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Effect of NA Conversion on the Binding of Nuclear Matter*

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There have been two types of approaches regarding the effect of the NA conversion on the nuclear matter binding. They are by means of (i) the two-channel formalism, and (ii) three-body forces. They lead to qualitatively different results: the effect has been found to be repulsive in the former but attractive in the latter. The relationships between these two approaches is examined and an important difference concerning the treatment of three-body correlations is emphasized.

O efeito da conversão NA na ligação da matéria nuclear tem sido abordado de duas maneiras distintas, a saber, (i) o formalismo do canal duplo, e (ii) o formalismo das forças a três corpos. Os resultados obtidos são qualitativamente diferentes, o efeito sendo repulsivo no primeiro caso e attrativo no segundo. Examina-se a relação entre esses dois formalismos e destaca-se uma importante diferença com relação ao tratamento da correlação de três corpos.

1. INTRODUCTION

In recent years, there has been a surge of interest in the role played by the nucleon isobar $\Delta(1236)$ in various aspects of nuclear physics. One of such problems is concerned with the effect of A on the binding of nuclear matter, and a number of papers on this effect have appeared.

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All the calculations so far done can be classified into two types, which we will refer to as i) the two-channel formalism (TCF)[1], and ii) the one-channel formalism (OCF) with three-body (3N)-forces[2,3]. Although they both aim at estirnating the effect of A, the results of these two types of approaches are quite different. The binding energy of nuclear matter is reduced in the TCF, whereas it has been found to increase due to 3N-forces. The purpose of this paper is to examine the relationship between the two types of approaches, and emphasize that they significantly differ in the way three-body correlations are treated.

2. RELATION BETWEEN ONE – AND TWO-CHANNEL FORMALISMS

Let us examine the relation between the OCF and TCF and see how the same A-effect is treated in the two approaches. Much of what we are going to do below is a straightforward application of what was done for the $\Lambda\Sigma$ conversion[4]. The following processes in the two-body sistem

$$\text{(a) } NN \to NN, \quad \text{(b) } NN \leftrightarrow NA, \quad \text{(c) } NA \to NA \tag{1}$$

can be described in the TCF by means of a two-cornponent wave function

$$\Psi = \begin{pmatrix} \psi_N \\ \psi_\Delta \end{pmatrix} \tag{2}$$

where the suffixes N and A refer to NN and NA channels, respectively. We ignore the AA-combination for simplicity. The Schrödinger equation takes the form

$$(T_N + v_N)\psi_N + v_{N\Delta}\psi_\Delta = E\psi_\Delta \quad , \tag{3}$$

$$(T_\Delta + v_\Delta)\psi_\Delta + v_{\Delta N}\psi_N = (E-\Delta m)\psi_\Delta \quad , \tag{4}$$

where T_N and T_Δ are the kinetic energies in the NN and NA channels, respectively, and Δm ($\approx$ 300 MeV) is the NA mass difference. The units

are such that $c = \hbar = 1$. The potentials v_N, $v_{N\Delta}$ and $v_{\Delta N}$, and v_Δ correspond to the transitions (a), (b) and (c) in Eq. 1), respectively.

If the energy is well below the NA threshold, one can reduce the above TCF to the OCF by eliminating ψ_Δ. That is, from (4) one obtains

$$\psi_\Delta = - (T_\Delta + v_\Delta + \Delta m - E)^{-1} v_{\Delta N} \psi_N \ , \tag{5}$$

which is put into (3) to yield the Schrödinger equation in the OCF:

$$(T_N + V)\psi_N = E\psi_N \tag{6}$$

where we now deal with a one-component wave function ψ_N. The OCF potential, V, is related to v_N etc. by

$$V = v_N - v_{N\Delta}(T_\Delta + v_\Delta + \Delta m - E)^{-1} v_{\Delta N} \tag{7}$$

which we write as

$$V = v_N - v_{N\Delta} G_\Delta v_\Delta + v_{N\Delta} G_\Delta v_\Delta G_\Delta v_{\Delta N} - \cdots \ , \tag{8}$$

where $G_\Delta = (T_\Delta + \Delta m - E)^{-1}$. Even if the TCF potentials v_N etc. are simple local potentials, V in the OCF is in general a complicated potential, depending on the energy E. However, if $E << \Delta m \approx 300$ MeV it would not be very misleading to replace G_Δ by $(\Delta m)^{-1}$.

Next, let us consider a system which consists of three or more particles. The OCF potential V is related to v_N etc., in the TCF, by

$$V = \sum_{i,j} \{ v_N^{(ij)} - v_{N\Delta}^{(ij)} G_\Delta v_{\Delta N}^{(ji)} \} - \sum_{i} \sum_{j \neq k} v_{N\Delta}^{(ij)} G_\Delta v_{\Delta N}^{(ik)} + \cdots \ , \tag{9}$$

where e.g. $v_N^{(ij)}$ acts on the nucleon pair $(i \neq j)$. The two terms in the curly brackets are two-body forces, while the third term represents a 3N-force acting on nucleons (i,j,k). Note that we are assuming only two-body forces in the TCF. The OFC potential which is equivalent to the TCF potentials takes therefore the form

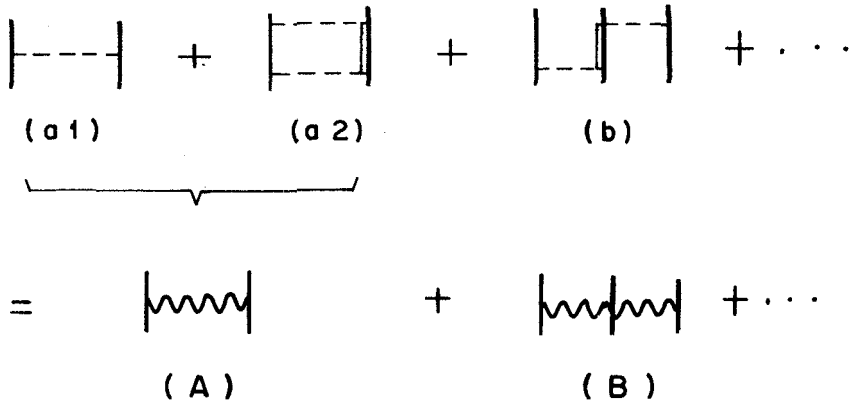

Fig.1 - Correspondence between diagrams in the TCF and OCF.

Table 1 - The expectation value of $\langle W \rangle$ (in MeV) of the TPE 3N-force given by Eqs. (13 ∿ 15) for various values of the cutoffs c and d (in F). The density of nuclear matter has been taken to be $\rho = .170\ F^{-3}$ or $p_F = 1.36\ F^{-1}$. A positive value of $\langle W \rangle$ means a repulsion. Note that the sign changes depending on d.

	d(f)	0.	0.4	0.6	1.0	1.4
c(F) =	0.6	6.24	4.35	1.70	-1.65	
	1.0	4.22	3.80	3.06	1.07	-0.30

$$V = \sum_{i,j} v^{(ij)} + \sum_{i,j,k} v^{(ijk)} + \ldots . \qquad (10)$$

Eqs.(9) and (10) are illustrated in Fig.1, where a dashed line corresponds to a TCF potential, while a wavy line to an OCF potential. Note there the correspondences

$$(a1) + (a2) = (A) , \quad (b) = (B) . \qquad (11)$$

In short, if one starts with the TCF with two-body forces only, and eliminates the A-component, one is led to the OCF with many-body forces.

Now, suppose that these few particles are imbedded in infinite nuclear matter. For simplicity, let us first assume that nuclear matter is represented by a Fermi gas. Then, the intermediate states such that the nucleon momentum p is less than the Fermi momentum p_F are suppressed because of the Pauli principle. Hence, the contributions from (a1) and (b) of Fig. 1 are reduced. In fact, if we take a one-pion-exchange type force for v's which flips the nucleon isospin, we find that the diagram (b) has no contribution to the binding of nuclear matter. In the OCF, however, (A) and (B) are note suppressed by the Pauli principle effect. This is because they are *instantaneous* interactions, the intermediate states containing A having been eliminated. Note thateven if (b) is completely suppressed, (B) remains unchanged and does contribute to the binding energy. If the two calculations in OCF and TCF are both correctly done, they should agree in the end. What happens is that

$$\langle \text{Pauli correction of (a2)} \rangle \approx \langle \text{3N-force (B)} \rangle , \qquad (12)$$

where $\langle\ \rangle$ stands for the expectation value in nuclear matter. In the approximation $G_\Delta = (\Delta m)^{-1}$, Eq.(12) holds exactly.

The Fermi gas model for nuclear matter is unrealistic. There are strong correlations between nucleons such that nucleons cannot be very close to each other, and this has to be taken into account in any realistic

calculation. In the presence of nucleon-nucleon correlations, the relation between the two types of calculations, i.e., the TCF and the OCF with 3N-forces, becomes complicated. Of courses, if both calculations are done correctly, they should again give the same answer. In practice, however, they usually yield difference results. This implies that two different approximations have been used. The problem then is to see where the difference comes from, and decide which approach is preferable.

3. TWO-PION-EXCHANGE THREE-BODY FORCE

The nature of the difference between the TCF and OCF, in the presence of nucleon-nucleon correlations, can be illustrated by means of the two-pion-exchange (TPE) 3N-force. This 3N-force has been derived after eliminating the A and also the pion degrees of freedom[3,5] and it corresponds to the 3N-force which arises from diagram (b) of Fig. 1 when the dashed line represents the pion exchange. If we denote the TPE 3N--force which arises from the pion exchanges $1 \leftrightarrow 3 \leftrightarrow 2$ by $W(1,2;3)$, its expectation value in the Fermi gas model turns out to be[6,7]

$$\langle W(1,2;3)\rangle = \frac{1}{4} C_p p^2 \iint d^3x d^3y \; D^2(p_F z) \; .$$

$$. \{1 + (3\cos^2\theta_{xy} - 1) T(x) T(y)\} Y(x) Y(y) \; . \qquad (13)$$

Here,

$$\vec{x} = \vec{r}_1 - \vec{r}_3, \; \vec{y} = \vec{r}_2 - \vec{r}_3, \; \vec{z} = \vec{x} - \vec{y}, \; \cos\theta_{xy} = (\vec{x}.\vec{y})/xy \; ,$$

and

$$T(x) = 1 + \frac{3}{\mu x} + \frac{3}{(\mu x)^2} \; , \qquad Y(x) = \frac{e^{-\mu x}}{\mu x} \; ,$$

$$D(p_F z) = 3 j_1(p_F z)/(p_F z) \; . \qquad (14)$$

The constants involved are the pion mass μ and nuclear matter density ρ and Fermi momentum p_F. For the strength parameter C_p of the poten-

tial, let us take the same value as those in previous calculations[2,7], namely C_P = 0.61 MeV. Note that we have not considered the pionic form factor in the above $W(1,2;3)$ for simplicity. Its inclusion does not affect our conclusion.

In deriving (13), nucleon-nucleon correlations have not been considered. In order to simulate the effect of the correlations, let us incorporate the following factor in the integral of (13);

$$f(x,y;z) = \theta(x-c)\theta(y-c)\theta(z-d) \ , \qquad (15)$$

where $\theta(x-c)$ = 1 (O) if $x>c$ ($x<c$). This factor suppresses the integrand when any of the nucleon-nucleon distances is very small. The reason why we introduce two cutoff distances, c and d, will become clear below.

Table 1 shows $\langle W(1,2;3)\rangle$ for various values of c and d. The point to be emphasized is that for a given value of c, the expectation value varies considerably as d varies. This is due to the factor $(3\cos^2\theta_{xy}-1)$ in (13) which may change sign depending on the configuration of the three particles, in particular, depending on the ratios d/c. The calculation with d=0 corresponds to the Pauli correction of the diagram (a2) in the TCF. The reason why the contribution in this case is repulsive is simple. The diagram (a2) gives rise to an attraction which is now suppressed by the Pauli principle. Hence, the Pauli correction of the TPE two-body force is repulsive. In this case, the cutoff c takes care of the correlation between the two nucleons in (a2), but no three-body correlations is taken into account. Even in a more sophisticated calculation, in which ladder diagrams are summed by means of the two-body Bethe-Goldstone equation[8], three-body correlation is not directly taken care of. They enter only indirectly through the background potential in which nucleons move.

4. CONCLUDING REMARKS

Although our analysis is admittedly very crude, it strongly suggests the importance of handling three-body correlations carefully. In a more

sophisticated calculation, effects other than that of the Pauli principle will appear. In fact, Day and Coester[1] pointed out that the so-called dispersion effects is more important than the Pauli principle effect. But this dispersion effect may also be sensitive to three-body correlations. Then, would the OCF with 3N-forces be really satisfactory? We have to remember that the OCF is after all an approximate substitute for the TCF. It is surely better to treat the NA conversion explicitly in the many-body calculation rather than to eliminate it, as in the OCF, before doing the rnany-body calculation. Our contention is that, if the effect of A is very sensitive to three-body correlations as we have indicated, the many-body calculation in the TCF should be done in such a way that the effects of A and three-body correlations were simultaneously taken care of in a consistent manner. A natural avenue of approach would be to solve the three-body Bethe-Goldstone equation in the TCF. If this could be done, it would be obviously better than any of the OCF and TCF calculations so far done.

Day and Coester[1] summarized the results of nuclear matter calculations done so far including their own in TCF. They plotted the saturation points in an energy-density plane, and showed thatall the points so far obtained lie in a narrow band while the empirical saturation point is clearly outside of the band. In view of this grave situation, it appears highly desirable to do a calculation which incorporates effects of A and three-body correlation sirnultaneously.

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