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On Therre Cosmologial Models with Hoyle's Hypothesis*

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A critica1 analysis points out that the Einstein field equations have a wider conceptual content than that of Mach's principle. It is suggested that the principle of Mach signifies the determination of the cosmic thrree-geometry by the inertial term in the energy-momentum tensor and this is formulated through a scalar equation. This equation may give Whitrow's relation. An alternative scalar equation is proposedinciuding the cosmic constant. Both scalar equations furnish a possible argument for introducing the Pryce-Hoyle tensor in two cosmological models, one of which al ready publ ished (Prog. Theor. Phys) 53, 1365, 1975). The models are neither of the "big-bang" type nor oscillatory, and do not exhibit very high red-shifts. With the assumption of a time varying G and c (Lett.Nuovo Cimento 15, 595, 1976), it is possible to have a third model with zero cosmological constant and high red-shifts. For the three models considered, the cosmic pressure is perfectly determined from the field equations and the two hypotheses proposed. The pressure should not be identified with the usual matter and radiation pressure. Numerical values are presented for the third model.

Com base em uma análise crítica mostra-se que as equações de campo de Einstein possuem um conteudo conceitual mais amplo do que aquele inerente ao principio de Mach. Sugere-se que esse princípio signifique a determinação da geometria cósmica tridimensional pelo termo inercial do tensor energia-momento, e isso é formulado mediante uma equação esca-

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lar. Essa equação pode dar a relação de Whitrow. Uma equação escalar alternativa é proposta incluindo a constante cosmológica. Ambas equações escalares fornecem um possível argumento para a introdução do tensor de Pryce-Hoyle em dois modelos cosmológicos, um dos quais já publicado (Prog. Theor. Phys. 53, 1365, 1975) Nenhum dos modelos considerados podem ser do tipo "big-bang" e nem oscilatórios. Ambos modelos não apresentam "red-shifts" muito elevados. Coma hipótese de G e c variáveis com o tempo (Lett. Nuovo Cimento, 15, 595, 1976), é possível se ter um terceiro modelo com constante cosmológica zero e "red-shifts" elevados. Para os três modelos considerados, a pressão cósmica é perfeitamente determinada a partir das equações de campo e das duas hipóteses propostas. A pressão não deve ser identificada com a habitual pressão cinética e de radiação. Valores numéricos são apresentados para o terceiro modelo.

1. MACH'S PRINCIPLE AND ITS CONSEQUENCES

Two fundamental ideas are at the basis of Mach's principle as formulated in Einstein's original paper on the general theory of relativity [1], to wit:

a) Since an absolute space, independent of matter, has no physical meaning, local accelerations are referred to an inertial frame defined by the distribution of cosmic masses.

b) In order to save the validity of the principle of causality, local inertial forces - say, centrifugal or Coriolis - are effects of gravitational interaction of local accelerated objects with the cosmic background masses.

We can enumerate a set of six propositions that directly or indirectly result from Mach's principle which are the following.

1.1) THE PRINCIPLE OF GENERAL RELATIVITY

An immediate consequence of (a) is that there exists no privileged frame of reference in the universe. Thus all cosmic frames are to beviewed as identical to each other and this means that the bizarre Newtonian concept of an absolute space independent of matter should be rejected.

1.2) THE PRINCIPLE OF EQUIVALENCE

As a consequence of (b), it should be possible, by a suitable transformation of coordinates, to interchange inertial forces and gravitational ones, that is, inertia and gravitation are to be considered equivalent.

The extremely accurate von Eötvös experiment[2], repeated by Dicke with a hundred-fold greater precision[3], provides a very important basis for that theoretical equivalence. The experiment shows that inertial and gravitational effects are independent of the nature of bodies. This ensures equivalence of inertia with gravitation everywhere, for, if different bodies locally swept by a gravitational field were not equally accelerated - say a proton and a neutron - the ratio of theír masses would be a function of position. In other terms, if the proton mass is nota function of space coordinates, the neutron mass ought to be so. Therefore, if there is equivalence between inertial and gravitational forces for the proton, that would not be the case with the neutron.

The equivalence principle has received more recently an additional experimental basis in the experiments performed by Hughes, Robinson, and Lopes[4], and Drever[5]. These laboratory observations were suggested by Salpeter and Cocconi[6] based on a mistaken conclusion inferred by Einstein[7] from item (b) of Mach's principle. According to that inference, which certain authors have wrongly identified with Mach's principle, as for example Weinberg[8], the concentration of nearby masses would alter the value of local inertial masses. Thus, the inertial mass of a body would not be a scalar quantity, but a tensor magnitude, and as Salpeter and Cocconi pointed out, the concentration of mass at the center of the Galaxy could produce in the solar neighbourhood an asymmetry in mass of the order of $\Delta m/m \sim 10^{-7}$. The aforementioned experiments gave a negative result within an accuracy of $\Delta m/m \sim 10^{-20}$, a result which refutes Einstein's inference and reinforces the equivalence principle, since no gradient for inertial mass is observed.

1.3) THE PRINCIPLE OF CO-VARIANCE

Since a general principle of relativity follows from Mach's principie, the form of the laws of physics should not depend on the choice of the coordinates and **reference frames,** as is the case in Newtonian dynamics in which the laws of **movement** are invariant for inertial systems of reference only. The laws of physics should by covariant, i.e., their form should not be altered by arbitary coordinate transformations.

1.4) FORMULATION OF THE LAWS OF PHYSICS IN A RIEMANIANN SPACE-TIME

Suppose a laboratory swept by a gravitational field. Considering it an infinitesimal local region in space-time, phenomena observed in it follow the laws of special relativity. Hence, we have a local Minkowskian metric[1]:

$$ds^2 = -(dX^i)^2 + (dX^0)^2 \tag{1}$$

A non-accelerated observer sees this infinitesimal space-time interval with different coordinates x^α obeying the linear transformations:

$$dX^\alpha = a^\alpha_\beta dx^\beta \ , \quad \alpha,\beta = 1,2,3,0 \ , \tag{2}$$

where the a^a_β are functions of the X^a which - according to the equivalence principle - depend only on the acceleration of the laboratory, or, on the gravitational field only. Inserting (2) into (1) there comes':

$$ds^2 = g_{\beta\gamma} dx^\beta dx^\gamma . \tag{3}$$

This is a metric form of a Riemannian space-time, where the metric ten-, sor $g_{\beta\gamma}$ represents the gravitational properties of the field. The equivalente principle leads to the connection of gravitation with, the Riemannian space-time, that is, gravitation has to do with geometry, not with the nature of bodies.

Another argument can be developed as presented by Dicke[9]. Since the equivaience of inertia with gravitation demands an interchange of inertial properties with gravitational ones through coordinate transformations, the simplest Lagrangian for a particle, in the variational principle, should be a scalar which mixes the gravitational field with dynamical magnitudes, to wit:

$$g_{\mu\nu}u^{\mu}u^{\nu} \ , \qquad (4)$$

where $g_{\mu\nu}$ represents the gravitational field, and u^{α} the components of the four-velocity. The variational principle

$$\delta\int g_{\mu\nu}u^{\mu}u^{\nu}d\tau = 0, \qquad (5)$$

leads to the equations

$$\frac{d}{d\tau}\, g_{\alpha\beta}u^{\beta} - \frac{1}{2}\, g_{\beta\gamma,\alpha}u^{\beta}u^{\gamma} = 0 \ , \qquad (6)$$

which represent geodesics. Therefore, the gravitational tensor $g_{\mu\nu}$ is the Riemann metric tensor.

1.5) DETERMINATION OF SPACE-TIME BY THE ENERGY-MOMENTUM TENSOR

According to the equivalence principle, the source term for gravitation is the total rest mass distribution. Since the metric tensor has the physical meaning of gravitation, and the properties of space-time represent gravitational properties, then rest mass determines geometrical properties. Einstein enlarged this proposition in the sense that not only inertial energy determines space-time geometry, but also other forms of energy. The properties of space-time are determined by the distribution of matter, or by the energy-momentum tensor. Of course, this proposition, although suggested by Mach's principle, is not entirely derived from it. Einstein's view is related to that of greek philosophers who maintained that the nature of space is an inseparable property of matter, but it does not express necessarily the more restricted idea contained in Mach's principle, namely, the equivalence of inertial forces with gravitational forces. Hence, Einstein's equations of general relativity have a wider content than that of Mach's principle. If we are to expect the agreement of the Einstein equations with the principle of Mach, restrictions must be imposed on them. A physical theory cannot be constructed solely on differential equations: boundary and initial conditions are also needed[9,11]. We know that the energy-momentum tensor of general relativity does not determine uniquely the space-time geometry through the Einstein equations, which shows beyond doubt that restrictions are necessary in order that the theory fulfils the requirements of Mach's principle.

1.6) THE COSMOLOGICAL PRINCIPLE: ISOTROPIC AND HOMOGENEOUS COSMIC SPACE-TIME. OBJECTIONS TO THE STANDARD COSMOLOGICAL MODELS

The cosmological principle as we know it reduces the $g_{\mu\nu}$ matrix to diagonal terms only; $g_{00} = 1$, the space part of the metric being multiplied by a function of time. This cosmic metric, the so-called Robertson-Walker metric, represents a space-time defined by co-moving coordinates and a universal time identical to all co-moving observers. This type of matrix represents a restriction in cosmic space-time which accomplishes in part the principle of Mach.

Co-moving coordinates define everywhere a cosmic reference system which is inertial[12], that is, local accelerations are related to this universal frame connected with the cosmic mass distribution. This means that every observer is referred to a space-type three-dimensional hypersurface determined by a homogeneous and isotropic distribution of matter. Thus, local inertial effects are related to this hypersurface. It is apparent that requisite (a) of Mach's principle is verified in the domain of world-models with a Robertson-Walker space-time. Nonetheless, requisite (b) offers difficulties, at least for cosmological models with zero and negative curvatures, i.e., Euclidean and pseudo-spheric spaces.

The Euclidean and pseudo-spheric cosmologies are necessarily infinite in the mass content. The assumption of a dual solution, interior and exterior, presupposes $p=0$ at the boundary, and this contradicts the everywhere zero pressure gradient of uniform models. An infinite mass distribution cannot be compatible with requisite (b) of Mach's principle, since an interaction between a local accelerated body and an infinite mass distribution cannot be defined. It seems that closed uniform world-models, that is, spherical universe models, are the convenient choice to fulfill the requirements of Mach's principle[7,10], since a completely universal isotropy for a finite mass distribution bypasses the above objection.

However, even the standard models with positive curvature are not satisfactory for other reasons. As we know, the Friedmann and Lemaître cosmological models, except for the Lemaître models with infinite contraction past time, are subject to singularities at zero cosmic time. Such singularities represent zero volume, infinite density and infinite velocity of expansion, so that at zero time the standard models are unphysical.

Furthermore, a serious difficulty pointed out by Misner[13] is the physical situation created by the particle-horizon[14] present in these models. Before time t_0 of the particle horizon, interaction between the fundamental particles (co-moving ponderable matter) is not possible onaccount of the expansion velocity which is higher than the velocity of light.

This situation cannot justify the uniformity of the universe models subsequent to the singularity, for there is no obvious physical mechanism capable of producing uniformity of density and pressure after the particle-horizon epoch.

Hence, an additional term in the energy-momentum tensor, that can preclude singularity and justify universal isotropy and homogeneity may be introduced. The Pryce-Hoyle tensor as demonstrated by Hoyle and Narlikar[15] is a possible way out from the above theoretical impasse, and we will consider it further.

2. A SUGGESTION FOR A MATHEMATICAL FORMULATION OF MACH'S PRINCIPLE: A SCALAR EQUATION

As we know, the cosmological principle is a restriction imposed on the Einstein equations such that a universal time is defined which is separated from a homogeneous and isotropic three-space, on account of the homogeneous and isotropic distribution of the cosmic ponderable matter.

This suggests an additional relation to Einstein's equations, namely, a relation that signifies the determination of the three-geometry through the ponderable matter distribution, i.e., through the inertial energy term of the energy-momentum tensor. This seems to represent mathematically the principle of Mach since it postulates the determination of a cosmic inertial frame apart from the cosmic time coordinate, which isan idea compatible with the separation between space and time in the Robertson-Walker rnetric. Furthermore, this determination of the three-geometry by inertia alone, agrees with item (b) which states an interaction without reference to the time coordinate. Thus items (a) and (b) of Mach's principle, namely, determination of a cosmic inertial frame and gravitational interaction without reference to time, can be represented by that relation.

A way to accomplish this would be first of all, to separate the three-geometry terms in the Einstein equations from the space-time terms.This is done in the equations of Fourès-Bruhat[16].

The Einstein (0,0) equation for uniform space-time in Fourès-Bruhat's form is:

$$2\Lambda - \left[{}^{(3)}R + K^2 - K^{i}_{j} K^{j}_{i}\right] = - 2\kappa T_{00} \quad , \; i,j = 1,2,3, \qquad (7)$$

where ${}^{(3)}R$ is the three-space scalar curvature, and K^{i}_{j} is the extrinsic curvature, i.e., the tensor that tells how the space-like hypersurface is curved in the four-space. It can be seen in Fourès-Bruhat's equations that energy and energy flow determine three-space and the embedding of the three-space in the four-space. In the original Einstein equations , it is the four-space which is directly determined by the matter tensor. In Fourès-Bruhat's equation we have a separation of the three-space from the four-space, which means that the matter tensor determines simultaneouly two distinguishable geometrical concepts. This raises the possibility that part of the matter tensor may determine only one of the geometrical concepts. Hence this possibility and the above suggestion raised by the Robertson-Walker metric may be formulated through the following scalar equation:

$$\kappa\rho^{a} = (\alpha/6)\ {}^{(3)}R\ , \tag{8}$$

where a and a are constants to be determined. Relation (8) states that inertial energy alone determines the cosmic three-geometry, i.e., the cosmic co-moving inertial system of a closed three-geometry is determined by the inertial content of the universe. This proposition is apparently Machian. Developing the scalar curvature we have:

$$\kappa\rho^{a} = \alpha/R^{2}(t)\ . \tag{9}$$

If we postulate $a=1$, this yelds

$$GM/c^{2}R(t) = \alpha\pi/4\ , \tag{10}$$

which is Whitrow's relation[17] for a closed cosmological model, which we used before in a special closed universe model with matter injection[18,19].

In other words, if we keep Einstein's κ constant, or else both G and c constant, it is necessary to have $M(t)$. A second hipothesis comes about, namely, the inclusion of the Pryce-Hoyle energy density in the matter tensor. This, as we have shown[19], ensures the perfect determination of $R(t)$, $\rho(t)$, $p(t)$, and the time varying Hoyle field in the model universe considered.

In relation (8), we could of course make a negative in order to have a negative curvature for the three-space. But as we have pointed out, the open universes are not "sympathetic" towards the principle of Mach. Besides, Wheeler[11] has shown that the Fourès-Bruhat equations can be deduced from a special variational principle, whose well-definition requires that the three-space be closed.

Observe that equation (10) can be expressed as follows:

$$(4/\alpha\pi)\left[GM^{2}/R(t)\right] = Mc^{2}\ . \tag{11}$$

This relation has the remarkable form of an equivalence of inertia with gravitation, such that a cosmic total gravitational energy is defined for a closed expanding universe. The idea contained in (11) reinforces the assumption that relation (8) represents the principle of Mach. This shows the convenience of assuming the hypothesis a=1 in relation (8). Observe that, for this special cosmological model, a modification of the energy-momentum tensor through the addition of a scalar term which represents a negative energy density is essential. In a different context of ideas, Brans and Dicke[20] claimed also the necessity to include anadditional term in the matter tensor in order to obtain a formalism that could accomplish, at least in part, the requirements of Mach's principle.

We note that, in the thirties, Einstein[21] suspected that the status of the classical energy-momentum tensor was unsatisfactory, as it was a phenomenological representation of matter which, as such, was a crude substitute for a representation that would include all known properties of matter. He said then that the lefthand side of his equations was made of a "fine marble", whereas the righthand side was of a "low grade wood".

A very important advantage obtained by the inclusion of the Hoyle-Pryce tensor in the equations of general relativity is that it precludes singularities[22,23,19]. The absurdities inherent to the singularities of the standard models are perfectly avoided in cosmological models with matter injection processes. The application of the Pryce-Hoyle scalar field to non-steady state cosmological models was presented for the first time by Nariai[23] who analyzed several models including spherical and pseudo--spherical types.

Ours[19] is a spherical world-model which presupposes relation (9).

3. ANOTHER POSSIBILITY FOR THE SCALAR EQUATION. THE ROLE PLAYED BY THE COSMIC CONSTANT IN A SPECIAL COSMOLOGICAL MODEL WITHOUT A BIG BAND, BUT WITH A MULTI-BANG

The cosmic constant A may be incorporated into the energy-momentum tensor in such a way that the pressure and density of the redefined tensor are given by

$$\bar{p} = p - \Lambda\kappa^{-1} \ , \ \bar{\rho} = \rho + \Lambda\kappa^{-1} \ . \tag{12}$$

This visualization introduces a different concept of the inertial content, which includes a constant term having nothing to do with observable matter. The principle of Mach applied to this visualization suggests therefore the following scalar equation:

$$\kappa\bar{\rho} = \frac{\alpha}{6} \ {}^{(3)}R \ , \tag{13}$$

or, in other terms,

$$\kappa\rho = \alpha/R^2(t) - \Lambda \ . \tag{14}$$

Relation (14) gives the modified form of Whitrow's relation, to wit:

$$\frac{GM}{c^2R} = \frac{\pi}{4}(\alpha - \Lambda R^2) \ . \tag{15}$$

It is clear that if we assume as before that G and c should beconstant, or else κ constant, then M should be a function of cosmic time. Therefore, a cosmological model presupposing (13) and the above hipothesis demands the Pryce-Hoyle field. We assume as before a Robertson - Walker metric with a positive curvature:

$$ds^2 = -R^2(t)(1 + r^2/4)^{-2}(dr^2 + r^2\sin^2\theta d\phi^2 + r^2 d\theta^2) + dt^2 \ , \tag{16}$$

and the following set of equations:

$$\begin{aligned} R_{00} - \frac{1}{2} g_{00} R &= -\kappa\bar{T}_{00} \ , \\ R_{11} - \frac{1}{2} g_{11} R &= -\kappa\bar{T}_{11} \ , \\ \lambda^{\mu}{}_{;\mu} &= f^{-1} n(t) \ , \end{aligned} \tag{17}$$

where $n(t)$ is the number of particles produced per unit proper volume, and

$$\bar{T}_{\mu\nu} = (\bar{\rho} + \bar{p})u_\mu u_\nu - g_{\mu\nu}\,\bar{p} - f(\lambda_\mu\lambda_\nu - \frac{1}{2}\,g_{\mu\nu}\lambda_\alpha\lambda^\alpha) \;; \tag{18}$$

h_μ is the Pryce-Hoyle field.

integration of the last equation (17), and substitution of ρ according to (14), gives

$$R^3\lambda' = (mf\kappa)^{-1}R^3\,(\alpha/R^2 - \Lambda) + \text{const.} \tag{19}$$

Development of the first two equations (17) gives

$$(\alpha-3)R^{-2} = 3(R'/R)^2 + \frac{\kappa f}{2}\,(\lambda')^2 \;. \tag{20}$$

From these two relations, the function $R(t)$ can be determined from the following integral:

$$t = \sqrt{3}\int_{R_0}^{R} \{(\alpha-3)R^4 - [\theta + \Gamma\,(\alpha R - \Lambda R^3)]^2\}^{-1/2}.R^2 dR \;, \tag{21}$$

where

$$\theta = (\kappa f/2)^{1/2}\;.\;\text{const}\;,\quad \Gamma = (\kappa f/2)^{1/2} \;. \tag{22}$$

The integral (21) is somewhat different from the one of our previous cosmological model[19], especially in that for the present case it is not possible to have a divergent function $R(t)$, on account of the negative coefficient of the sixth power term. The assumption we made before, of a slowing down process of matter injection, is automatically met in the present formulation. Of course oscillatory solutions are possible provided we make θ equal to zero, which happens also with the previous model. Oscillatory solutions while mathematically possible are nonetheless physically unsound for these cosmologies because: expansion in these models is not the effect of an initial explosion starting from a singularity as it is the case with the standard models. As can be seen from equation (21) as well as in the integral (12) of our previous paper[19], a singularity at zero time is impossible. On the other hand, the expansion process is not a consequence of an initial high pressure in the big bang, but it is an effect continually sustained by the matter injection process; the energy rate varies at the expense of work done by a negative stress[19]. For such models, the pressure, as can be seen from the field equations, is completely determined (which is not the case with the standard models) and does not signify a kinetic or radiation stress, but a negative pressure having to do with the metric field and the h-field. Such completeness raises the question as to the temperature of the model universe at zero cosmic time. It seems that a very high temperature, which presupposes high kinetic and radiation pressures, should not occur in our two models. Otherwise, there would be significant parameters for the expansion of the model universe in a theory which already includes a complete set of functions that completely describe the expansion process, which is a situation of incompatibility. Therefore, the very high temperatures due to kinetic and radiation stresses, at the big bang phase of the standard models, ought to be absent from our model universes. Nevertheless, high temperatures are not excluded from the sites where matter injection takes place (quasi stellar objects and radio galaxies). The start of the expansion for these model universes is not a compact fire ball in a big bang, but a multi bang with localized high temperatures imbedded in a rather cool small cosmic volume. In oscillatory standard models, the heavy matter which is synthetized at the interior of stars is reprocessed during the hot phases following collapsing periods. In our models, however, such non-localized hot phases are not present, which means that the synthesis of heavy elements is an irreversible process. Hence, the oscillatory solutions should be discarded.

The only possible solution for the expansion function $R(t)$, in the present model, should be of a convergent type. If we impose the conditions

$$R'_\infty = R''_\infty = 0\ , \tag{23}$$

where $R_\infty = R(t\to\infty)$, every subsequent time derivative should also be zero. This ensures the divergence of the integral (21), which is necessary for a convergent function $R(t)$. Function $R(t)$ and $\lambda(t)$, for this world-model, have a behaviour equivalent to that present in our previous model[19].

The behaviour of the expansion function demands that

$$\alpha = 3 \ . \tag{24}$$

From condition (23), it follows that

$$\begin{aligned} \Gamma &= 2\sqrt{\alpha - 3}\ \ R_\infty\ (\alpha - 3\Lambda R_\infty^2)^{-1} \\ \theta &= -\sqrt{\alpha - 3}\ R_\infty^2\ (\alpha + \Lambda R_\infty^2)(\alpha - 3\Lambda R_\infty^2)^{-1} \ . \end{aligned} \tag{25}$$

According to (22), Γ must be positive, which means that:

$$\alpha > 3\Lambda R_\infty^2 \ . \tag{26}$$

Let us make,

$$\Lambda = a\alpha / R_\infty^2 \ . \tag{27}$$

Let us assume also at zero time the following condition:

$$(\alpha - 3)R^4 - \left[\dot{\theta} + \Gamma\ (\alpha R - \Lambda R^3)\right]^2 \geq 0 \ . \tag{28}$$

Inserting (25) and (27) into (28), we have

$$-4a^2Y^6 + \left[(1-3a)^2 + 8a\right]Y^4 - 4(1+a)aY^3 - 4Y^2 + 4(1+a)Y - (1+a)^2 \geq 0 , \tag{29}$$

Now, from (24), (25) and (27), it follows the condition:

$$-1 < \alpha < \frac{2}{3} \ . \tag{30}$$

It can be seen from inequality (29) that the present model universe, as well as our previous one, cannot have very high red-shifts. Bearing in mind that the red-shifts z obey the law,

$$1 + z = R_p / R \ , \tag{31}$$

where R_p is the present value of the scaling function, it follows then

$$Y^{-1} = (R_\infty/R_p)(1+z). \tag{32}$$

If we choose as an example, the value (-1/2) for a, the highest red-shift possible for this model universe should be of the order of 3. It is known that very few measured red-shifts are above 2.0 and below 3.52, the highest one measured. This fact poses several questions, which lead Sandage[24] to infer among other possibilities that the low red-shift cut-off could signify the birth time for quasars. Our two models with matter injection may afford a possible alternative interpretation, namely: since quasars are assumed as being "white holes"[18,19], the red-shift cut-off may represent the beginning of the matter injection process, or the zero time of the universe's expansion. Of course an objection may be raised as regards the very high red-shifts predicted for the cosmic black-body radiation interpreted through the primordial fireball theory. This objection may be bypassed however on grounds pointed out before[19].

We have shown recently[25] that there is a third possibility for the general equations (15), (16), (17), namely: constants A and θ equal to zero in a context of equations where the velocity of light and Newton's G are proved to be time varying functions, such that the Einsteinian κ is kept constant. In this new formulation of the cosmological theory, the expansion function $R(t)$ is necessarily divergent, and its derivative convergent to zero, which means that the condition for a decaying activity of the white holes is still maintained. In this theoretical framework, high red-shifts are possible, and the cosmic background thermal radiation may be interpreted as originating at zero when matter injection sites were at the peak of their energetic emission and in a rather small cosmic volume. Therefore, at the origin of cosmic time, not a *big bang* but a *multi bang*, with localized high temperatures giving birth to the present isotropic cosmic thermal microwave radiation, is that which took place. This third possibility offers still advantages as to numerical calculations, since the uncomfortable constants A and θ are absent.

From the two modified Einstein equations (17), we obtain the energy balance equation

$$\frac{d}{dt}\{[\rho - f(\lambda')^2]R^3\} + \{p - f(\lambda')^2\}\frac{dR^3}{dt} = 0 \ , \tag{33}$$

which is identical to that of our previous model.

Equation (33) poses the idea that the variation of the energy rate with time is connected with work done by a negative stress along cosmic expansion, as we have pointed out before.

4. THE MEANING OF THE COSMIC STRESS. THE POSSIBILITY FOR TIME VARYING G AND c

The third possibility refers to a model universe with equations[19] plus the hypotheses of time varying gravitational "constant" and velocityof light, as well as zero Λ and zero θ (Ref.25).

From equations (7) of Ref.19, and from condition $\Lambda=\theta=0$, it follows that

$$\kappa p/c^2 = \frac{\kappa}{2} f(\lambda')^2 - R^{-2} - 2(R''/R) - (R'/R)^2 \tag{34}$$

where

$$c^2 = (BR)^{-1} \ , \tag{35}$$

and B a dimensional constant. Developing (34) and including (35), we have

$$p = (\kappa B)^{-1}\ [(2\Gamma^2/3R^5) - (\alpha/3R^3)] \ . \tag{36}$$

As we have pointed out before, $R\to\infty$ in this model. It can be seen from the initial condition,

$$R_0' = R'(0) = 0 \ , \tag{37}$$

that Γ should of the order of R_0 i.e.,

$$\Gamma^2 = (\alpha-3)R_0^2 \tag{38}$$

Since $3<\alpha<10$, even the initial value of p should be negative or of low positive value. Thus, not only the overall pressure in (33) is negative but p is also negative, for it should assume negative values along cosmic expansion. Hence, the cosmic stress p does not have the usual meaning of a kinetic plus radiation pressure. This comes about from the fact that in the present theory the introduction of Whitrow's relation precludes the indetermination of the pressure, implying its determination through the field equations. In the standard models, the function of state $p(\rho)$ is postulated in place. A nonconventional pressure, as it is the present case, is not such a surprising result. For, McCrea[26] has called attention to the fact that the analogy between the matter tensor in general relativity and in Newtonian fluid dynamics is misleading, since negative values for the stress can be obtained in the latter. This happens with de Sitter's non empty static universe and in the present theory. Inspection of (36) shows that the pressure is solely determined from the contribution of the Pryce-Hoyle term and the inertial one, and these are already well determined through the field equations, the covariant divergence of A_μ and Whitrow's relation. Of course, the inertial term in the equation may be corrected for the contribution of radiation and thermal energy and the stress should assume the following expression:

$$p = \frac{(\kappa B)^{-1}}{3}\{(2\Gamma/R^5)-(\alpha/R^3)-[\kappa(u+u_\gamma)/R]-\kappa(u'+u'_\gamma)\}\ , \tag{39}$$

where u and u_γ are the thermal and radiation energy densities. The known forms of energy contribute negatively to the pressure.

The conservation law in models with the Pryce-Hoyle tensor refers to an energy-momentum tensor which includes the usual form of the matter tensor plus the negative energy density given by the Hoyle tensor. Relation (33) comes also from

$$T^{\mu\nu}{}_{;\nu} = 0\ , \tag{40}$$

where $T^{\mu\nu}$ is of the form given by (18). We see in (33) that the process of matter injection is determined through a work done by a negative stress. This idea was presented for the first time by McCrea in his alternative interpretation of the Hoyle cosmological model[26].

A question may be raised as to the role played in this model by the kinetic and radiation pressures at the start of the expansion. According to what was seen, the initial bang cannot be associated with the usual intuitive picture of an explosion produced by conventional pressure. The mental picture of an initial multibang with high kinetic and radiation temperatures concentrated in the white-holes may favour the mathematical formalism of the present theory. The expansion process of our theory has to do, not with an initial explosion, but with a continuous process of matter injection, which represents a cosmic repulsion[27] operating along cosmic time, and having its maximum at zero time.

The third model, with variable c and G, presents certain features as to numerical values, which demand a model universe with a present density, two orders of magnitude higher than the usual adopted value obtained from galactic mass alone. Should present observational data on intergalactic matter be confirmed, the present adopted density should be considered too low. Values of the order of 10^{-28} g.cm^{-3} can be obtained for the third model, provided we introduce a correction in relations (1) and (2) of that model[25].

The relations considered should be

$$\begin{aligned} (4\pi/3)r^3 &= (1/n)2\pi^2R^3 , \\ A_0 &= \pm 3\sqrt{N/n_1} \,.\, e/(8\pi\varepsilon_0 r) , \end{aligned} \tag{41}$$

where

$$n_1 < n , \tag{42}$$

and not $n_1=n$, as previously stated[25]. This difference between n_1 and n comes from the assumption that the Euclidean regions coincide with the surrounding milieu of the white-holes and do not include the less denser regions of intergalactic space. This assumption presupposes the hypothesis that the charged milieu is the denser neighbourhood of the white-holes and that the intervening space is neutral.

If we assume n/n_1 constant along cosmic expansion, a should be given by:

$$a = \left(\frac{9n}{4\pi}\right)^{1/3} \left(\frac{1}{n_1}\right)^{1/2} . \tag{43}$$

Since the cosmic density p and white-hole density ρ_1 are given by

$$\begin{aligned} \rho_1 &= (N/n_1)\ m_p/(\frac{4\pi}{3} r^3) , \\ \rho &= Nm_p/(n \frac{4\pi}{3} r^3) , \end{aligned} \tag{44}$$

it follows that

$$\rho_1/\rho = n/n_1 , \tag{45}$$

which agrees with (42).

If we bear in mind observational data[28,29,30] for galactic and intergalactic densities, we may adopt:

$$n/n_1 \sim 10^4 . \tag{46}$$

Adopting values for a, in the range of 4 to 10, considering (38), $R_p >> R_0$, and the value of the Hubble parameter, i.e., 55 $km.s^{-1}\ Mpc^{-1}$, such orders of magnitude can be obtained:

$$R_p \sim 10^9 \text{ pc} ,$$

$$\rho \sim 10^{-28} g.cm^{-3} .$$

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