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Regge Trajectories for Narrow Resonances

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The existence of narrow mesonic resonances is seen to be compatible with asymptotic results on Regge trajectories obtained previously on rather general grounds.

Mostra-se a compatibilidade entre a existência de ressonâncias mesônicas estreitas e comportamentos assintóticos obtidos previamente, para trajetórias de Regge mesônicas, em base a argumentos bastante gerais.

1. INTRODUCTION

In the course of the last few years, we tried to derive consequenkesof the analyticity and unitarity of the scattering amplitudes on rising Regge trajectories in an essentially model-independent frame[1]. The main result obtained was that the widths of the resonances interpolated by the trajectories grew linearly with the mass of the resonance. A crucial test for our result was thought to be the eventual discovery of a resonance at the same time massive and narrow.

Would the recent finding of the psion family[2] point, in this sense, to the breakdown of the standard analyticity[3]? In the following analysis, we show that the answer is "not necessarily".

According to an investigation of Chang and Nelson[4], the psions are consistent with a Regge classification under the $O(4)$ group, in the sense

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of Freedman and Wang[5]. The details are not relevant to our discussion, but the result that the narrow resonances sit on Regge trajectories that are linear in s is very important. As the proposed mechanisms to explain the inhibition of strong decays are not much dependent on the growth of the mass of the resonance, we will consider, for simplicity, the case of narrow resonances of very high mass. Is the existence of these particles compatible with the usual assumptions of analyticity and unitarity ?

2. THE MODEL

We start by assuming that the mesonic trajectory $\alpha(s)$ has the following properties:

a) it is analytic in the complex s-plane cut along the real axis above the physical threshold and continuous on the real axis;

b) it is real analytic, i.e., $\alpha^*(s)=\alpha(s^*)$;

c) it grows slower than an exponential in s, for $|s|\to\infty$ on the upper half-plane of the physical sheet;

d) $$\lim_{s\to\pm\infty} \frac{\alpha(s)}{(-s)^{\varepsilon}\,|\ell n(-s)|^{\beta}} = -C_{\pm}\ , \qquad (1)$$

ε and β being real numbers.

Under these conditions, the Phragmén-Lindelöf[6] theorem can be applied to the function $\alpha(s)/(-s)^{\varepsilon}|\ell n(-s)|^{\beta}$, with the consequence that $C_+ = C_- = C$, a positive constant, if the trajectory is to rise with s.

For large values of s, it follows that

$$\mathrm{Re}\,\alpha(s) = -C\cos(\pi\varepsilon)s^{\varepsilon}(\ell n\, s)^{\beta} + \beta\pi C\sin(\pi\varepsilon)s^{\varepsilon}(\ell n\, s)^{\beta-1}\ , \qquad (2)$$

$$\mathrm{Im}\,\alpha(s) = C\, s^{\varepsilon}(\ell n\, s)^{\beta-1}\left[\sin(\pi\varepsilon)\ell n\, s + \pi\beta\cos(\pi\varepsilon)\right]\ . \qquad (3)$$

The widths of the resonances interpolated by $\alpha(s)$ are

$$\Gamma(s) = \frac{\text{Im}\ \alpha(s)}{\sqrt{s}\ \text{Re}\ \alpha'(s)} \tag{4}$$

the prime denoting differentiation with respect to s. Using (2) and (3), one gets

$$\Gamma(s) = \sqrt{s}\ \text{[illegible]}\ s. \left| \sin(\pi\varepsilon)\ \ell n\ s + \beta\pi \cos(\pi\varepsilon) \right| \ .$$

$$. \left| -\varepsilon \cos(\pi\varepsilon)(\ell n\ s)^2 + \beta\ \ell n s. \left[\pi\varepsilon \sin(\pi\varepsilon) - \cos(\pi\varepsilon)\right] + \beta(\beta-1)\pi \sin(\pi\varepsilon) \right|^{-1} . \tag{5}$$

The unitarity requirement (from potential theory) of positivity of the imaginary part gives origin to the restrictions

$$\frac{1}{2} \leq \varepsilon \leq 1 \quad , \tag{6}$$

where the extreme values are included only if $\beta \neq 0$.

Assume, according to Chang and Nelson[4], that the trajectory is very close to a straight-line, taking ε to be very close to 1. Trajectories of this kind have been extensively studied and are, in our opinion, good candidates to be the "real life" trajectories that interpolate the resonances known before the psions. Their prominent feature is that, as follows from Eqs. (2-5),

$$\Gamma(s) = -\frac{\tan(\pi\varepsilon)}{\varepsilon} \sqrt{s} \quad , \tag{7}$$

that is, a width of a very general form which grows linearly with the mass and does not depend on any trajectory parameter other than ε. The problem then is: putting the psions on trajectories of this type that are almost linear means giving them widths of the same order of the widths of "ordinary" resonances. Is this an evidence that the psions lie on trajectories of different analyticity properties?

Not so. Observe, in fact, that we can get very near a linear trajectory in a different way. Take, in Eq. (1), $\varepsilon=1$ and $\beta \neq 0$. The trajectory is then, for large s, linear except for a logarithrnic factor. Frorn Eq. (5), we have now for the width,

$$\Gamma(s) = -\beta\pi \frac{\sqrt{s}}{\ln s + \beta}, \qquad (8)$$

and for the trajectory,

$$\mathrm{Im}\,\alpha(s) = -\beta C\pi s(\ln s)^{\beta-1}, \qquad (9)$$

$$\mathrm{Re}\,\alpha(s) = Cs(\ln s)^{\beta}. \qquad (10)$$

By taking, in Eq.(8), a small, negative value for β, one can see that trajectories are obtained which are essentially linear (that is, whose real parts are linear in s except for a logarithmic dependence), whose irnaginary parts are positive and srnall cornpared to the real ones, and whose asyrnptotic widths can be orders of magnitude srnaller than the widths given by Eq.(7). Recall that the pararneter ϵ can, in the context of considerably realistic models, be connected to the intercept of trajectories, and, ço, is not really free[7]. It is, therefore, really essential that narrow widths be obtained through a different mechanisrn than a clever choice of ϵ.

It is this mechanisrn that we clairn to have found

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