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Analysis of the Frequency Modulation of Double Heterostructure Junction Lasers*

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Previous analysis of the interaction of ultrasonic waves with the homojunction laser modes is extended to the case of double heterostructure lasers. For this structure the extra degree of freedom, provided by the choice of the active region thickness, allows to optimize the modulation bandwidth. It is shown that for an optimal value of the active region thickness (0.26um), a bandwidth of about 8 GHz can be obtained.

Estende-se uma analise anterior, da interação de ondas ultrassônicas com os modos de laser de homojunção, ao caso dos lasers de heteroestrutura dupla. Nesse caso, pode-se escolher a espessura da região ativa de maneira a otimizar a banda de modulação. Mostra-se que uma banda de 8GHz pode ser obtida com o valor otimizado da espessura da região ativa (0,26 um).

1. INTRODUCTION

It has been demonstrated that the variation of the dielectric constant produced by ultrasonic waves provides a means to directly modulate the optical frequency of a semiconductor laser[1-3]. In a previous paper[4], hereafter called I, a detailed analysis of this modulation method was

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carried out for homostructure lasers, then the only available lasers. In the present paper, this analysis is extended to the double heterostructure[5-7]; these lasers, besides having the known advantage of CW room temperature operation, provide an extra degree of freedom since the thickness of the active region can be arbitrarily chosen. This thickness can be optimized to maximize the modulation bandwidth, or any other desired parameter. The same perturbation method used in I is also applied here, after solving for the laser modes. Because of the spread of the optical field in the neighbouring layers, when the active region is extremely thin, the maximum modulation frequency does not increase monotonically as the active region gets thinner; a maximum is reached for a few thousand angstroms of thickness, the exact value depending on the aluminum content of the neighbouring regions.

2. ANALYSIS OF THE FREQUENCY MODULATION

Let us assume that the index of refraction can be defined as uniform in the active region with the value n_2 and also uniform and identical on the two neighbouring regions with a value n_1. Also, we shall assume these neighbouring regions to be semi-infinite, which in practice means that the optical field does not extend significantly into the substrate or the contacting layers. Thus as shown in Fig. 1 we have

$$n(x) = \begin{cases} n_1 & \text{for } |x| > d/2 \, , \\ n_2 & \text{for } |x| < d/2 \, , \end{cases} \tag{1}$$

where d is the thickness of the active region (pure GaAs region).

In the actual case, small variations of the index of refraction both inside and outside of the active region, due to temperature and carrier density inhomogeneities, are small and neglecting them will not significantly alter the conclusions of this paper.

As in I, we assume that the effect of sound waves is to add a small perturbation to the dielectric constant proportional to the instantaneous

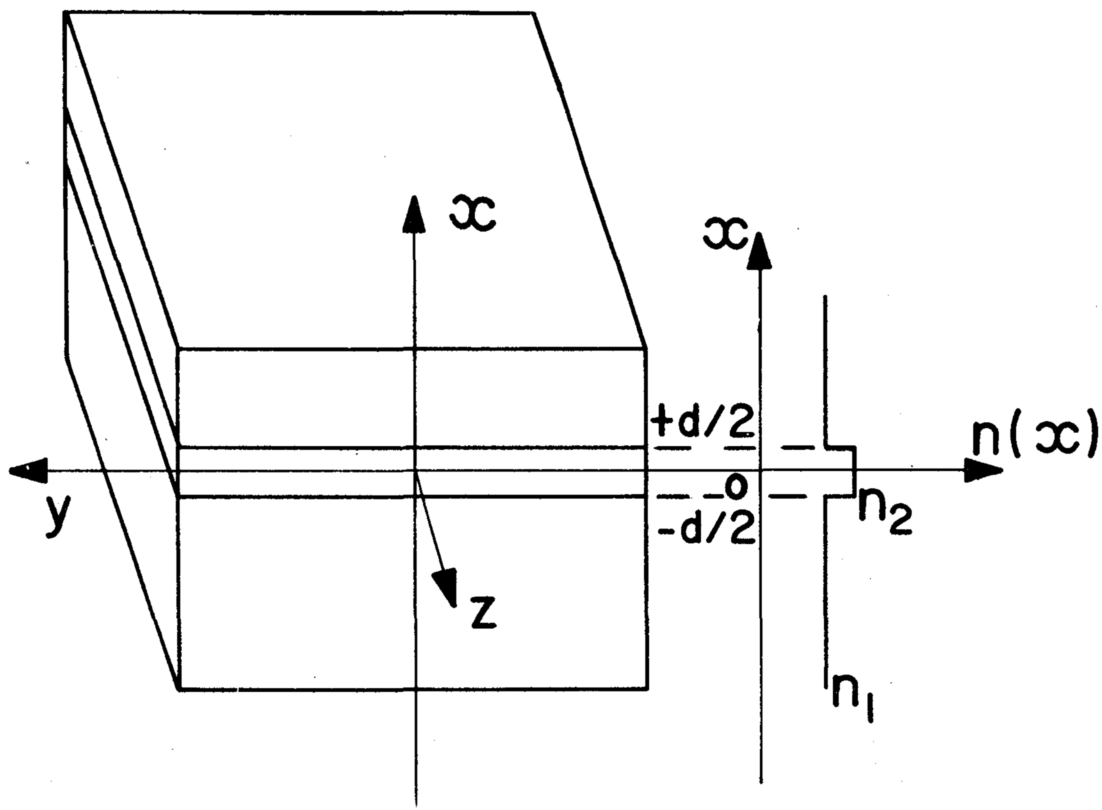

Fig.1 - Double heterostructure laser and refractive index profile along the x-direction.

sound pressure at any point of the laser. The sound waves will be assumed travelling in the x-direction, so the dielectric constant can be represented by

$$\varepsilon(x,t) = \varepsilon^{(0)}(x) + A\cos(\omega_s t + \frac{\omega_s}{c_s}x), \qquad (2)$$

where

$$\varepsilon^{(0)}(x) = (\mu/\mu_0)\varepsilon_0 n^2(x) \qquad (3)$$

and A is a constant proportional to the peak sound pressure; ε_0 is the vacuum dielectric constant, c_s the sound velocity, ω_s the angular sound frequency, and μ/μ_0 the relative magnetic permeability.

Since for any reasonable pressure $A \ll \varepsilon^{(0)}(x)$, the instantaneous shift in the optical frequency can be calculated by first order perturbation theory, assuming a quasi-static approximation. As in I, the perturbed mode frequency will be given by

$$\nu(t) = \nu_0 + A\frac{d\nu}{d\varepsilon}\,\frac{\int_{vol} E^2(\vec{x},t)\cos(\omega_s t + \frac{\omega_s}{c_s}x)\,d\vec{x}}{\int_{vol} E^2(\vec{x},t)\,d\vec{x}}, \qquad (4)$$

where $d\nu/d\varepsilon$ is the derivative of mode frequency relative to a spacially uniform variation of the dielectric constant $\varepsilon^{(0)}$, and $E(\vec{x},t)$ is the optical field of the unperturbed problem. Since the sound waves as defined in Eq.(2) do not depend on the y- and z-directions, only the x dependente of the optical field should be considered because the y and z dependences cancel out in Eq. (4).

For the unperturbed problem of theslab waveguide, we assume an electric field in the y-direction of the form

$$E(x,z,t) = E_0 X(x)\exp\{\pm jk\gamma z\}\exp\{-j\omega t\}, \qquad (5)$$

$\omega = 2\pi\nu$ being the angular frequency of the radiation; $k=\omega/c=\omega\sqrt{\varepsilon_0\mu_0}=2\pi/\lambda$ is the free space propagation constant, c the speed of light in vacuum, λ the free-space wave-length, and γ a dimensionless wave number associated with waves propagating in the z-direction.

The solutions for odd and even modes are

$$X(x) = \begin{cases} c_{1,2} \begin{Bmatrix} \cos \frac{\alpha d}{2} \\ \sin \frac{\alpha d}{2} \end{Bmatrix} \exp\{\beta(x+d/2)\} & \text{for } x \le -\frac{d}{2}, \\ c_{1,2} \begin{Bmatrix} \cos \alpha x \\ \sin \alpha x \\ \sin \alpha x \end{Bmatrix} & \text{for } |x| \le \frac{d}{2}, \\ c_{1,2} \begin{Bmatrix} \cos \frac{\alpha d}{2} \\ \sin \frac{\alpha d}{2} \end{Bmatrix} \exp\{-\beta(x-d/2)\} & \text{for } x \ge \frac{d}{} \end{cases} \qquad (6)$$

where

$$\alpha = k\left[n_2^2 - \gamma^2\right]^{1/2}, \quad \beta = k\left[\gamma^2 - n_1^2\right]^{\frac{1}{2}} = \left[(n_2^2-n_1^2)k^2-\alpha^2\right]^{1/2} \qquad (7)$$

and c, and c_2 are normalization constants.

Continuity of the field at the two dielectric interfaces gives the well known[a] eigenvalue equations of the symmetric slab waveguide:

$$\tan(\alpha d/2) = \beta/\alpha \qquad \text{for even modes}, \qquad (8a)$$

$$\cot(\alpha d/2) = -\beta/\alpha \qquad \text{for odd modes}. \qquad (8b)$$

For a given set n_1, n_2, d, and h, we can obtain the roots α_m ($m = 0,1,2,3\ldots$) by an iterative method. We verify that Eq.(8a) always has at least one solution corresponding to the fundamental mode m = 0.

Eq.(8b) will only have a solution if the dimensionless parameter $V = \frac{1}{2}\left[n_2^2 - n_1^2\right]^{1/2}.kd$ is greater or equal to $\pi/2$. This value corresponds to *the cutoff of the first odd* **mode,** *which will be guided only if* $d \geq d_c$, *where*

$$d_c = \frac{\lambda}{2\left[n_2^2 - n_1^2\right]^{1/2}} \tag{9}$$

Substituting the electric field expression given by **Eqs. (6)** *and (12) in Eq. (4), and after carrying out the integrations, we obtain the perturbed* **mode** *frequencies in terms of* α, β, *and the sound propagation* **constant** $k_s = \omega_s / c_s$:

$$\nu_{\substack{\text{even}\\ \text{odd}}}(t) = \nu_0 + \Delta\nu_{\substack{\text{even}\\ \text{odd}}} \cos \omega_s t \ , \tag{10}$$

with

$$\Delta\nu_{\text{even}} = A \frac{d\nu}{d\varepsilon} \frac{2 \dfrac{\beta d \cos u - u \sin u}{(\beta d)^2 + u^2} \cos^2 \dfrac{\alpha d}{2} + \dfrac{\sin u}{u} + \dfrac{1}{2}\left(\dfrac{\sin u^+}{u^+} - \dfrac{\sin u^-}{u^-}\right)}{2 \cos \dfrac{\alpha d}{2} \cdot \left(\dfrac{1}{\beta d} \cos \dfrac{\alpha d}{2} + \dfrac{1}{\alpha d} \sin \dfrac{\alpha d}{2}\right) + 1} , \tag{10a}$$

$$\Delta\nu_{\text{odd}} = A \frac{d\nu}{d\varepsilon} \frac{2 \dfrac{\beta d \cos u - u \sin u}{(\beta d)^2 + u^2} \sin^2 \dfrac{\alpha d}{2} + \dfrac{\sin u}{u} - \dfrac{1}{2}\left(\dfrac{\sin u^+}{u^+} - \dfrac{\sin u^-}{u^-}\right)}{2 \sin \dfrac{\alpha d}{2} \cdot \left(\dfrac{1}{\beta d} \sin \dfrac{\alpha d}{2} - \dfrac{1}{\alpha d} \cos \dfrac{\alpha d}{2}\right) + 1} , \tag{10b}$$

where we have defined $u = k_s d/2$, $u^+ = u + \alpha d$ *and* $u^- = u - \alpha d$.

We verify that when k_s *goes to zero, the limit of* $\Delta\nu$ is $A\, d\nu/d\varepsilon$, *which is the maximum frequency shift of the* **mode.**

3. APPLICATION TO GaAs Aℓ DH LASERS

We performed numerical calculation in the case of an $A\ell_x Ga_{1-x}As/GaAs/A\ell_x Ga_{1-x}As$ double heterostructure laser with x = 0.3. We used the measured values[9] of the refractive index at 1.38 eV (h = 8984 Å), n_1 = 3.38 and n_2 = 3.59. In this case the cutoff for the first order mode givenby Eq.(1'6) is d_c = 0.37 µm. This aluminum concentration quite corresponds to the maximum step of the refractive index normally used on lasers. The eigenvalues a_0 and α_1 of Eqs. (8a), and (8b), were numerically solved by computer for $0 < d \leq 1$ µm, and we dalculated the normalized shift of the optical frequency mode $\Delta\nu/A\,(d\nu/d\varepsilon)$ given by Eq.(10). Fig.2 shows the normalized optical frequency shift or peak frequency modulation vs the sound wave number k_s, or equivalently sound frequency f_s, for same values of d for the fundamental mode (m=0).

These curves show that the sound modulates effectively the lasers frequency up to very high frequencies, if the active region is somewhere between 0.2 and 0.6 ym. Physically, this corresponds to the tighter confinement of the optical field, so that up to severa1 GHz the sound pressure can still be regarded as approximately constant over the field region.

Another interesting point is the difference in shape of curves 1(d = 0.1 µm) and 3 (d = 0.6 um). They cover about the same modulation range because they correspond to optical fields approximately of same width; their different shape is caused by differences in the shape of the optical field. In the thin active region, most of the optical field is outside the region (exponential shape), while in the thick one most is in the active region (sinusoidal shape).

In Fig.3, it is represented the normalized frequency shift for the fundamental (even) and the first order (odd) modes. Two characteristics are noticeable in the figure. First, that the effective bandwidth is larger for the fundamental mode; this is to be expected because, for the same size of the active region, the first order mode spreads further, thus increasing the averaging effect over the sound pressure (Eq.4). A second characteristic is the fact that the curve crosses the abcissa axis and

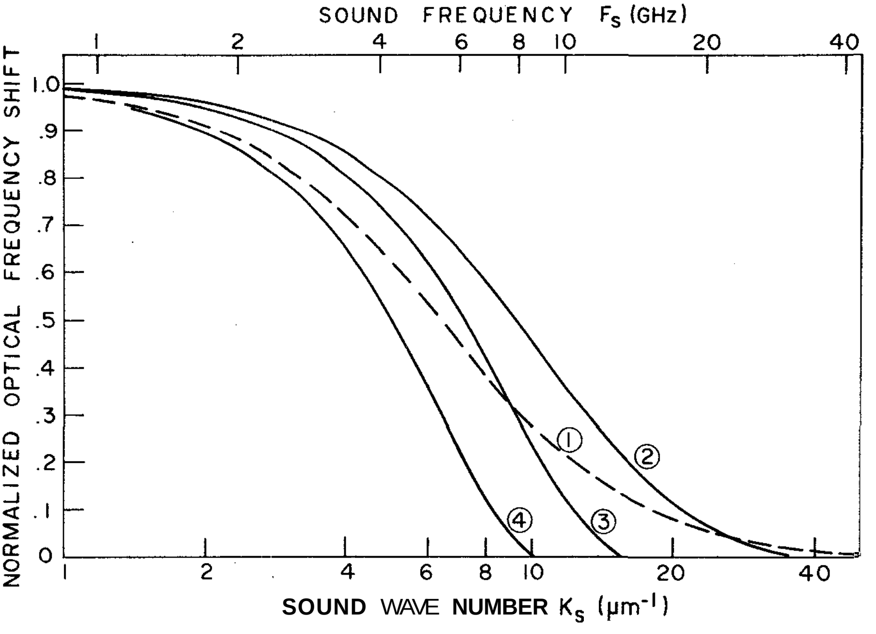

Fig.2 - Calculated optical frequency shift as a function of sound wave number, or equivalently sound frequency, for a $Al_xGa_{1-x}As/GaAs/Al_xGa_{1-x}As$ DH laser with x = 0.3, in the case of the fundamental mode ($m = 0$). For the sound frequency scale, it is assumed a sound wave propagating with the velocity c_s = 5376 m/s corresponding to a (111) junction plane. Numbers 1, 2, 3 and 4 correspond respectively to 0.1, 0.2, 0.6 and 1 µm, active region thickness.

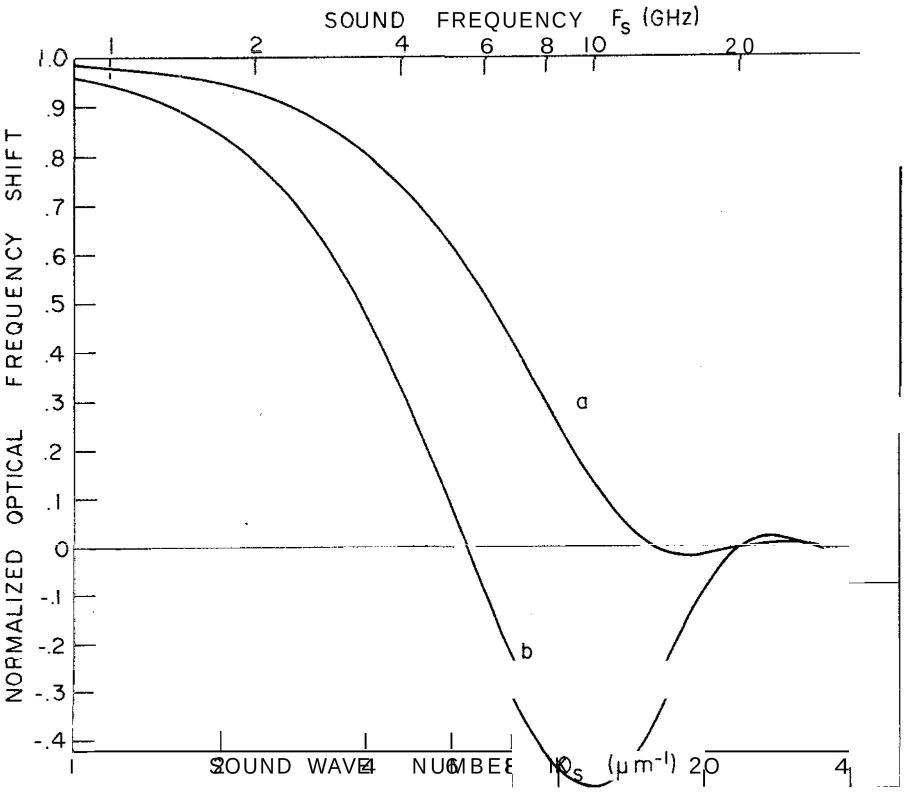

Fig.3 - Calculated optical frequency shift as a function of sound wave number or equivalently sound frequency for a $Al_xGa_{1-x}As/GaAs/Al_xGa_{1-x}As$ DH laser with x = 0.3, for d = 0.6 µm. Curve (a) corresponds to the fundamental mode (m=0), and curve (b) to the first order mode (m=1). Same assumption as in Fig.2 for the sound frequency scale.

goes negative. This means that the modulation goes to an opposite phase, that is, when the sound pressure is maximum on the center of the active region, the average pressure as seen by the modes (Eq.4) is negative because the neighbouring half wavelengths of the sound wave have a bigger effect than the one in the center. This effect is particularly large on the first order mode because the optical field is zero at the center, thus reducing the modulation effect of the pressure at that point.

On Fig.4, we have calculated the bandwidth $k_s^{(0.5)}$ vs the active region thickness for the fundamental and first order modes, where we have defined the bandwidth $k_s^{(0.5)}$ (or $f_s^{(0.5)}$) as the value of k_s (or f_s) for which the optical frequency shift is divided by two. The bandwidth shows a maximum of 9.51 μm^{-1} at d = 0.26 µm for the fundamental mode and a maximum of 3.85 μm^{-1} at d = 0.6 µm for the first order mode. For the optimized value d = 0.26 µm which is smaller than the first ordermodecutoff value, the laser will only support the fundamental mode.

Using the elastic constants of GaAs (Ref. 10), the longitudinal sound velocities are $c_s[111]$ = 5376 m/s and $c_s[100]$ = 4712 m/s for sound waves propagating in directions perpendicular to the (111) and (100) planes which generally correspond to the usual junction plane in GaAs DH lasers. With these values, we found a maximum bandwidth of about 8 GHz and 7 GHz respectively for a (111) and a (100) junction planes, in the case of the fundamental mode, which means that the orientation of the junction plane should be taken into account if one wants to optimize the bandwidth.

4. CONCLUSION

Sound modulation of a DH laser has been theoretically demonstrated tobe optimized for a certain value of the optical cavity thickness. This optimized thickness has been determined to be of the order of .0.26 µm for the given values of n_1, n_2 and λ, corresponding to available GaAs DH lasers.

The use of sound waves to frequency modulate a semiconductor laser was shown to be even more effective for double heterostructure lasers than

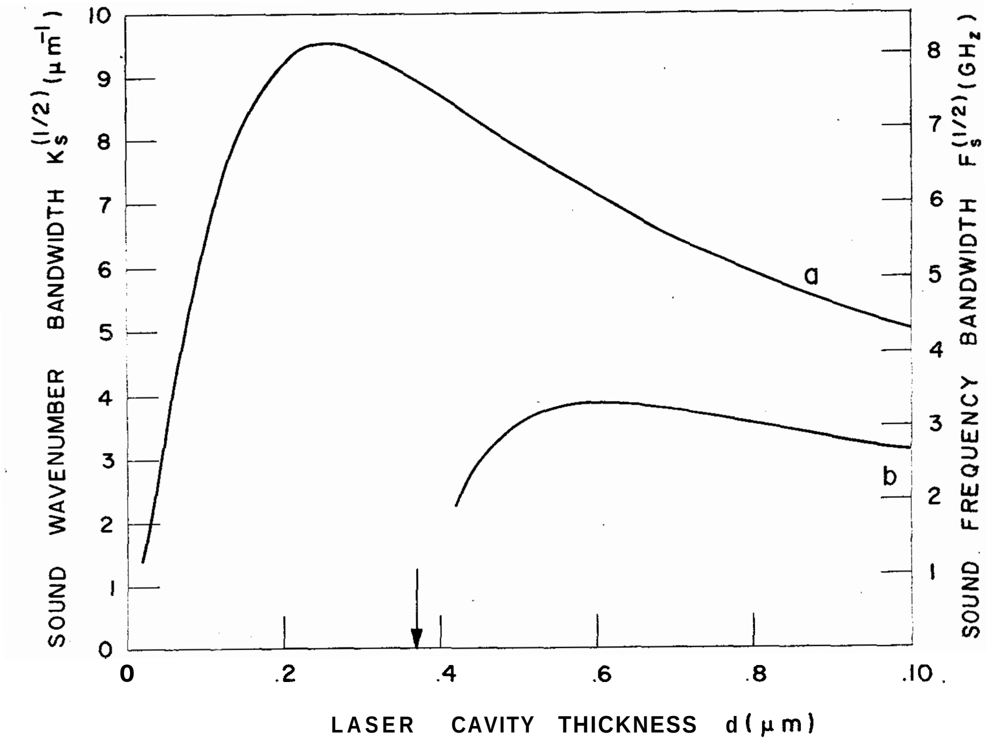

Fig.4 - Sound wave number bandwidth as a function of laser active region thickness. Curve (a) corresponds to the fundamental mode with a maximum at 0.26 ym, and curve (b) corresponds to the first order mode with a maximum at 0.60 µm. For the latter mode, the curve starts above the cutoff (0.37 µm) indicated by an arrow. The bandwidth is defined by the value for which the optical frequency shift is divided by two. Same assumption as in Fig.2 for the sound frequency scale.

for homostructure ones. The added degree of freedom allows us to optimize the laser for maximum modulation bandwidth which can reach 8 GHz for the aluminum concentration chosen in the cladding layers. For different parameters (n_1, n_2, λ), the optimal value of d can be directly determined using the optimal value of the dimensionless parameter $V=1.100$.

It should be noted that frequency rnodulation is the only method that does not couple to the "spiking" r-esonance of the laser, because it does not change neither the electron nor the photon populations[4,11]. Thus, frequencies in the GHz regions can be obtained without the distortion found in other methods of modulation.

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