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A Geometrical Approach to Free-Field Quantization*

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A geometrical approach to the quantization of free relativistic fields is given. Complex probability amplitudes are assigned to the solutions of the classical evolution equation. It is assumed that the evolution is strictly classical, according to the scalar unitary representation of the Poincar group in a functional space. The theory is equivalent to canonical quantization.

Apresenta-se uma abordagem geométrica da quantização de campos relativisticos livres, onde amplitudes complexas de probabilidade são atribuídas às soluções da equação clássica de campo. Supõe-se que a evolução seja estritament clássica, sendo dada pela representação escalar unitária do grupo de Poincaré em um espaço funcional. A teoria é equivalente à quantização canônica.

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1. INTRODUCTION

To give the motivation which led us to the ideas of quantization treated in this article, we shall first analyse the very simple case of a free particle in one dimension in Schrödinger quantum mechanics. The theory is based on the canonical commutation relation

$$[x,p] = i\hbar \, , \qquad (1.1)$$

where x and p are the position and momentum operators in the Hilbert space convenient for the description of the particle. We can show that such a rule is a consequence of the assumption of having the group of spatial translations unitarily represented in such space. Indeed, let 0 and 0' be two observers translated relative to each other by a distance a. The quantum-mechanical states of the particle will be $|r>$, $|s>$ ... and $|r_a>$, $|s_a>$..., respectively. As usual, we require that such states be connected by a unitary operator $U(a)$,

$$|r_a> = U(a) \; |r> \quad ,$$

which satisfies the group property

$$U(a)U(b) = U(a+b) \; .$$

From this, it follows that it satisfies the differential equation

$$\frac{dU(a)}{da} = - \frac{i}{\hbar} \, pU(a)$$

for som Hermitian operator p. The solution of this equation is clearly

$$U(a) = e^{-\frac{iap}{\hbar}}$$

and we now identify p, the infinitesimal generator of spatial translations, as the momentum operator for the particle. Owing to the probabilistic interpretation of the theory, we must demand the expectation values of the position operator viewed by the two observers to be connected by

$$\langle x\rangle = \langle x\rangle' + a ,$$

from which, assuming orthonormality of the states before the transformation $\langle r|s\rangle = \delta_{rs}$, it follows that

$$x + a = e^{-\frac{iap}{\hbar}}\, x\, e^{\frac{iap}{\hbar}}$$

and thus,

$$[x,p] = i\hbar . \qquad (1.1)$$

This suggests that the symmetries might play an important role in the quantization process, at least when no interaction exists. It is also well known that when we "realize" the abstract Hilbert space, say in the coordinate representation, the quantum-mechanical states become complex probability amplitudes defined over a classical attribute of the classical state, namely the position of the particle. In what follows, we shall develop the lines of a quantum-mechanical theory for free relativistic fields based on such ideas.

2. GENERAL DESCRIPTION OF THE METHOD

Let χ be a classical field obeying a certain Poincaré-covariant evolution equation,

$$A\chi = 0, \quad \chi \equiv \chi_t(\vec{x}) . \qquad (2.1)$$

For simplicity, we assume A to be a first order differential operator in the space-time coordinates. At a certain instant of time, say $t = 0$, the classical field state is represented by an arbitrary function $\chi(\vec{x})$. We shall postulate that the quantum-mechanical field state is a complex probability amplitude $F[\chi]$ for finding the field described by a particular classical state χ. We further assume a purely classical evolution for such amplitudes, that is,

$$F_t[\chi_t(\vec{x})] = F[\chi(\vec{x})] \ , \qquad (2.2)$$

since there is no interaction.

It turns out, as we will see later on, that it is often convenient to parametrize the family of solutions of the classical evolution equation by the Fourier components of the *initial* classical data $\chi(\vec{x})$, since we are interested in a Poincaré-covariant formalism. We then generalize Eq.(2.2) to

$$F'[f'(k)] = F[f(k)] \ , \qquad (2.3)$$

where the prime stands for an arbitrary transformation of the Poincaré group, and $f(\vec{k})$ are the Fourier components of χ.

Mathematically, we can say that the quantum-mechanical field states are functionals of the solutions of the classical evolution equation which define a functional space (FS) which here plays the role of the Hilbert space of ordinary quantum mechanics. In this FS, we have a scalar unitary representation of the Poincaré group.

The next step is to seek for explicit expressions for the relevant operators as functional differential operators in the FS, and then find their spectra. This scheme is carried out in the following Sections for the free Klein-Gordon, electromagnetic and Dirac fields.

3. THE KLEIN-GORDON FIELD

The free real scalar field is described by the equation

$$(\Box + m^2)\Phi = 0, \quad \Phi \equiv \Phi_t(\vec{x}), \quad \Box = \eta^{\mu\nu}\partial_\mu\partial_\nu, \quad \eta^{\mu\nu} = \mathrm{diag}(+...) \ . \qquad (3.1)$$

As usual, we expand an arbitrary solution as

$$\Phi_t(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\omega_k} \left[e^{i(\omega_k t - \vec{k}.\vec{x})} a(\vec{k}) + e^{-i(\omega_k t - \vec{k}.\vec{x})} a^*(\vec{k}) \right] ,$$

where $\omega_k = \sqrt{\vec{k}^2 + m^2}$ ($\hbar = c = 1$). This equation can be rewritten as

$$\Phi_t(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\omega_k} e^{i\vec{k}.\vec{x}} \left[a_t(-\vec{k}) + a_t^*(\vec{k}) \right] , \tag{3.2}$$

where

$$a_t(\vec{k}) = e^{i\omega_k t} a(\vec{k})$$

gives the oscillatory evolution of the Fourier components of Φ. The quantum-mechanical field states will then be functionals of the type $F[a(\mathrm{k}), a^*(\mathrm{k})]$. Their evolution is given by

$$F_t[a_t(\vec{k}), a_t(\vec{k})] = F[a(\vec{k}), a^*(\vec{k})]$$

or

$$e^{-it\underset{\sim}{H}} F[a_t(\vec{k}), a_t^*(\vec{k})] = F[a(\vec{k}), a^*(\vec{k})] , \tag{3.3}$$

where we have written $\exp(-it\underset{\sim}{H})$ in the hope of finding a Hermitian operator $\underset{\sim}{H}$ to represent the Hamiltonian of the quantized field. Differentiation at $t = 0$ yields

$$\underset{\sim}{H} = \int d^3k \, \omega_k \left[a(\vec{k}) \frac{\delta}{\delta a(\vec{k})} - a^*(\vec{k}) \frac{\delta}{\delta a^*(\vec{k})} \right] .$$

We note here that this representation will turn out to be unitary when we define a scalar product in which $\underset{\sim}{H}$ is a Hermitian operator.

The same procedure applied to spatial translations, namely,

$$F_{\vec{x}}\left[a_{\vec{x}}(\vec{k}),a^{*}_{\vec{x}}(\vec{k})\right] = F\left[a(\vec{k}),a^{*}(\vec{k})\right] ,$$

yields, for the momentum operator, the expression

$$\underset{\sim}{\vec{P}} = \int d^3k\ \vec{k}\left[a(\vec{k})\ \frac{\delta}{\delta a(\vec{k})} - a^*(\vec{k})\ \frac{\delta}{\delta a^*(\vec{k})}\right] .$$

Let us now consider the eigenvalue equations,

$$\begin{aligned} \underset{\sim}{H}F_{\vec{k}'} &= \omega' F_{\vec{k}'} , \\ \underset{\sim}{P}F_{\vec{k}'} &= \vec{k}' F_{\vec{k}'} . \end{aligned} \tag{3.4}$$

We can easily show that the functional

$$F_{\vec{k}_1\ldots\vec{k}'_m}\left[a(\vec{k}),a^*(\vec{k})\right] = \prod_{j=1}^{m} a(\vec{k}_j) \prod_{\ell=1}^{m} a^*(\vec{k}'_\ell)$$

is a simultaneous eigenstate of $\underset{\sim}{H}$ and $\underset{\sim}{P}$ with energy-momentum eigenvalues given by

$$\begin{aligned} \omega' &= \sum_{j=1}^{n} \omega_{k_j} - \sum_{\ell=1}^{m} \omega'_{k_\ell} , \\ \vec{k}' &= \sum_{j=1}^{n} \vec{k}_j - \sum_{\ell=1}^{m} \vec{k}'_\ell , \end{aligned} \tag{3.5}$$

which are, of course, unsatisfactory since there would be no minimum energy state for the quantized field and the momentum would fail to be a strictly additive quantity. To overcome such difficulties, we restrict the FS to the *physical* space spanned by analytic functionals, that is, by functionals with no dependence on $a^*(\vec{k})$. The Hamiltonian and momentum operators then reduce to

$$\underset{\sim}{H} = \int d^3k\ a(\vec{k})\ \frac{\delta}{\delta a(\vec{k})} , \tag{3.6}$$

$$\underset{\sim}{\vec{P}} = \int d^3k\ \vec{k}a(\vec{k})\ \frac{\delta}{\delta a(\vec{k})} . \tag{3.7}$$

The number operator can be defined as

$$\underset{\sim}{N} = \int d^3k \; a(\vec{k}) \frac{\delta}{\delta a(\vec{k})} \quad .$$

The eigenstates of $\underset{\sim}{H}$ and $\underset{\sim}{\vec{P}}$ are

$$F_{\vec{k}_1 \ldots \vec{k}_n} [a(\vec{k})] = \prod_i a(\vec{k}_3) \quad , \tag{3.8}$$

with eigenvalues

$$\omega' = \sum_j \omega_{k_j} \quad , \quad \vec{k}' = \sum_j \vec{k}_j \quad . \tag{3.9}$$

General field states are obtained as usual by superposition, i.e.

$$F[a(\vec{k})] = \int \frac{d^3k_1 \ldots d^3k_n}{\omega_{k_1} \cdots \; k_n} f(\vec{k}_1 \ldots \vec{k}_n) \prod_{j=1}^{n} a(\vec{k}_j) \quad , \tag{3.10}$$

where F and f are Lorentz **scalars,** and f is completely **symmetric** for the interchange of its arguments.

We **will** now define an inner product $<F,G>$ of arbitrary field states F and G , compatible with the probabilistic interpretation of the **theo-**ry, namely

a) positive-definiteness,
b) $<F,G> = <G,F>^*$,
c) normalizability of an arbitrary state,
d) relativistic invariance under the **Poincaré** group,
e) linearity.

(3.11)

Following the analogy with ordinary quantum mechanics, we **would** like to define the inner product as a functional **integral**[2,3,4] over the **manifold** of the a's. Indeed, **we** can define **it** as

$$<F,G> = \int \delta a \; \exp\left(- \int \frac{d^3k}{\omega_k} |a(\vec{k})|^2\right) F^*[a(\vec{k})] G[a(\vec{k})] \quad , \tag{3.12}$$

where we have been careful to introduce the weight gaussian functional, as well as to define properly the measure δa as

$$\delta a = \frac{da_r}{\sqrt{\pi}} \frac{da_i}{\sqrt{\pi}}$$

with $a = a_r + ia_i$ (where $i = \sqrt{-1}$), so as to assure the **existence** of (3.12). Hence, for arbitrary states F and G we will have

$$<F,G> = \int \frac{d^3k_1 \ldots d^3k'_n}{\omega_{k_1} \ldots \omega_{k'_n}} f^*(\vec{k}_1,\ldots,\vec{k}_n)\; g(\vec{k}'_1,\ldots,\vec{k}'_n) < \prod_{j=1}^{n} a(\vec{k}_j), \prod_{\ell=1}^{n} a(\vec{k}'_\ell)>.$$

By applying the definition (3.11) to twon-particle basis states, we find

$$< \prod_j a(\vec{k}_j), \prod_\ell a(\vec{k}'_\ell)> = \frac{1}{n!} \prod \omega_{k_j} \sum_{\sigma_\ell} \delta(\vec{k}_j - \vec{k}'_{\sigma_\ell})$$

where a sum is made ove **all permutations** σ_ℓ of the **momenta** $\vec{k}'_\ell$. We finally get

$$<F,G> = \int \frac{d^3k_1 \ldots d^3k_n}{\omega_{k_1} \ldots \omega_{k_n}} f^*(\vec{k}_1,\ldots,\vec{k}_n) g(\vec{k}_1,\ldots,\vec{k}_n), \qquad (3.13)$$

in view of the **symmetry** of f and g in its arguments.

Hence, our definition (3.12) is equivalent to a very good-looking one Eq.(3.13), which clearly satisfies all the desired properties (3.11). We point out here that definition (3.12) is a generalization, to the case of functionals, of Bargmann's[5] definition of inner product in a space of analytic functions of complex arguments. It can also be shown that

$$<F,\underset{\sim}{H}G> = <\underset{\sim}{H}F,G> \quad ,$$

so that the representation we are dealing with is unitary. As a final remark to conclude this Section, we note that the *vacuum* (the zero quanta state) has a vanishing energy since it is represented by a constant functional.

4. ELECTROMAGNETIC FIELD

The free electromagnetic field, in the Lorentz gauge, is classically described by the wave equation

$$\Box A_\mu = 0 \ ,$$

where A_μ is the electromagnetic four-vector potential In order to carry out the quantization, we shall further assume the radiation gauge which reduces the above equation to

$$\Box \vec{A} = 0 \ , \ \nabla . \vec{A} = 0 \tag{4.1}$$

of which a general solution can be expanded as

$$\vec{A}_t(\vec{x}) = \frac{1}{(2\pi)^3} \int \frac{d^3k}{\omega_k} \left[\vec{\alpha}_t(-\vec{k}) + \alpha_t^*(\vec{k})\right] e^{i\vec{k}.\vec{x}} \tag{4.2}$$

where $\omega_k = |\vec{k}|$ and the transversality condition now reads $\vec{k}.\vec{\alpha}=0$. We can expand $\vec{a}$ as

$$\vec{\alpha}(\vec{k}) = p(\vec{k})\hat{k}_\theta + q(\vec{k})\hat{k}_\phi \ , \tag{4.3}$$

where $\hat{k}_\theta$ and $\hat{k}_\phi$ are unit vectors in the $\vec{k}$ space, pointing in the directions of increasing θ and ϕ, respectively. Just as we have done before, we assume the *initial* quantum mechanical field state to be a functional of the *initial* classical state, which we label again by its Fourier components: $F[p(\vec{k}),q(\vec{k})]$. We have now assumed analyticity once and for all, so as to avoid the problems discussed in connection with Eq. (3.5). By repeating the same steps followed in the foregoing Section, we find

$$\underset{\sim}{H} = \int d^3k\ \omega_k \left[p(\vec{k})\ \frac{\delta}{\delta p(k)} + q(\vec{k})\ \frac{\delta}{\delta q(\vec{k})} \right] , \tag{4.4}$$

$$\underset{\sim}{\mathrm{P}} = \int d^3k\ \vec{k} \left[p(\vec{k})\ \frac{\delta}{\delta p(\vec{k})} + q(\vec{k})\ \frac{\delta}{\delta q(\vec{k})} \right] . \tag{4.5}$$

We see that two independent functions $p(\vec{k})$ and $q(\vec{k})$ naturally appear. This reflects the existence of two degrees of freedorn for the photon, corresponding to its two possible helicity states. (Time-like and longitudinal photons are ignored from the start). As a result, both p and q must be present in the observables of the theory.

It is straightforward to see that the state

$$F_{\vec{k}_1 \ldots \vec{k}_n ; \vec{k}'_1 \ldots \vec{k}'_n} \left[p(\vec{k}), q(\vec{k}) \right] = \prod_j \ (\vec{k}_j) \prod_\ell q(\vec{k}'_\ell) \tag{4.6}$$

is a simultaneous eigenfunctional of $\underset{\sim}{H}$ and $\underset{\sim}{\vec{P}}$, with eigenvalues

$$\omega' = \sum_j \omega_{k_j} + \sum_\ell \omega_{k_\ell} \ , \ \vec{k}' = \sum_j \vec{k}_j + \sum_\ell \vec{k}_\ell \ . \tag{4.7}$$

Arbitrary states are obtained by superposition

$$F\left[a(\vec{k},\lambda)\right] = \int \frac{d^3k_1 \ldots d^3k_n}{\omega_{k_1} \ldots \omega_{k_n}}\ f(\vec{k}_1,\lambda_1 ; \ldots ; \vec{k}_n,\lambda_n) \prod_{j=1}^{n} a(\vec{k}_j,\lambda_j) , \tag{4.8}$$

where we have introduced the notation $a(\vec{k},1) = q(\vec{k})$ and $a(\vec{k},2) = p(\vec{k})$. A sum over repeated λ indexes is understocd. The function f is symmetric under permutations of its argurnents.

The scalar product is now generalized to

$$<F,G> = \int \delta a \, \exp(-\sum_{\lambda=1}^{2} \int \frac{d^3k}{\omega_k} |a(\vec{k},\lambda)|^2 \quad F^*[a(\vec{k},\lambda)]G[a(\vec{k},\lambda)] \quad , \qquad (4.9)$$

with the measure δa defined as

$$\delta a = \prod_{\lambda} \frac{da_r(\lambda)}{\sqrt{\pi}} \frac{da_i(\lambda)}{\sqrt{\pi}} .$$

It can be shown that this product is equivalent to

$$<F,G> = \int \frac{d^3k_1 \ldots d^3k_n}{\omega_{k_1} \ldots \omega_{k_n}} f^*(\vec{k}_1,\lambda_1 \ldots \vec{k}_n,\lambda_n) g(\vec{k}_1,\lambda_1 ; \ldots ; \vec{k}_n,\lambda_n) \quad , \qquad (4.10)$$

which obviously satisfies all desired properties.

5. DIRAC SPINOR FIELD

We shall now apply this method to the quantization of a field of a different geometrical character with respect to the Poincaré group, namely the Dirac spinor field. It is classically described by the free equation

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0 \quad . \qquad (5.1)$$

It is well known that such an equation has positive as well as negative frequency solutions, ψ^+ and ψ^-, respectively. We take $\chi \equiv \psi^+$ to represent particles, while ψ^-, subjected to the charge conjugation operation, represents antiparticles,

$$\Phi = i\gamma^2\chi^* \equiv \chi_c \quad .$$

We make a Fourier expansion of Φ and χ ,

$$\chi_t(\vec{x}) = (2\pi)^{-3/2} \int \frac{d^3k}{\omega_k} \; a_r(\vec{k}) \; u^r(\vec{k}) \; e^{ik.x}, \quad k.x = k_\mu x^\mu \tag{5.2}$$

$$\Phi_t(\vec{x}) = (2\pi)^{-3/2} \int \frac{d^3k}{\omega_k} \, b_s(\vec{k}) \; v_c^s(\vec{k}) \; e^{ik.x} \tag{5.3}$$

wherea sum over $r = 1,2$ and $s = 1,2$ is implied. Theexpansions are done in such a way as to assure that the a's and b's are Lorentz scalars.

Just as before, the quantum-mechanical field states will be postulated to be analytic functionals of the initial data $[a_r(\vec{k}),\, b_s(\vec{k})]$. Now we must take into account the fact that we are dealing with fermions. The principle of indistinguishability of identical particles then tells us that the states must be antisymmetric under interchange of the a's among themselves, the same holding for interchanges of the b's. Hence, we must construct the functionals representing the states as anticomnuting products of the a's and b's, like the wedge product

$$\bigwedge_{j=1}^{n} a_{r_j}(\vec{k}_j) = \frac{1}{n!} \sum_{\sigma_j} \varepsilon(\sigma_j) \prod_{j=1}^{n} a_{r_j}(\vec{k}_j) \;, \tag{5.4}$$

where we sum over the permutations σ and $\varepsilon(\sigma_j)$ is the parity of the permutation σ_j. In the left hand side, we adopt the increasing orderfor the index j.

In order to obtain the Hamiltonian and momentum operators, we now require the space-time translation group to be represented in FS. We shall use the geometrical motivation of having an infinitesirnal functional variation expressed as

$$\delta F = \int d^3k \left[\delta a_r(\vec{k}) \wedge \frac{\delta F}{\delta a_r(\vec{k})} + \delta b_s(\vec{k}) \wedge \frac{\delta F}{\delta b_s(\vec{k})} \right]$$

to define the functional derivative of the wedge product as

$$\frac{\delta}{\delta a_r(\vec{k})} \bigwedge_{j=1}^{n} a_{r_j}(\vec{k}_j) = \sum_{j=1}^{n} (-)^{j-1} \delta(\vec{k}-\vec{k}_j)\, \delta^r_{r_j} \bigwedge_{\substack{\ell=1 \\ \ell\neq j}}^{n} a_{r_\ell}(\vec{k}_\ell) .$$

We then obtain, by the procedure already familiar from the preceeding examples.

$$\underset{\sim}{H} = - \int d^3k\, \omega_k \left[a_r(\vec{k}) \wedge \frac{\delta}{\delta a_r(\vec{k})} + b_s(\vec{k}) \wedge \frac{\delta}{\delta b_s(\vec{k})} \right] \tag{5.5}$$

and

$$\underset{\sim}{\vec{P}} = \int d^3k\, \vec{k} \left[a_r(\vec{k}) \wedge \frac{\delta}{\delta a_r(\vec{k})} + b_s(\vec{k}) \wedge \frac{\delta}{\delta b_s(\vec{k})} \right] . \tag{5.6}$$

We can define a number operator by

$$\underset{\sim}{N} = \int d^3k \left[a_r(\vec{k}) \wedge \frac{\delta}{\delta a_r(\vec{k})} + b_s(\vec{k}) \wedge \frac{\delta}{\delta b_s(\vec{k})} \right] \equiv$$

$$\equiv \underset{\sim}{N}_{\text{particles}} + \underset{\sim}{N}_{\text{antiparticles}} \tag{5.7}$$

so that the charge operator $\underset{\sim}{Q}$ will be

$$\underset{\sim}{Q} = e \int d^3k \left[a_r(\vec{k}) \wedge \frac{\delta}{\delta a_r(\vec{k})} - b_s(\vec{k}) \wedge \frac{\delta}{\delta b_s(\vec{k})} \right] . \tag{5.8}$$

Simultaneous eigenstates of $\underset{\sim}{H}$ and $\underset{\sim}{\vec{P}}$ can also be found, namely

$$F^{r_1 \ldots r_n ; r'_1 \ldots r'_m}_{k_1 \ldots k_n ; k'_1 \ldots k'_m} [a,b] = \bigwedge_{j=1}^{n} a_{r_j}(\vec{k}_j) \bigwedge_{\ell=1}^{m} b_{r'_\ell}(\vec{k}'_\ell) , \tag{5.9}$$

with energy-momentum eigenvalues given by

$$\omega' = \sum_j \omega_{k_j} + \sum_\ell \omega_{k'_\ell} ,$$
$$\vec{k}' = \sum_j \vec{k}_j + \sum_\ell \vec{k}'_\ell . \tag{5.10}$$

We can also define the scalar product here by suitably generalizing the definitions of the preceeding Sections.

6. CONCLUSION

The formalism here developed for free-field quantization is equivalent, for such fields, to canonical quantization. Let us take, as an example, the Hamiltonian of the Klein-Gordon field, Eq. (3.6). By making the correspondence

$$a(\vec{k}) \to a^+(k) ,$$

$$\frac{\delta}{\delta a(\vec{k})} \to a(\vec{k}) .$$

where a^+ and a are the creation and annihilation operators, respectively, this Hamiltonian can be cast into the corresponding expression of the canonical formalism. The above association is, of course, permissible since

$$\left[\frac{\delta}{\delta a(\vec{k})} , a(k') \right] = \delta(\vec{k} - \vec{k}') .$$

Therefore, at least as long as one deals with free fields, the scheme here developed seems to work. It is thus natural to investigate the applicability of the ideas which inspired the method to the realistic case of interacting fields. For example, suppose we have a particular classical theory of interacting fields, say Electrodynamics, or perhaps a simpler theory. Suppose we have defined the convenient FS for the theory and we want to quantize it in such a way as to preserve all its symmetries. Would more complicated (possibly non-linear) representations of these symmetries in the corresponding FS provide an alternative approach to the quantized theory? Furthermore, would the resulting theory be mathematically more satisfactory than, for instance, standard QED? Aside from pointing out such a possibility, the present work does not present, of course, any new results.

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