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Inelasti Effect in the Siclewise Dispersion Relations for the Electromagnetic Form Factors of the Nucleon*

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We present a model for the inelastic effects.of the sN S-matrix in the Hilbert problem associated with the calculation of the rNN vertex and the I=1/2, J=1/2 electroproduction amplitudes. The model is applied to the calculation of the isocalar anomalous magnetic moment of the nucleon.

Apresentamos um modelo para os efeitos inelásticos na matrix *S* do sistema rN, para o problema de Hilbert associado ao cálculo do vértice nNN e das aplitudes de eletroprodução com *I=1/2* J=1/2 Aplica-se o modelo ao cálculo da parte isoescalar do momento magnético anômalo do núcleon.

1. INTRODUCTION

Theanalyticity in the nucleon a of the electromagnetic form factors of the nucleon, at fixed momentum transfer squared $q^2 \leq 0$, allows for the dispersion relation of the Pauli form factor[1]:

$$F_2(q^2) = \frac{1}{\pi}\int_{W_0}^{\infty} dW \left\{\frac{\operatorname{Im} F_2(+W;q^2)}{W-m} + \frac{\operatorname{Im} F_2(-W;q^2)}{W+m}\right\} ,$$

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where it is assumed that no subtraction is needed. The threshold W_0 is the sum of the nucleon and pion masses, $W_0 = m + \mu$.

The isotopic spin structure of the electromagnetic current leads one to consider isoscalar and isovector form factors. They will be normalized at $q^2 = 0$ by $F_2(0) = \kappa . e/2m$, where e is the charge of the proton and κ is the anomalous magnetic moment, with experimental values $\kappa^S = -0.060$ (isoscalar) and $\kappa^V = 1.853$ (isovector).

In the lowest intermediate state approximation, the unitarity equation reads:

$$\text{Im}\, F_2(\pm W;q^2) = \Phi(\pm W;q^2)\; K(\pm W)\; M^*(\pm W;q^2) \;,$$

where $\Phi(\pm W;q^2)$ is a phase space factor, $K(\pm W)$ is the πNN form factor with one nucleon off shell, and $M(\pm W;q^2)$ is a certain combination of the $I = 1/2$, $J = 1/2$ multipole amplitudes of pion electro- or photoproduction $\gamma N \to \pi N$. The isospin structure of these amplitudes naturally depends on the form factors considered.

The first calculation of these "sidewise" dispersion relations was done by Drell and Pagels[2] with the so-called *threshold dominance model*. Their approximation consists in assuming a pointlike πNN vertex $K(\pm W) = g$, the pion nucleon coupling constant, and in taking the Born approximation for the photoproduction amplitudes. Furthermore, they use zero pion mass kinematics and have to introduce a cut-off to avoid divergent integrals.

In the subsequente work of Ademollo, Gatto and Longhi[3], the πNN vertex was given some structure by assuming elastic unitarity:

$$\text{Im}\, K(\pm W) = \exp\{i\delta(\pm W)\}\; \sin\delta(\pm W) . K^*(\pm W) \;,$$

where $\delta(\pm W)$ stands for the P11, if $+W$, and S11, if $-W$, pion nucleon phase shifts, respectively. The multipoles are obtained alço from elastic unitarity in the s-channel: $\text{Im}\, M = \exp\{i\delta\} \sin\delta . M^*$, with Born terms and Δ-resonance exchange to account for the crossed channel singularities.

Love and Rankin[4] used a πNN form factor which is actually the same as ours, and took the multipoles from experimental data. The dispersion relations were integrated up to the upper limit of the available data. They included also an estimate of the intermediate ηN state which is strongly coupled to the S11 state. Their calculation, however, suffers from the ambiguities in the phenomenological analysis of the photo production data.

The threshold dominance model was critically analyzed and it was shown that the low-energy contribution to the dispersion integral was seriously overestimated due to the use of zero pion mass kinematics.

In our approach, we assume saturation of unitarity by the lowest intermediate state: $\mathrm{Im}\, F_2 = \Phi\, \mathrm{Re}\{K M^*\}$, what amounts to $\mathrm{Im}\, F_2 = \Phi K M^* + \rho$ with $\mathrm{Re}\, \rho = 0$. In Section 2, we shall discuss our calculation of the πNN form factor taking the inelasticity of the πN P11 and S11 waves into account. The electroproduction amplitudes are treated in a similar way in Section 3, and we apply our model to the calculation of the isoscalar magnetic moment in Section 4.

2. THE PION-NUCLEON FORNI FACTOR

The form factors $K(\pm W)$ are shown to be boundary values of a real analytic function $k(z)$ in the complex z-plane cut along the real axis from $\pm W_0$ to $\pm\infty$:

$$K(\pm W) \equiv K_+(\pm W) = k(\pm(W + i0)),\ \text{for}\ W \geq W_0\ ,$$

and

$$K_-(\pm W) = k(\pm(W - i0)) = (K_+(\pm W))^*\ . \tag{1}$$

By definition, the pion-nucleon coupling constant is

$$g = k(m)\ . \tag{2}$$

Omitting the $\pm W$ dependence, the unitarity equation reads

$$(1/2i)(K_+ - K_-) = (1/2i)(S - 1)\ K_- + R, \tag{3}$$

where $S = \eta \exp\{2i\delta\}$ is the S-matrix for πN scattering in the P11 state, if $(+W)$, and in the S11 state if $(-W)$; the contribution of intermediate states others than the πN one is denoted by R.

For given functions $S(\pm W)$ and $R(\pm W)$, Eq.(3) leads to an inhomogeneous Hilbert problem:

$$K_+ = S\ K_- + 2iR\ , \tag{4}$$

which can be solved following the standard methods of Muskhelishvili[5] provided certain regularity conditions on the functions $S(\pm W)$ and $R(\pm W)$ are fulfilled.

However, the general solution of the Hilbert problem (4), with supplementary condition (2), is seen not to satisfy the Schwarz reflection property (1) for arbitrary S and R.

Since we shall assume S to be given by the experimental data on πN scattering in the P11 and S11 states, the requirement (1) imposes in fact a condition on the function R which has to be satisfied for the problem to have a solution. This condition takes the form of a quite complicated functional relation between S and R but it can be displayed in a simpler form by the following device: taking the sum and difference of (4), and its complex conjugate, we obtain

$$K_+ = -\frac{1 - S}{1 - S^*} K_- - \frac{4\ \mathrm{Im}\ R}{1 - S^*}\ , \tag{5a}$$

$$K_+ = \frac{1 + S}{1 + S^*} K_- + \frac{4i\ \mathrm{Re}\ R}{1 + S^*}\ . \tag{5b}$$

The first of these boundary conditions amounts to the statement that the righthand side of (3) is real, while the second one is nothing else than the well known Goldberger-Treiman trick consisting in taking half the sum over in- and out-states in the unitarity equation. Each of these boundary conditions defines a Hilbert problem which we shall write as

$$\mathrm{H}_j : \quad K_+ = G_j K_- + g_j , \tag{5c}$$

where $j = 1$ corresponds to (5a), and $j = 2$ to (5b).

The advantage of this formulation of the boundary condition isthat each of the Hilbert problems H_j has the following property:

a) The kernet G_j has modulus one, so that

$$\phi_j = (1/2i) \ln G_j \quad \text{is real} \ ; \tag{6a}$$

. b) The inhomogeneous term satisfies

$$g_j = - g_j^* \, G_j \ . \tag{6b}$$

As we shall show later, these properties permit to express the Schwarz reflection property in a rather simple way.

The phases ϕ_j are given in terms of the phase shifts and inelasticities by

$$\phi_1 = \arctan\{\frac{1 - \eta \cos 2\delta}{\eta \sin 2\delta}\} \ , \ \phi_2 = \arctan\{\frac{\eta \sin 2\delta}{1 + \eta \cos 2\delta}\} \ , \tag{7}$$

where the determination of the arctan is chosen such that ϕ_j varies continuously with W over the interval $[W_0, \infty]$. In the elastic region, $\eta=1$ and $\phi_1 = \phi_2 = \delta$.

According to Levinson's theorem, the phase shifts $\delta(\pm W)$ tend to an integral multiple of π at infinite energy so that, if $1 - \eta(\pm W)$ is different frorn 0 at ∞ , $\tan\phi_1(\pm\infty) = \infty$ and $\tan\phi_2(\pm\infty) = 0$. The experimental data[6] seem to favour $\phi_1(\pm\infty) \approx \pi/2$ and $\phi_2(\pm\infty) = 0$. In Muskhelishvili's terminology, this implies that all the endpoints of the discontinuity contour ($\pm W_0$ and ∞) are special ends.

The so-called fundamental solutions of the associated homogeneous problems are

$$X_1(z) = \frac{z - m}{2m}\,\Omega_1(z) \;, \tag{8a}$$

$$X_2(z) = \Omega_2(z) \;, \tag{8b}$$

where

$$\Omega_j(z) = \exp\left[\frac{z - m}{\pi}\int_{W_0}^{\infty} dW' \left(\frac{\phi_j(+W')}{(W' - m)(W' - z)} - \frac{\phi_j(-W')}{(W' + m)(W' + z)}\right)\right]. \tag{9}$$

The boundary values of $X_j(z)$, denoted by $X_{j,+}(\pm W)$ and $X_{j,-}(\pm W)$, are different from zero and bounded on rhe cuts $[\pm W_0, \pm\infty]$.

The general solution of the inhomogeneous Hilbert problem, which is bounded at $\pm W_0$ by $|k(z)| <$ const. $|z - (\pm W_0)|^{-\alpha}$, and at ∞ by $|k(z)| <$ const. $|z|^{\alpha}$, $0 \leq \alpha < 1$, is given by

$$k_j(z) = X_j(z)\{\, A_j(z) + g\, P_j(z)\}, \tag{10}$$

where $P_j(z)$ is a polynomial in the variable $(z + m)/(z - m)$,

$$P_1(z) = (z + m)/(z - m) + C\,. \tag{11a}$$

$$P_2(z) = 1. \tag{11b}$$

These polynomials are constructed in such a way that $k(m) = g$. The function $X_j(z)\, A_j(z)$ is a particular solution of the inhomogeneous problem, A_j being given by

$$A_j(z) = \frac{z - m}{\pi}\int_{W_0'}^{\infty} dW' \left(\frac{r_j(+W')}{(W' - m)(W' - z)} - \frac{r_j(-W')}{(W' + m)(W' + z)}\right) \;, \tag{12}$$

where the functions $r_j = \frac{g_j}{2i X_{j,+}}$ are real due to (6b). The lower integration limit W_0' is the inelastic threshold.

It is clear that the functions $X_j(z)$ and $A_j(z)$ are real analytic functions so that the only condition imposed by the Schwarz relation (1) is that the arbitrary constants, which may appear in the polynomials $P_j(z)$, must be real.

The requirements (1) and (2) are thus fulfilled but, in order that the initial boundary condition (3) be satisfied, we must demand that the general solution of the Hilbert problem H_1 satisfies also the other boundary condition (5b). This yields a condition on R in the form

$$4i \text{ Re } R = (1 + S^*) K_{1,+} - (1 + S) K_{1,-} , \qquad (13a)$$

where $K_{1,\pm}$ depend functionally on Im R and S.

Equivalently we could start with the general solution of H_2 and demand that

$$4 \text{ Im } R = - (1 - S^*) K_{2,+} - (1 - S) K_{2,-} . \qquad (13b)$$

In practice, $R(\pm W)$ is not very well known. As a simple model, we shall assume the inelastic contribution $R(\pm W)$ to be given by the intermediate σN and ηN states which are strongly coupled to the πN channel through the S11 and P11 resonances. Furthermore, the $\sigma N \to \pi N$ and $\eta N \to \pi N$ and amplitudes will be approximated by the sum of their Born terms and the resonance exchange with some phenomenological propagators $\Delta_{P11}(W^2)$ and $\Delta_{S11}(W^2)$, while the σNN and ηNN vertices will have only the resonance exchange terms. With these approxirnations, it comes up that Im $R(\pm W)$ is proportional to interference terms between different resonance propagators or between Born terms and resonance propagators, while Re $R(\pm W)$ is proportional to the squares of these propagators.

From this model, we shall only use the result: $|\text{Im } R| << |\text{Re } R|$. It seems thus preferable to start from the Hilbert problem H_1 neglecting Im R so that the general solution (10) becomes

$$k(z) = g\, P_1(z)\, X_1(z). \tag{14}$$

In other words, our model of $R(\pm W)$ consists in taking Im $R(\pm W) = 0$ and Re $R(\pm W)$ can, if one wishes, be computed from (13a).

The solution (14) still contains an arbitrary real constant C which can be fixed by demanding that $k(z)$ should tend to zero at ∞. In this case, C = -1 and (14) reads

$$k(z) = g\, \Omega_1(z)\ , \tag{15}$$

which is the solution proposed by Love and Rankin.

That $k(z)$ should tend to zero at ∞ is suggested by a sum rule of Love[7]:

$$\int_{W_0}^{\infty} dW' \{\mathrm{Im}\ K(+W';0) - \mathrm{Im}\ K(-W';0)\} = 0 \quad ,$$

$$\int_{W_0}^{\infty} dW'\ W' \{\mathrm{Im}\ K(+W';0) + \mathrm{Im}\ K(-W';0)\} = 0 \ .$$

These relations hold for a zero value of the momentum squared of the pion, and they can be generalized to negative values of this momentum squared but not to the physical value μ^2.

It was the aim of this Section to show what kind of approximations are involved in the solution of Love and Rankin.

The boundary values of the solution (14) are:

$$K_+(\pm W) = g\ \exp\{i\phi(\pm W)\}\ \ \omega(\pm W). \tag{16}$$

The index 1 has been dropped so that $\phi(\pm W)$ is given by the first of equations (7), and $\omega(\pm W)$ is obtained from (9) with $z = \pm W$ and with the integral taken as a Cauchy principal value.

The numerical integration was performed over the entire energy interval putting $\phi(\pm W) = \pi/2$ for values of W greater than the upper limit of the available data (2189 MeV in our case).

3. THE ELECTROPRODUCTION AMPLITUDES IN THE $I = 1/2$, $J = 1/2$ STATES

3a. GENERAL METHOD

Once the kinematical singularities have been removed, the amplitudes $M(\pm W;q^2)$ are seen to be boundary values on the real axis of an analytic function $m(z)$, at fixed real values of $q^2 \leq 0$. The function $m(z)$ has, besides the s-channel unitarity cut from $\pm W_0$ to $\pm\infty$, additional singularities due to the Born terms and the exchanges in the crossed channels of $\gamma N \to \pi N$.

We shall assume that $m(z)$ can be written as the sum of two terms

$$m(z) = m_L(z) + m_R(z) \quad , \tag{17}$$

where the first one $m_L(z)$ contains all but the physical s-channel singularities. It will be given by a model which in fact defines the above separation.

The s-channel unitarity yields, for $W \leq W_0$,

$$\mathrm{Im}\, M = (1/2i)\ (S - 1)\ M^* + R' \quad ,$$

or, since m_L is real in the s-channel cut,

$$M_{R,+} = S\, M_{R,-} + (S - 1)\ m_L + 2i\ R' \quad , \tag{18}$$

with obvious notations.

Again we note that, for given S and m_L, the Hilbert problem (18) has no solution for arbitrary R' satisfying the constraint

$$M_{R,+} = (M_{R,-})^* .$$

In analogy with the πNN vertex problem, we take Im R' = 0 so that we are lead to the inhomogeneous Hilbert problem:

$$M_{R,+} = \exp\{2i\phi\} M_{R,-} + 2i \sin\phi.\exp\{i\phi\} m_L , \quad (19)$$

where $\phi(\pm W)$ is the same as in (16).

The πNN problem was solved using $k(m) = g$ and assuming a suitable behaviour of $k(z)$ at ∞, as appears in the subtraction procedure used to obtain (8) and (9). In the case of the multipole calculation also, some additional information is needed, First, we suppose that the cross sections for electroproduction go to constants at infinite energy, implying that $M(\pm W)$ should be bounded at infinity. The second information we use concerns the behaviour of partial wave amplitudes near the origin. Using helicity formalism, it is straightforward to generalize the results of Freedman and Wang[8], for scattering of spinless particles, to the $\gamma N \rightarrow \pi N$ process

We obtained the following result for our $I=1/2$, $J=1/2$ multipoles:

$$M(\pm W) \underset{W\rightarrow 0}{\sim} C_\pm (W^2)^{1/2 - \alpha(0)} , \quad (20)$$

where $C_\pm$ are constants, and where $\alpha(0)$ is the value, at zero energy, of the nucleon Regge trajectory exchanged in the s-channel. A typical value is $\alpha(0) \simeq -0.39$.

The fundamental solution of the associated homogeneous problem is now chosen as

$$X'(z) = \frac{z}{m} \Omega'(z) , \quad (21)$$

with

$$\Omega'(z) = \exp\{ \frac{z}{\pi} \int_{W_0}^{\infty} dW' (\frac{\phi(+W')}{W'(W'-z)} - \frac{\phi(-W')}{W'(W'+z)})\}$$

$$= \Omega(z)/\Omega(0) \ .$$

The general solution of (19) for a "well behaved" model of $m_L(z)$ is

$$m_R(z) = X'(z) \ \{A_L(z) + a_0 \frac{m}{z} + a_1\} \ , \qquad (22)$$

where a, and a, are arbitrary real constants, and

$$A_L(z) = \frac{z}{\pi} \int_{W_0}^{\infty} dW' (\frac{\psi(+W')}{W'(W'-z)} - \frac{\psi(-W')}{W'(W'+z)}) \qquad (23)$$

with

$$\psi(\pm W') = \sin\phi(\pm W') \ \frac{m_L(\pm W')}{x'(\pm W')} \ , \ X'(\pm(W' + i0)) = \exp\{i\phi(\pm W')\} \ x'(\pm W') .$$

The total multipole is then obtained as

$$M_+(\pm W) = \exp\{i\phi(\pm W)\} \ [\cos\phi(\pm W) \ m_L(\pm W)$$

$$+ x'(\pm W)\{a_L(\pm W) + a_0 \frac{m}{\pm W} + a_1\}] \ , \qquad (24)$$

where $a_L(\pm W)$ has the same form as *(23)* with the integral taken as a Cauchy principal value.

The above calculation scheme is in fact an adaptation of the early work of Dennery, Zagury, and Adler[9], for the calculation of the $I=3/2$, $J=3/2$ multipoles which dominate the electroproduction process.

The new feature we have developed here is the approximate treatment of the inelastic resonant caracter of the $I=1/2$, $J=1/2$ pion-nucleon states which is essential in our problem.

By construction, $m_R(z)$ is analytic in the neighbourhood of the origin:

$$m_R(z) \underset{z\to 0}{\to} a_0 + a_1 \frac{z}{m} + 0(z^2) \ .$$

The "left-hand" contribution $m_L(z)$ is certainly not analytic near $z=0$, but if we have an asymptotic expansion on the real axis of the form

$$m_{L,+}(\pm W) \underset{W\to 0}{\sim} c_0 + c_1 \frac{W}{m} + \dots \ ,$$

then the constants a_0 and a_1 can be fixed by the assumed behaviour near the origin in (20):

$$\begin{aligned} a_0 + c_0 &= 0 \ , \\ a_1 + c_1 &= 0 \ . \end{aligned} \tag{25}$$

The main, well known, advantage of this calculation scheme is the linearity in $m_L(z)$. The model for $m_L(z)$ will generally consist in a sum of Born terms and contributions arising from different resonance exchanges in the crossed channels:

$$m_L(z) = \sum_j \gamma_j \, m_{L,j}(z) \ , \tag{26}$$

where the γ_j's are products of strong interaction coupling constants with hadronic electromagnetic form factors.

Each $m_{L,j}(z)$ will give a contribution $m_j(z)$ to the total multipole and

$$m(z) = \sum_j \gamma_j \, m_j(z) \ . \tag{27}$$

The arbitrary constants $a_{0,j}$ and $a_{1,j}$ which appear in each term will be fixed by the behaviour of $m_{L,j}(z)$ near the origin according to (25).

3b. CONVERGENCE QUESTIONS

The above calculation scheme works as long as the functions occurring in the boundary condition (19) satisfy the Hölder-continuity requirements inherent to the formulation of the Hilbert problem. In particular, the inhomogeneous term

$$g'(\pm W) = 2i \sin\phi(\pm W).\exp\{i\phi(\pm W)\}\, m_L(\pm W)$$

should tend sufficiently fast to constant values at infinity:

$$|g'(\pm W) - g'(\pm\infty)| < \frac{\text{const.}}{|W|^{\varepsilon}}, \qquad (28)$$

ε being a positive constant.

We shall assume that $\phi(\pm W)$ tends very fast to its asymptotic value $\pi/2$ so that, for sufficiently large W, $g'(\pm W) \sim -2\, m_L(\pm W)$. Since the function $m_L(z)$ is given by some model, we must verify whether condition (28) is satisfied. Our model consists in Born terms and vector meson exchange in the t-channel, and the worst kind of behaviour we encounter is of the following type:

$$m_L(\pm W) \underset{W\to\infty}{\sim} A\left(\frac{W}{m}\right)^2 + B\left(\pm\frac{W}{m}\right) + O\left(\frac{1}{W}\ln W\right).$$

We shall also need the behaviour near the origin which is:

$$m_L(\pm W) \sim \alpha + \beta\left(\pm\frac{W}{m}\right) + O(W^2), \text{ when } W \to 0 .$$

Clearly, with such a model the integral (23) diverges and we have to make two subtractions. Since we have some information about the behaviour of the multipole near the origin, it seems natural to choose $z = 0$ as subtraction point. One way to proceed is to introduce the function

$$\tilde{m}_R(z) = (m_R(z) - (m_R(0) + m_R'(0)\, z))\left(\frac{m}{z}\right)^2, \qquad (29a)$$

where $m_R(0)$ and $m'_R(0)$ are the values of the function m_R and of its first derivative at the origin. This function is analytic in the cut complex z-plane, and satisfies the boundary condition

$$\tilde{M}_{R,+} = \exp\{2i\phi\}.\tilde{M}_{R,-} + 2i \sin\phi.\exp\{i\phi\}.\tilde{m}_L \, , \tag{30}$$

with

$$\tilde{m}_L(z) = \{m_L(z) + m_R(0) + m'_R(0)\ z\}(\frac{m}{z})^2 \quad . \tag{29b}$$

The solution so obtained contains, besides the two arbitrary constants of the type a,, a, in (22), the two subtractíon constants $m_R(0)$ and $m'_R(0)$.

An alternative way is to introduce the functions

$$\tilde{m}_R(z) = m_R(z)\ (\frac{m}{z})^2, \quad \tilde{m}_L(z) = m_L(z)\ (\frac{m}{z})^2, \tag{31}$$

so that $\tilde{m}_R(z)$ obeys the same boundary condition (30) on the cut, but has an additional singularity at the origin (a double pole). The general solution of this modified Hilbert problem is:

$$\tilde{m}_R(z) = X'(z)\{\tilde{A}_L(z) + a_0(\frac{m}{z})^3 + a_1(\frac{m}{z})^2 + a_2(\frac{m}{z}) + a_3\} \, , \tag{32}$$

where $\tilde{A}_L(z)$ is given by the analogous expression to (23) with $m_L(z)$ replaced by $\tilde{m}_L(z)$. The boundary values on the cut are:

$$\tilde{M}_+(\pm W) = e^{i\phi(\pm W)} \Big[\cos\phi(\pm W)\ \tilde{m}_L(\pm W) +$$

$$x'(\pm W)\ \{\tilde{a}_L(\pm W) + a_0\ (\pm\frac{m}{W})^3 + a_1\ (\pm\frac{m}{W})^2 + a_2(\pm\frac{m}{W}) + a_3\}\Big] \ . \tag{33}$$

We shall need the asymptotic behaviour of the functions $x'(\pm W)$ and $\tilde{a}_L(\pm W)$:

$$x'(\pm W) \sim \pm x_\infty \{1 + x_1\ (\pm\frac{m}{W}) + O(1/W^2) \quad ,$$

where x_∞ and x_1 are real constants which we can compute from the phases $\phi(\pm W)$, and

$$\tilde{a}_L(\pm W) \sim b_0 + b_1 \left(\pm\frac{m}{W}\right) + O(1/W^2);$$

See note 10.

The constants b_0 and b_1 naturally depend on the model for $m_L(z)$. The total multipole then behaves at ∞ as

$$\tilde{M}_+(\pm W) \sim \pm i x_\infty((b_0 + a_3) + ((b_0 + a_3)x_1 + b_1 + a_2)\left(\pm\frac{m}{W}\right) + O(1/W^2)),$$

and, since $W\,\tilde{M}_+(\pm W)$ must be bounded at infinity, the constants a_2 and a; are fixed by

$$a_2 + b_1 = 0 \text{ and } a, + b, = 0 . \tag{34}$$

The two other constants are fixed by the assumed behaviour at the origin (20),

$$\tilde{M}_+(\pm W) \underset{W\to 0}{\sim} (a_0 + \alpha)\left(\pm\frac{m}{W}\right)^2 + (a_1 + \beta)\left(\pm\frac{m}{W}\right) + O(W^0)$$

so that (20) implies

$$a, + a = 0 \quad \text{and} \quad a, + \beta = 0. \tag{35}$$

The number of necessary subtractions will naturally depend on the particular $m_{L,j}(z)$ considered.

4. CALCULATION OF THE ISOSCALAR ANOMALOUS MAGNETIC MOMENT

In this case, the model for $m_L(z)$ will be given by the contribution of the Born terms due to nucleon exchange in the s- and u-channel and by the exchange of the ρ meson in the t-channel. The gauge invariance of this approximate amplitude is assured by the method of Ball[11] which can be extended to $q^2 \neq 0$, or by the equivalent treatment of Adler[9].

The isospin structure of the isoscalar multipole is such that the pion exchange in the t-channel or the $\Delta(1236)$ exchange in the u-channel does not contribute. We obtain:

$$m_L(z) = e\, g^2/2m \left[m_{L,1}(z) + \kappa^S m_{L,2}(z) + \lambda \frac{G^V}{g} m_{L,3}(z) + \lambda \frac{G^T}{g} m_{L,4}(z) \right] , \tag{36}$$

where the first two terms come from the Born approximation and the last two are due to ρ exchange. The vector and tensor coupling constants of ρNN are G^V and G^T, and λ is the $\rho\pi\gamma$ coupling. This model leads to a multipole of the type (27) which together with the result for $K(\pm W)$ is substituted in the dispersion relation of the isoscalar form factor at $q^2 = 0$. The following result is obtained:

$$\kappa^S = g^2/4\pi \left(a + b\kappa^S + c\lambda \frac{G^V}{g} + d\lambda \frac{G^T}{g} \right) , \tag{37}$$

where the numerical values of the calculated constants are:

$$a = -0.084 \ ; \quad b = -1.93 \ ; \quad c = 4.71 \ ; \quad d = 8.70 \ . \tag{38}$$

No attempt was made to evaluate errors bars on these numbers in view of the rather complicated way they were obtained. Like every result of a dispersion relation calculation, Eq. (37) gives only a relation between amplitudes of different processes which in our case are form factors at $q^2 = 0$. Amongst the wide sample of experimental values of the strong interaction coupling constants, we choose those of Köpp and Söding (A), of Bugg (B) and of Schierholz (C), Ref. 12:

	$g^2/4\pi$	G^T/G^V	G^V/g
(A)	14.4	3.66	.29 ± .03
(B)	14.7	2.5 ■ 1.	.36 ± .09
(C)	14.4	4.78	.20

With these numbers we obtain

$$\kappa^S = \alpha + \beta\,\lambda' \quad ,$$

where A' = sign($\lambda\, G^V/g$) $|\lambda|$, and the constants a and β are given by:

	a	β	
(A)	-0.42	5.30	.55
(B)	-0.42	4.76	2.60
(C)	-0.42	4.72	

Clearly the contribution of the ρ meson cannot be neglected since otherwisewe shouldobtaina valueof -0.42 for κ^S to becompared with the experimental value of -0.060.

The value of the ρπγ coupling λ, which is directly linked to the ρ→πγ decay, is not available from experimental data since this decay mode is largely dominated by the ρ→πγ mode. However, in the vector meson dominance model, one can relate the ρ→πγ decay to the processes ρ→ππ and π→γγ. Such a calculation was done by Gourdin[13] with the result:

$$|\lambda| = .117 \pm .007 \text{ or } .126 \pm .009 \ ,$$

depending on the values used for the ρ→ππ and π→γγ couplings.

If we look carefully at Eq. (39), with the numerical values (40), it appears that a small error in A' would affect seriously the result for κ^S.

It seems thus preferable to compute λ' from (39) with the well known value of κ^S. This yields the following values of h':

(A) $\lambda' = .068 \pm .007$

(B) $\quad .075 \left\{ {+.09 \atop -.025} \right.$

(C) $\quad .076$

These values differ roughly by a factor 1.5 from those obtained by Gourdin in the VMD model. The order of magnitude is the same.

5. COMMENTS

A similar calculation can in principle be done for the isovector anomalous magnetic moment. The model for $m_L(z)$ would be given by the Born terms (including the π exchange), the ω exchange in the t-channel and the $\Delta(1236)$ exchange in the u-channel. However, if we want to treat the resonances in the "stable particle" approximation with simple propagators and coupling constants, it appears that the A contribution yields too divergent a behaviour and prevents us to determine the subtraction constants as was done in 3b). More realistic models for this contribution should be used as e.g. the model for the $I=3/2$, $J=3/2$ multipoles in the work of Adler[9] or Bijtebier[14].

The discrepancy of our values of the $\rho\pi\gamma$ coupling with the one obtained in the VMD model is not too serious in view of the approximations involved in both calculations. A good test for our model would be the comparison of the implicit values we obtain for the isoscalar multipole in Section 4 with recent multipole analysis of the photoproduction process.

Finally, we want to mention a similar calculation by Bluvstein, Cheskov and Dubovik[15] for the electromagnetic form factors at large values of $|q^2|$. It seems to us that the inelastic effects which show up in the use of the phase $\phi_1(\pm W)$ instead of $\phi_2(\pm W)$ or $\delta(\pm W)$, are not taken properly into account by these authors.

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