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Electromagnetic Structure of a Bound Nucleon*

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The effect of binding on the electromagnetic (e.m.) structure of a nucleon in a nucleus is examined by means of a model consisting of a single nucleon which is bound in a harmonic oscillator potential and also coupled to the pion field through the Chew-Low interaction. We consider the "two-pion contribution" to the e.m. structure. This is the part which is probably most susceptible to the binding effect. By the binding effect we mean the one which arises because the nucleon wave functions, in the intermediate state as well as in the initial and final states, are distorted by the potential in which the nucleon is bound. This may be compared to a similar correction to the impulse approximation for pion-nucleus scattering, Unlike the latter which is likely to be quite appreciable, we find that the binding correction to the e.m. structure of the nucleon is negligibly small. The socalled quenching effect due to the Pauli principie when there are other nucleons is also discussed.

Examina-se o efeito de ligação na estrutura electromagnética (e.m.) de um nucleon, em um núcleo, por meio de um modelo consistindo de um único nucleon que está ligado em um potencial de oscilador harmônico, e também acoplado ao campo de pions, através da interação de Chew-Low. Aquí, consideremos a "contribuição de dois pions" para a estrutura e.m.. Essa é a parte provavelmente a mais sensível ao efeito de ligação. Por efeito de ligação, entendemos o fato das funções de onda do nucleon, no estado intermediário bem como nos estados inicial e final, serem dis-

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torcidas pelo potencial no qual o nucleon esteja ligado. Isso pode ser comprado a uma correção semelhante à aproximação de impulso no espalhamento pion-núcleo. Todavia, ao contrário desse último caso em que o efeito é provavelmente muito apreciável, encontramos para a correção de ligação para a estrutura e.m. do nucleon uma quantidade desprezível. O assim chamado efeito "quenching" devido ao princípio de Pauli, quando há outros nucleons presentes, é discutido.

1. INTRODUCTION

From the meson-theoretical point of view the electromagnetic (e.m.) structure of the nucleon is due to the cloud of mesons which are constantly being emitted and absorbed by the nucleon. The simplest and yet importante contribution comes from the "two-pion process" which is depicted by the Feynman diagram of Fig.1. For a free nucleon this effect is calculated by using plane waves for the nucleon in the intermediate state as well as in the initial and final states. If however the nucleon is bound in an external potential, one should use bound state wave functions for the nucleon, in place of plane waves, and hence the structure of the bound nucleon will become different from that of the free nucleon. This is what we mean in this paper by the binding effect. This resembles the Lamb shift, in which the self-energy of a bound particle becomes different from that for a free one, due to the binding effect. Also, this effect should be distinguished from those which are related to quenching due the Pauli principle or to exchange currents ; the former does not require the presence of other nucleons.

Recently, it has been shown by means of a model calculation, which is very similar to the one given in the present paper, that the binding effect on the pion-nucleon scattering amplitude is likely to be very significant[1]. Although the binding effect on the e.m. structure of the nucleon in the above sense is usually neglected, it does not seem obvious that it is really negligible.

The purpose of this paper is to examine the binding effect by considering a nucleon which is bound by a harmonic oscillator potential and

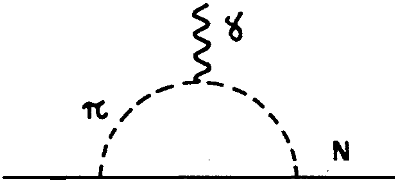

Fig.1. The two-pion contribution to the e.m. structure of the nucleon.

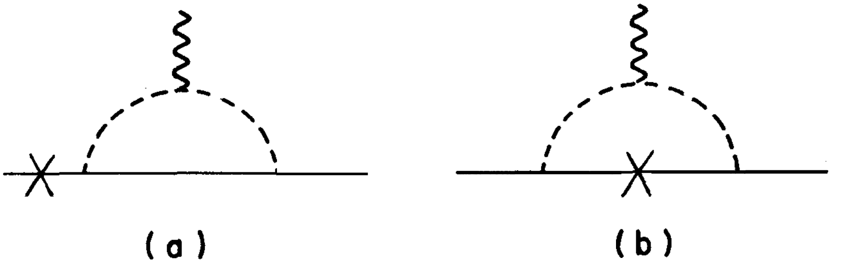

Fig.2. The cross indicates the action of the external potential.

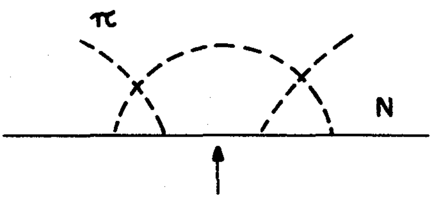

Fig.3. A typical diagram for pion-nucleon scattering. The arrow indicates the intermediate state through, which a large binding correction results.

is coupled to the pion field through the Chew-Low interaction. We calculate the e.m. form factor and then the charge and magnetic moments of the nucleon, due to' the two-pion diagram of Fig. 1, for free and bound nucleons. Unlike the pion scattering from a bound nucleon, we find that the binding effect on the e.m. structure of the nucleon is indeed very small. We also discuss the quenching effect due to the Pauli principle. In Sec.2, we define the model and obtain expressions for the charge and magnetic moment of the nucleon. Numerical results are given in Sec.3. We use units such that $c = \hbar = 1$.

2. DESCRIPTION OF THE MODEL AND CALCULATION

We consider a nucleon which is bound by a "shell-model potential" $V(r)$, and is also couples to the pion field through the Chew-Low interaction. Here we are assuming that all the effects of meson exchanges between nucleons in a nucleus are represented by the potential $V(r)$. The Hamiltonian consists of three parts:

$$H = H_N + H_\pi + H_I \ , \tag{2.1}$$

where

$$H_N = p^2/(2m) + V(r) \ , \tag{2.2}$$

while H_π and H_I are the standard Hamiltonians in the Chew-Low theory[2].

The e.m. structure of the nucleon due to the two-pion diagrarn (Fig.1) can be obtained by comparing the S-matrix element for the diagram with

$$S = -2\pi i\ \delta(0)\ e\ \{\rho(r)A_0(\vec{r}) - \vec{j}(\vec{r})\ .\ \vec{A}(\vec{r})\}\ , \tag{2.3}$$

where A_0 and $\vec{A}$ are the sacalar and vector potentials for the external e.m. field, and ρ and $\vec{j}$ are the charge and current densities of the nucleon, respectively. The charge density ρ consists of a part due to the e.m. structure of the nucleon itself and a part due to the nucleon density in the bound state. The calculation of S is very similar to that of Miyazawa[3] for a free nucleon, the only difference being that now the nucleon propagator is $[2\pi i(p_0 - E_\nu + i\varepsilon)]^{-1}$, where $E_\nu(\nu \equiv n,\ell,m)$ is an eigenvalue of H_N, i.e.,

$$H_N\psi_\nu = E_\nu\psi_\nu \qquad (E_0 \text{ is set to be zero}) , \tag{2.4}$$

and also the vertex part is given by the matrix element of H_I between two nucleon-states. We will confine ourselves to the case where the nucleon is in the ground state (ν=0) both in the initial and final states. Then we need matrix element $\langle\nu|H_I|0\rangle$, where ν stands for the intermediate state of the nucleon. In evaluating this matrix element, it is convenient to write the nucleon wave functions as

$$\psi_\nu = f_{n\ell}(r)\ Y_{\ell m}(\hat{r}) \ , \quad \hat{r} \equiv (\theta,\phi) \ . \tag{2.5}$$

Then, $\langle\nu|H_I|0\rangle$ can be reduced to the form

$$\langle\nu|H_I|0\rangle = i(4\pi)^{-1/2}\ (g/\mu)(2\pi)^{-3/2}\ .$$
$$. \ i^\ell \sum_\alpha \int d\vec{k}(2\omega)^{-1/2}\ G_\nu(k)\tau_\alpha\vec{\sigma}\cdot\vec{k}\{a_k - (-)^\ell a_k^\dagger\} \ , \tag{2.6}$$

where g is the renormalized coupling constant $((g^2/4\pi) = 0.08)$, $\omega_k = (k^2 + \mu^2)^{1/2}$ and

$$G_\nu(k) = \int r^2 dr\ j_\ell(kr)\ f_\nu(r)\ f_0(r) \ . \tag{2.7}$$

Other notations are the same as in Refs. 2 and 3. The S-matrix element is given by[3]

$$S = -\ 2\pi i\ \delta(0)\ e\ \tau_3\ \frac{8\pi(g/\mu)^2}{(2\pi)^6(2\pi i)} \sum_\nu \int_{-\infty}^{\infty} d\xi \int d\vec{k} \int d\vec{k}' \ .$$
$$\frac{(\vec{\sigma}.\vec{k})(\vec{\sigma}.\vec{k}')e^{i(\vec{k}'-\vec{k}).\vec{r}}\ Y^*_{\ell m}(\hat{k})\ Y_{\ell m}(\hat{k}')\ G_\nu(k)\ G_\nu(k')}{(\xi^2 - \omega^2 + i\varepsilon)(\xi^2 - \omega'^2 + i\varepsilon)(\xi - E_\nu + i\varepsilon)}$$

$$. \{2\xi A_0(\vec{r}) - (\vec{k} + \vec{k}') . \vec{A}(\vec{r})\} \quad . \tag{2.8}$$

Comparing this S with (2.3), we find

$$\rho(r) = \frac{4(g/\mu)^2}{(2\pi)^5} \tau_3 \sum_\nu \int d\vec{k} \int d\vec{k}' \; .$$

$$\frac{\vec{k}.\vec{k}' e^{i(\vec{k}'-\vec{k}).r} \; Y^*_{\ell m}(\hat{k}) \; Y_{\ell m}(\hat{k}') G_\nu(k) \; G_\nu(k')}{(\omega + \omega')(\omega + E_\nu)(\omega' + E_\nu)} \tag{2.9}$$

and

$$\vec{J}(\vec{r}) = \frac{2(g/\mu)^2}{(2\pi)^5} \tau_3 \sum_\nu \int d\vec{k} \int d\vec{k} \; \frac{(\vec{k} + \vec{k}')(\sigma . \vec{k})(\sigma . \vec{k}') \; e^{i(\vec{k}'-\vec{k}).\vec{r}}}{\omega \omega'(\omega + E_\nu)(\omega' + E_\nu)}$$

$$. \left(1 + \frac{E_\nu}{\omega + \omega'}\right) \; Y^*_{\ell m}(\hat{k}) \; Y_{\ell m}(\hat{k}') \; G_\nu(k) \; G_\nu(k') \; . \tag{2.10}$$

The charge $Q = \int d\vec{r} \, \rho(r)$ is given by

$$Q = \tau_3 \frac{\alpha}{\mu^2} \sum_{n,\ell} (2\ell + 1) \int_0^\infty dk \; \frac{k^4 \; G^2_\nu(k)}{\omega(\omega + E_\nu)^2} \quad , \tag{2.11}$$

where $\alpha = (2/\pi)(g^2/4\pi)$. Here the summation for the quantum number m has been performed. This Q is the charge which corresponds to Fig.1, and not the entire charge of the nucleon. To obtain the magnetic moment κ, recall the relation $\kappa\sigma = \frac{1}{2} \int (\vec{r} \times \vec{j}) d\vec{r}$ and use the trick $\vec{r} \to -\frac{1}{2} i(\vec{\nabla}_{k'} - \vec{\nabla}_k)$, which leads to

$$\kappa = \tau_3 \; \frac{\beta}{\mu^2} \sum_{n,\ell} (2\ell+1) \int_0^\infty dk \; \frac{k^4 \; G_\nu{}^2(k)}{\omega^2(\omega + E_\nu)^2} \left(1 + \frac{E_\nu}{2\omega}\right) \tag{2.12}$$

where $\beta = (4/3\pi)(g^2/4\pi)$. In the following, the factor τ_3 in Q and κ will be suppressed.

So far we have not specified the "shell model potential" $V(r)$, which we now assume to be a harmonic oscillator potential,

$$V(r) = \frac{1}{2} m \eta^2 r^2 . \tag{2.13}$$

Then E_ν, f_ν and $G_\nu(k)$ are all explicitly known. In particular,

$$E_\nu = E_n = n\eta . \tag{2.14}$$

Since E_ν is independent of ℓ, the ℓ-summations in Eqs. (2.11) and (2.12) can be easily done (see Appendix) and we obtain

$$Q = \sum_{n=0}^{\infty} Q_n = \frac{\alpha}{\mu^2} \sum_{n=0}^{\infty} \frac{1}{n!} \int_0^\infty dk \, \frac{k^4 (b^2k^2/2)^n}{\omega(\omega + E_n)^2} \exp(-\frac{b^2k^2}{2}) \tag{2.15}$$

and

$$\kappa = \sum_{n=0}^{\infty} \kappa_n = \frac{\beta}{\mu^2} \sum_{n=0}^{\infty} \frac{1}{n!} \int_0^\infty dk \, \frac{k^4 (b^2k^2/2)^n}{\omega^2(\omega + E_n)^2} (1 + \frac{E_n}{2\omega}) \exp(-\frac{b^2k^2}{2}) \tag{2.16}$$

where $b^2 = 1/(m\eta)$.

The summations for n in the above formulas can be converted into integrations using

$$\sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{(\omega + n\eta)^m} \left(\frac{K}{\eta}\right)^n = \frac{1}{(m-1)!} \int_0^\infty d\lambda \, e^{-\lambda\omega} \lambda^{m-1} \exp\left(\frac{K}{\eta} e^{-\lambda\eta}\right), \tag{2.17}$$

where $m \geq 1$ is an integer. We then obtain

$$Q = \frac{\alpha}{\mu^2} \int_0^\infty dk \, \frac{k^4}{\omega} \int_0^\infty d\lambda \, e^{-\lambda\omega} \, \lambda \, \exp\left[\frac{K}{\eta} (e^{-\lambda\eta} - 1)\right] \tag{2.18}$$

and

$$= \frac{\beta}{\mu^2} \int_0^m dk \quad \int_0^\infty d\lambda \, e^{-\lambda\omega} \, \lambda (1 + \frac{K e^{-\lambda\eta}}{2\omega}) \, \exp\left[\frac{K}{\eta} (e^{-\lambda\eta} - 1)\right], \tag{2.19}$$

where $K = k^2/(2m)$ and $K/\eta = (bk)^2/2$.

Next, let us examine two limiting cases, the loose-binding limit $(\eta \to 0)$ and the tight-binding limit $(\eta \to \infty)$. In the loose-binding limit, the λ-integration in (2.18) and (2.19) can easily be done by expanding the integrands into power series of η. The first terms in Q and κ give

$$Q_L = \lim_{\eta \to 0} Q = \frac{\alpha}{\mu^2} \int_0^\infty dk \, \frac{k^4}{\omega (\omega + K)^2} \tag{2.20}$$

and

$$\kappa_L = \lim_{\eta \to 0} \kappa = \frac{\beta}{\mu^2} \int_0^\infty dk \, \frac{k^4}{\omega^2 (\omega + K)^2} \quad (1 + \frac{K}{2\omega}) \quad , \tag{2.21}$$

which we interpret as those for a free (unbound) nucleon. The second terms of the expansions lead to

$$\Delta Q_L = \frac{3\alpha}{2(\mu m b)^2} \int_0^\infty dk \, \frac{k^6}{\omega (\omega + K)^4} \tag{2.22}$$

and

$$\Delta\kappa_L = \frac{\beta}{(\mu m b)^2} \int_0^\infty dk \, \frac{k^6}{\omega^2 (\omega + K)} \, (1 + \frac{K}{4\omega}) \tag{2.23}$$

which represent binding corrections. In the tight-binding limit, on the other hand, only the $n=0$ terms in (2.15) and (2.16) remainand also $\exp(-b^2 k^2/2) \to 1$. Hence, we arrive at

$$Q_T = (\alpha/\mu^2) \int_0^\infty dk \ (k^4/\omega^3), \tag{2.24}$$

$$\kappa_T = (\beta/\mu^2) \int_0^\infty dk \ (k^4/\omega^4), \tag{2.25}$$

which are identical with the corresponding quantities in the *static* Chew-Low theory in which the nucleon is held fixed[3,4]. Note that Q_T and κ_T are obtained from Q_L and κ_L by ignoring the nucleon recoil energy, K. Also the so-called closure approximation in which the excitation energy E_n in (2.15) and (2.16) is ignored gives Q_T and κ_T.

So far we considered only one nucleon bound in the $n=0$ state. If there are other nucleons, the intermediate states which are occupied by them have to be excluded in the summations in (2.15) and (2.16). Note that, since the pion considered is charged, the charge of the nucleon in the intermediate state has to be different from that of the initial state. For example $n=0$ for ^{4}He and $n=0$ and 1 for ^{16}O have to be excluded. This is the so-called quenching effect due to the Pauli principle, and is equivalent to the effect of the exchange current[5]. Of course the two-pion process contributes only to the isovector part of the e.m. structure and hence its total contribution to an isoscalar nucleus such as ^{4}He and ^{16}O is null. It would be still of interest to see how the nucleons are individually modified.

3. RESULTS AND DISCUSSIONS

For all the κ-integrations in the preceeding Section, it should be understood that there is a cut-off factor $v^2(k)$. We present the results for

$$v^2(k) = \exp(-k^2/\Lambda^2), \tag{3.1}$$

with $\Lambda = 6\mu$. The cut-off parameter $\Lambda \lesssim 7\mu$ was found reasonable in the recent analysis of pion-nucleon scattering using the Chew-Low theory[6].

The pion-nucleon coupling constant is taken to be $g^2/4\pi=0.08$, and also $m=6.8\mu$, $\mu=138$ MeV are used.

For the harmonic oscillator potential,

$$\eta = 41\ A^{-1/3}\ \text{MeV} \quad , \tag{3.2}$$

leads to a reasonable nuclear size for the nucleus of mass number A (Ref.7). Eq.(3.2) gives $b=1.0\ A^{1/6}$F, and hence $b=1.3$ F for $A = 4$ and $b=1.6$ F for $A=16$. These values may be compared with $b=1.31$ F for ^{4}He and $b=1.76$ F for ^{16}O, which were obtained from analyses of the charge form factors[8]. As a typical value we choose $b=1.4$ F, which corresponds to $\eta=21$ MeV.

Table 1 shows the results for Q and κ for $b=0$, 1.4 F, and ∞ . The binding effect, which is the difference between $b=1.4$ F and $b=\infty$, is very small. It is about 1% for the magnetic moment, and slightly larger for the charge. These corrections are accurately given by (2.22) and (2.23). We have also examined the root-mean-square radius for the charge distribution and found that the binding correction is of the order of 1%. Therefore, we conclude that the binding effect on the nucleon structure in a nucleus is negligible.

TABLE 1

b(F)	0	1.4	∞	
Q	0.807	0.437	0.430	$(\Delta Q = 6.9 \times 10^{-3})$
κ	1.597	1.113	1.103	$(\Delta\kappa = 1.17\times 10^{-2})$

Table 1 - Charge Q and magnetic moment κ [in units of $e/(2m)$] . The binding effect on Q is $\Delta Q = Q(b=1.4\text{ F}) - Q(b=\infty)$.

The effect we have found is much smaller than that was estimated before by a different method[9]. As was pointed to us by Goebel (private communication), our previous calculation did not take account of the wave function renormalization [Fig.2 (a)] which largely cancels the effect considered [Fig.2 (b)]. The static approximation which we previously used is inadequate for handling the effect of Fig.2(a) because one of the nucleon propagators for it becomes infinite. Our present calculation is free from this difficulty because the nucleon motion is taken care of by using bound-state wave functions throughout. We therefore withdraw our previous suggestion that the binding effect in the sense as we have discussed may be very important in understanding the magnetic moments of ^3He and ^3H.

In a similar model calculation for pion-nucleus scattering at medium energies, it was found that the binding effect results in an increase of the pion-bound nucleon scattering cross section in the 3-3 resonance region by 50 to 70%. However, it was also pointed out that this large binding effect stems from processes in which the energy of the intermediate state can coincide with that of the initial state (see Fig.3). For the e.m. structure there are no such processes involved.

The Table 2 shows the first several terms in the n-summations. As pointed out at the end of Sec.2, some of these terms are quenched in a nucleus because of the Pauli principle. One can see from this table that the quenching effect can be quite appreciable, and an order of magnitude larger than the binding effect.

TABLE 2

n	0	1	2	3	4	total
$10^3 Q_n$	19	24	24	24	22	430
$10^3 \kappa_n$	97	105	95	83	72	1103

Table 2 - Contributions from intermediate states.

I would like to thank Professor C. J. Goebel for his comments on our previous calculation[9], Mr. R. V. Gerstenberger for collaboration in an earlier stage of the work and Mr. S. A. Abbas for his help in computation. Part of this work was done during my visit to the very hospitable **Departamento de** *Física, Universidade Federal de Pernambuco.* I would like to thank the National Research Council (CNPq) of Brazil and the NRC of Canada for giving me this opportunity under the Brazil - Canada Scientific Exchange Program.

APPENDIX

Our quantum numbers $\nu=(n,\ell,m)$ are the same as those in Ref. 10 which gives $f_\nu(r)$ explicitly. The integration in (2.10) for the harmonic oscillator can be done easily with the result

$$G_\nu(k) = c_\nu\,(bk)^n \exp(-b^2k^2/4)\ , \qquad (A.1)$$

$$c_\nu{}^2 = 1/\left[2^{2n+1}\ p!\,(p+\ell+\tfrac{1}{2})\,!\right]\ , \qquad (A.2)$$

where $p = (n-\ell)/2 > 0$. To do the R-summation (for a fíxed value of n) it is in fact unnecéssary to know c_ν explicitly. We only need

$$\sum_\ell\,(2\ell+1)c_\nu{}^2 = 1/(2^n n!)\,, \qquad (A.3)$$

which follows from the following identities:

$$1 = \sum_\nu <0|e^{-ik.r}|\nu><\nu|e^{ik.r}|0>$$

$$= \sum_{n,\ell}\ (2\ell+1)\ G_\nu{}^2(k)$$

$$= \sum_{n,\ell}\ (2\ell+1)c_\nu{}^2(bk)^{2n}\exp(-b^2k^2/2)\,. \qquad (A.4)$$

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