to compute χ_π at any energy value and, together with χ_f, calculate all the observables listed in the RHS of Eqs.(3.18). In Fig.3, the calculated $d\sigma_{el}/dt$, at s = 2809 GeV^2, is shown together with the experimental values. At other s values, we have similar curves and the agreement is excelient for all the ISR data. Fig.4 shows the slope parameter at t=0. The total, inelastic, and elastic cross sections, as functions of s are plotted in Fig.5.

In spite of a very simple parametrization for χ_f and χ_π, it is seen that the agreement of these results with the available data is more than satisfactory in all the ISR energy range and down to $s \simeq 200$ GeV^2. For $s < 200$ GeV^2, a deviation occurs which, in our opinion, is due essentially to the simplified choice of χ_f we have made. Looking at Fig.2, we see that, at $-t < 0.2$, a considerable devíation of F^2 from our curve does occur. If we have taken $\chi_f - \langle F^2 \rangle$, this would of course give rise to a more pronounced peak in $d\sigma_{el}/dt$ at low energies, without destroying the good agreement seen in Fig.3 for $-t \gtrsim 0.15$. On the other hand, this would evidently cause a flattening of σ_t, a_{inel} and σ_{el} as s decreases below ISR energies. At the same time, it would force a to become smaller, which is desirable, since our $\tilde{\rho}(u^2)$ in Eq.(2.18), with the a fixed above, is much narrower than $F(\kappa^2)$, although there is no a priori reason that they should coincide. In writing the source density, Eq.(2.15), we intended to consider not only the finite extension of the proton hadronic charge but also the finite range of their mutual interaction, which may be different from the former.

As to a' and $\sigma^{1,2}$, very few data are available and moreover these are frequently conflicting. Here we just mention that these quantities are slowly decreasing with s, due to the increase of χ_π and the prediction for σ_f^1 at s = 2809 GeV^2 is

$$\sigma^1 = 5.79 \text{ mb} \quad . \tag{4.6}$$

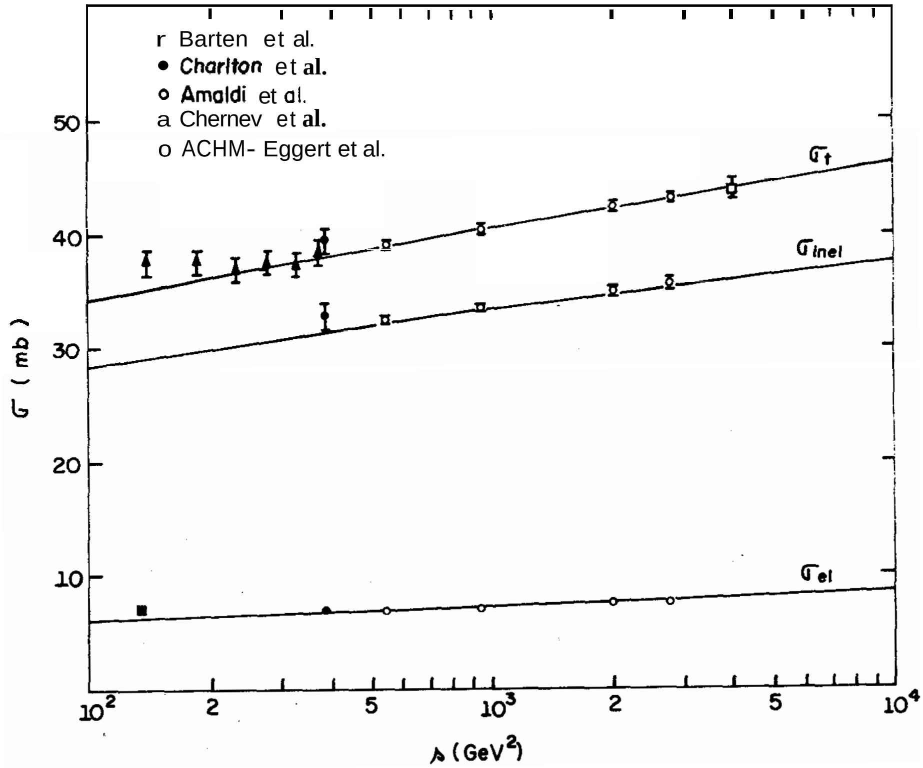

Fig.5. σ_t, σ_{inel} and σ_{el} as function of s, together with experimental data[1].

Later we will return to discuss this result, in connection with $\omega\, d\sigma/d\vec{k}$ and $\langle n\rangle$.

After having shown the most direct results predicted by the model, let us now turn to a discussion of other observables such as $\omega\, d\sigma/d\vec{k}$ and $\langle n\rangle$. As stated in the Introduction, our purpose is to construct a model which maintains the nice features of the bremsstrahlung analogy and which eliminates the difficulties A) to D) mentioned therein.

The quantities $\omega\, d\sigma/d\vec{k}$ and $\langle n\rangle$ are now given as a sum:

$$\omega \frac{d\sigma}{d\vec{k}} = P_f^1 \left(\omega \frac{d\sigma^1}{d\vec{k}}\right) + P_f^2 \left(\omega \frac{d\sigma^2}{d\vec{k}}\right) + P_\pi \left(\omega \frac{d\sigma_\pi}{d\vec{k}}\right)$$

$$= P_f \left(\omega \frac{d\sigma^1}{d\vec{k}} + \omega \frac{d\sigma^2}{d\vec{k}}\right) + P_\pi\, \omega \frac{d\sigma_\pi}{d\vec{k}} , \qquad (4.7)$$

and

$$\langle n\rangle = P_f^1 \langle n_1\rangle + P_f^2 \langle n_2\rangle + P_\pi \langle n_\pi\rangle$$

$$= 2P_f \langle n\rangle + P_\pi \langle n_\pi\rangle , \qquad (4.8)$$

where $P_f^1 = P_f^2 \equiv P_f$ are respectively the fragmentation probabilities of protons 1 and 2, and P_π the probability for pionization. Here $P_f^1 + P_f^2 + P_\pi = 2P_f + P_\pi > 1$, since mixed events are also possible and, in these cases, a double or triple counting is made.

Our choice of the parameters defining χ_π and χ_f, Eqs. (4.2) and (4.4), when considered together with Eqs. (3.18), indicates that even at the highest ISR energy, fragmentation is dominant over pionization, that is $P_f^1 > P_\pi$. Thus, in the energy range where data are available, $\omega\, d\sigma/d\vec{k}$ and $\langle n\rangle$ are much more influenced by fragmentation than pionization, and they will not be obtained by simply correcting Eqs. (2.7) and (2.13). Rather, contributions from pionization shall be regarded as small corrections to the basic results which correspond to fragmentation. Thus, a complete analysis of the problem requires a detailed description of how fragmentation takes place. This, however, will not be done here and we shall content ourselves in just presenting an argument which suggests that an appropriate account of **fragmentation** leads probably to experimentally consistent results.

Descriptions of fragmntation have been given by several authors in the past years, but here the version of Jacob and **collaborators**[13] will be considered. **In** those **papers**[13], they succeeded in reproducing both $<n>$ and $\omega\, d\sigma/d\vec{k}$ for several final particles, $\pi^{\pm}$, $K^{\pm}$, p, n,..., by **conveni-**ently defining: $\rho(M,p_{\perp}^2)$, the differentíal cross section for producing a cluster of **mass** M and transverse **momentum** $p_{\perp}$; $n(M)$, the average **multi-**plicIty for each kind of particle as a function of the cluster mass M; and finally $dD/d\vec{q}$, the normalized decay distribution. The agreernent with experiments is excellent, so we feel that with a similar paramtrization **we** can obtain, also in our mdel, where fragmntation is dominant below ISR energy, $\omega\, d\sigma/d\vec{k}$ and $<n>$ which are as good as in their calculation. Of course, the multiplicity distribution σ_n **will** now becom much **broa-**der.

In Jacob's calculation, the cross section

$$\sigma = \int\int \rho(M,p_{\perp}^2)\, dM dp_{\perp} ,$$

which gives a good **fit** for other quantities, is too large **compared** with the usually reported σ_{dif} (Ref.14). However, care must be taken **in** car-**rying** out **such** a comparison. According to our notation,

$$\sigma = \int d\vec{b}\, [1 - e^{-2\chi_f}] \qquad (4.9)$$

wouldbesuchacrosssection, whilea$_{dif}$ **should be compared with** σ^1 of **Eq.(3.18e),** or better twice this value, where an additional factor exp $\{-2(\chi_\pi + \chi_f)\}$ appears due to the beam absorption corresponding both to the **fragmentation** of the other particle, and to pionization. **In** our cal-culation, a = 12.7 mb, which is close to the **Jacob's** value (σ = 15 mb), whereas σ^1 is given by **Eq.**(4.6) which is much smaller, favouring our mo-**del.**

5. CONCLUSIONS

In this paper, a model for high-energy collisions has been studied, which includes two distinct and independent particle-production mechanisms: a) *pionization*, depicted as an excitation of the meson field by a classical source representing the incident particles in interaction, and b) *fragmentation*, described as a two-stage process consisting of an iocident-particle excitation followed by its subsequent decay. The use of eikonal approximation allowed us to write an S-matrix satisfying (s--channel) unitarity, which is convenient for studying absorption effects.

In spite of its simplicity, the model reproduces quite satisfactorily all the ISR data on $d\sigma_{el}/dt$, σ_t, σ_{inel} and σ_{el} and predicts a' which is comparable to the experimentally measured σ_{dif}. Accepting the success of fragmentation models in reproducing $\langle n \rangle$ and $\omega\, d\sigma/d\vec{k}$, it is concluded that the present scheme is also able to account for these observables.

As discussed in the preceding Section, these results may be improved further, both at lower energies and at larger t values, for $d\sigma_{el}/dt$, if we take a more realistic parametrization for χ_f.

Also, once fragmentation is well defined, it seems straightforward though laborious the extension of the model to the calculation of other quantities such as correlations and multiplicity distributions.

As to the large-k_T distribution of particles, we think that it strongly depends on the momentum-energy conservation constraints which we have not appropriately taken into account. As a consequence, we have a constant (energy independent) distribution, which is actually verified only in the small-k_T range.

Finally, we recognize that the work is of course incomplete to the extent that we simply borrowed some results on fragmentation, without working on it in more detail. This question is being studied now and will be discussed in a future publication.

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