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On the Zero Mass Limit of the Massive U(n) Thirring Model

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The zero mass limit of the massive Thirring model, with internal $U(n)$ symmetry, I studied in perturbation theory. We show that, on the curve in the space of coupling constants, where all vector and axial-vector currents are conserved, one may introduce an intrinsic parametrization for the interaction of the zero mass theory. This means that the interaction takes the current-current form.

Estuda-se, aqui, em teoria de perturbação, o modelo massivo de Thirring em seu limite de massa evanescente. Mostra-se que se pode, ao longo da curva no espaço das constantes de acoplamento onde todas as correntes vetoriais e axiais se conservam, introduzir uma parametrização intrínseca para a interação, no limite estudado. Significa isso que a interação toma a forma corrente-corrente.

1. INTRODUCTION

The massive Thirring model, with internal ~ (n)symmetry, has revealed itself as a very interesting laboratory to study questions related to asymptotic scale invariance. Since it does not appear to be exactly soluble, it is important to extract from it as much information as possible by making use of perturbation theory[1] One of the most important objects to study are the vector and axial-vector currents of the model, since they play a central role in any discussion, including the

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one aiming at an exact solubility[2]. In renormalized perturbation theory, on the other hand, the Lagrangian is not written in terms of currents, but in terms of four-fermion interactions multiplied by coupling constants g_i, $i=1,2,3$. They are not renormalization invariant and their numerical values do not tell us anything about the most important properties of the theory, namely, whether its massless limit possesses conserved axial currents.

It is, therefore, useful to eliminate the four-fermion interactions as much as possible in favour of current-current interactions, since, in the case the currents are conserved, their Green's functions can easily be obtained by integration of their Ward-Takahashi identities[3]. Besides this, the new coupling constants d_i, $i=1,2,3$, do now tell us how far we are away from, say, asymptotic scale invariance. We call, such a parametrization, intrinsic[4].

In Section 2, we study asymptotic scale invariance using a generalized Taylor subtraction scheme. In the third and final Section, we show that, out of all zero mass limit theories, the one which is scale invariant admits an intrinsic parametrization: we can rewrite its equation of motion in terms of limits of currents and fields, and in such a way that it takes the form one would naively expect.

2. PERTURBATIVE DISCUSSION

In this Section, methods of renormalized perturbation theory will be applied to the Thirring model[5], with an internal $U(n)$ symmetry, aiming at a classification of various situations according to the number of conserved currents.

The perturbative treatment can be done in a systematic way through normal product methods. We consider the model described by the following effective Lagrangian density:

$$L_{\text{eff}} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}\psi - m\bar{\psi}\psi + \frac{g_1-c_1}{2}(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi) + \frac{g_2-c_2}{2}(\bar{\psi}\gamma_{\mu}\vec{\lambda}\psi)(\bar{\psi}\gamma^{\mu}\vec{\lambda}\psi) + \frac{g_3-c_3}{2}(\bar{\psi}\psi)(\bar{\psi}\psi), \qquad (2.1)$$

where $\vec{X} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ are matrices of the fundamental representation of $SU(n)$, Ref.6. The finite constants, c_1, c_2 and c_3, are mass independent counter terms, fixed by normalization conditions to be specified later. The effective Lagrangian (2.1) is the most general one satisfying the requirements of renormalizability, Lorentz covariance, parity, charge conjugation and $U(n)$ symmetry. The Green's functions are given by the usual finite part of the Gell-Mann and Low formula, with a subtraction scheme which uses the forest formula with modified "Taylor" operators[7]:

Logarithmically divergent integrands require one subtraction:

$$\tau^{(0)} F(p,m) = F(0,\mu), \tag{2.2}$$

whereas linearly divergent integrands should be subtracted with $\tau^{(1)}$:

$$\tau^{(1)} F(p,m) = F(0,0) + p^{\mu} \left.\frac{\partial F}{\partial p^{\mu}}\right|_{\substack{p=0\\ m=\mu}} + m \left.\frac{\partial F}{\partial m}\right|_{\substack{p=0\\ m=\mu}} . \tag{2.3}$$

The subtraction scheme (2.2-3) is convenient for the derivation of homogeneous parametric differential equations for the N-point functions[5]. It is also clear that this subtraction scheme produces Green's functions which are well defined in the $m \to 0$ limit.

Defining the 2N-point vertex function $\Gamma^{(2N)}$ by

$$(2\pi)^2 \, \delta\left(\sum_{i=1}^{N} p_i + \sum_{i=1}^{N} q_i \right) \Gamma^{(2N)} (p_1, p_2, \ldots, p_N, q_1, q_2, \ldots, q_N)$$

$$= \int \prod_{i=1}^{N} dx_i dy_i \exp\{i \sum_{i=1}^{N} (p_i x_i + q_i y_i) <0| \mathrm{T} \prod_{i=1}^{N} \psi(x_i) \bar{\psi}(y_i) |0>^{\mathrm{prop}}\} ,$$

where the superscript "prop" indicates that only proper (amputated, one particle irreducible) diagrams are included, we obtain, from (2.2) and (2.3), the results

$$\left.\frac{\partial\Gamma^{(2)}}{\partial p^{\mu}}\right|_{\substack{p=0\\ m=\mu}} = i\gamma_{\mu}\ , \quad \left.\frac{\partial\Gamma^{(2)}}{\partial m}\right|_{\substack{p=0\\ m=\mu}} = -\,i, \quad \left.\Gamma^{(2)}\right|_{\substack{p=0\\ m=0}} = 0. \tag{2.4}$$

Using methods similar to those of Ref.7, the following differential equation can be established:

$$\left[\mu\frac{\partial}{\partial\mu} + \delta m\frac{\partial}{\partial m} + \sum_{i=1}^{3} \beta_{i}\frac{\partial}{\partial g_{i}} - 2N\gamma\right]\Gamma^{(2N)} = 0\ , \tag{2.5}$$

where 6, $\beta_i (i=1,2,3)$ and γ are mass independent functions of g_i, $i=1,2,3$, which can be obtained from the equation above by application of the normalization conditions satisfied by $\Gamma^{(2N)}$.

Normal products are introduced in the usual way. In particular, for proper functions containing only one special normal product vertex, the following notation will be used:

NORMAL PRODUCT	***NOTATION***	
$N_1\left[\bar{\psi}\gamma_{\mu}\psi\right]$	$\Gamma_{1,\mu}^{(2N)}$	
$N_1\left[\bar{\psi}\gamma^5\psi\right]$	$\Gamma_{2}^{(2N)}$	
$N_1\left[\bar{\psi}\gamma_{\mu}\lambda^{a}\psi\right]$	$\Gamma_{3,\mu,a}^{(2N)}$	
$N_1\left[\bar{\psi}\gamma^5\lambda^{a}\psi\right]$	$\Gamma_{4,a}^{(2N)}$	(2.6)
$N_2\left[(\bar{\psi}\gamma^5\psi)\,(\bar{\psi}\psi)\right]$	$\Gamma_{5}^{(2N)}$	
$N_2\left[(f_{abc}\bar{\psi}\gamma^{\mu}\gamma^5\lambda^{b}\psi)\,(\bar{\psi}\gamma_{\mu}\lambda^{c}\psi)\right]$	$\Gamma_{6,a}^{(2N)}$	
$N_2\left[(\bar{\psi}\gamma^5\lambda^{a}\psi)\,(\bar{\psi}\psi)\right]$	$\Gamma_{7,a}^{(2N)}$	

These normal product vertex functions satisfy differential equations analogous to (2.5), namely,

$$[D-(2N-2)\gamma]\begin{Bmatrix}\Gamma_{1,\mu}^{(2N)}\\ \Gamma_{2}^{(2N)}\\ \Gamma_{3,\mu a}^{(2N)}\\ \Gamma_{4,a}^{(2N)}\end{Bmatrix}=\begin{Bmatrix}t_1\Gamma_{1,\mu}^{(2N)}\\ t_2\Gamma_{2}^{(2N)}\\ t_3\Gamma_{3,\mu a}^{(2N)}\\ t_4\Gamma_{4,a}^{(2N)}\end{Bmatrix}.(\delta-1)\;, \qquad (2.7)$$

$$[D-(2N-4)\gamma]\begin{Bmatrix}\Gamma_{5}^{(2N)}\\ \Gamma_{6,a}^{(2N)}\\ \Gamma_{7,a}^{(2N)}\end{Bmatrix}=\begin{Bmatrix}\delta_1 p^{\mu}\varepsilon_{\mu}{}^{\lambda}\Gamma_{1,\lambda}^{(2N)}+\delta_2\Gamma_{5}^{(2N)}\\ \delta_3 p^{\mu}\varepsilon_{\mu}{}^{\lambda}\Gamma_{3,\lambda a}^{(2N)}+\delta_4\Gamma_{6,a}^{(2N)}+\delta_5\Gamma_{7,a}^{(2N)}\\ \delta_6 p^{\mu}\varepsilon_{\mu}{}^{\lambda}\Gamma_{3,\lambda a}^{(2N)}+\delta_7\Gamma_{6,a}^{(2N)}+\delta_8\Gamma_{7,a}^{(2N)}\end{Bmatrix}.(\delta-1)\;,$$

where $6_1,\ldots,\delta$, and $t_1,\ldots,t_4$, are functions of the g_i's, known from perturbation theory, and

$$D=\delta m\frac{\partial}{\partial m}+\mu\frac{\partial}{\partial\mu}+\sum_{i=1}^{3}\beta_i\frac{\partial}{\partial g_i}\;.$$

The model (2.1) has two conserved vector currents,

$$p^{\lambda}\Gamma_{1,2}^{(2N)}=i\sum_{k=1}^{N}\left[\Gamma^{(2N)}(\ldots,q_k+p,\ldots)-\Gamma^{(2N)}(\ldots,p_k+p,\ldots)\right]\;, \qquad (2.8a)$$

$$p^{\lambda}\Gamma_{3,\lambda a}^{(2\;)}=i\sum_{k=1}^{N}\left[\lambda^{a}\Gamma^{(2N)}(\ldots,q_k+p,\ldots)-\lambda^{aT}\Gamma^{(2N)}(\ldots,p_k+p,\ldots)\right], \qquad (2.8b)$$

whereas, for the axial currents, we have

$$(\alpha-e_1)p^\lambda\varepsilon_\lambda{}^\mu\Gamma_{1,\mu}^{(2N)} = -2m\Gamma_2^{(2N)} + \alpha_1\Gamma_5^{(2N)} +$$

$$+ i\alpha\sum_{k=1}^{N}\left[\Gamma^{(2N)}(\ldots,q_k+p,\ldots)\gamma_5^{p_k}\,\Gamma^{(2N)}(\ldots,p_k+p,\ldots)\right] \tag{2.8c}$$

$$(\eta-e_2)p^\lambda\varepsilon_\lambda{}^\mu\Gamma_{3,\mu a}^{(2N)} = -2m\,\Gamma_{4,a}^{(2N)} + \alpha_2\Gamma_{6,a}^{(2N)} + \alpha_3\Gamma_{7,a}^{(2N)}$$

$$+ i\eta\sum_{k=1}^{N}\left[\Gamma^{(2N)}(\ldots,q_k+p,\ldots)\gamma_5^{q_k}\lambda^a + \gamma_5^{p_k}\lambda^{aT}\Gamma^{(2N)}(\ldots,p_k+p,\ldots)\right] . \tag{2.8d}$$

The anomalies in (2.8c) and (2.8d) come from the two identities

$$2\alpha N_2\left[m\bar\psi\gamma^5\psi\right] = 2mN_1\left[\bar\psi\gamma^5\psi\right] + e_1N_2\left[\partial^\mu(\bar\psi\gamma_\mu\gamma^5\psi)\right] + \lambda_1N_2\left[(\bar\psi\gamma^5\psi)(\bar\psi\psi)\right] ,$$

$$2\eta N_2\left[m\bar\psi\gamma^5\lambda^a\psi\right] = 2mN_1\left[\bar\psi\gamma^5\lambda^a\psi\right] + e_2N_2\left[\partial^\mu(\bar\psi\gamma_\mu\gamma^5\lambda^a\psi)\right]$$

$$+ \lambda_2 f_{abc}N_2\left[(\bar\psi\gamma^\mu\gamma^5\lambda^b\psi)(\bar\psi\gamma_\mu\lambda^c\psi)\right] + \lambda_3N_2\left[(\bar\psi\gamma^5\lambda^a\psi)(\bar\psi\psi)\right] , \tag{2.9}$$

where

$$\alpha_1 = 2(g_3 - c_3)\alpha - \lambda_1 , \quad \alpha_2 = 2i(g_2 - c_2)\eta - \lambda_2 , \quad \alpha_3 = 2(g_3 - c_3)\eta - \lambda_3 ,$$

and

$$\gamma^5\alpha = 2\frac{\partial}{\partial m}\langle 0|TmN_1\left[\bar\psi\gamma^5\psi\right](0)\tilde{\bar\psi}(0)\tilde\psi(0)|0\rangle^{\text{prop}}\Big|_{m=\mu} ,$$

$$e_1\gamma_\mu\gamma^5 = \frac{2\mu}{i}\frac{\partial}{\partial q^\mu}\langle\ |TN_1\left[\bar\psi\gamma^5\psi\right](0)\tilde{\bar\psi}(q/2)\tilde\psi(q/2)|0\rangle^{\text{prop}}\Big|_{\substack{q=0\\ m=\mu}} ,$$

$$2n(n-1)\lambda_1 = -\mu\langle 0|TN_1[\bar\psi\gamma^5\psi](0)(\tilde{\bar\psi}(0)\gamma^5\tilde\psi(0))\tilde{\bar\psi}(0)\tilde\psi(0)|0\rangle^{prop}\Big|_{m=\mu}$$

and analogous formulae for η, e_2, A_1 and λ_3.

One interesting feature of the renormalized theory is the possibility of asymptotic conservation of the axial currents, though classically this is impossible. Formulae (2.7) and (2.8) can be used to obtain a number of interesting results:

i) By applying D to both sides of (2.8a) and (2.8b), and using (2.7), we obtain

$$(\delta-1)t_1 = (\delta-1)t_3 = 2\gamma \ ,$$

so that, at an eigenvalue $\bar g_0 = (g_{10}, g_{20}, g_{30})$, defined by

$$\beta_1 = \beta_2 = \beta_3 = 0,$$

all currents will scale canonically in the asymptotic region. Such a line of fixed points has been investigated, by Mitter and Weisz[1], in the neighborhood of the origin $g_1=g_2=g_3=0$.

ii) There exists a surface, S_1, in the space of coupling constants g_1, g_2, g_3, passing through the origin, $g_1=g_2=g_3=0$, and on which $\beta_1 = 0$. On S_1, the isoscalar axial current[8] is asymptotically conserved.

Applying the differential operator

$$\tilde D = m\frac{\partial}{\partial m} + \mu\frac{\partial}{\partial\mu} + \sum_{i=1}^{3}\beta_i\frac{\partial}{\partial g_i} \qquad (2.10)$$

to both sides of (2.8c), and using

$$\frac{\partial\Gamma^{(2N)}}{\partial m} = -\Delta_0\Gamma^{(2N)},$$

and

$$(\alpha-e_1)(-\tilde{p}^{\lambda}\Delta_0\Gamma_{1,\lambda}^{(2N)}) = i\alpha\sum_{k=1}^{N}\Big[\Delta_0\Gamma^{(2N)}(\ldots,q_k+p,\ldots)\gamma_5^{q_k} +$$

$$+\gamma_5^{p_k}\Delta_0\Gamma^{(2N)}(\ldots,p_k+p,\ldots)\Big]-2\big[-\alpha+m(\Delta_0-t_2)\big]\Gamma_2^{(2N)}+\alpha_1\Delta_0\Gamma_5^{(2N)} ,$$

where A, is the soft mass vertex insertion, we obtain the equations

$$\alpha\,\tilde{D}(1-\frac{e_1}{\alpha}) - (\delta-1)\alpha_1\delta_1 = 0,$$

$$1 - 2\gamma - \frac{\tilde{D}\alpha}{\alpha} - (\delta-1)\alpha = 0, \tag{2.11}$$

$$\alpha_1\frac{D\tilde{\alpha}}{\alpha} - \tilde{D}\alpha_1 + 4\gamma\alpha_1 - (\delta-1)\alpha_1\delta_2 = 0$$

We now choose g_3-c_3 so that $\alpha_1 = 0$ and, furthermore, c_1 is chosen so that e_1/α depends only on g_1 (this is always possible in perturbation theory, since $(e_1/\alpha) = A(g_1-c) +$ higher order terms, with $A\neq 0$). Thus , from the first of the equations (2.10) we get

$$\tilde{D}\frac{e_1}{\alpha} = 0 \quad \text{implying} \quad \beta_1(g_1,g_2,g_3(g_1,g_2)) = 0 .$$

iii) There exists another surface, S_2, also passing through the origin, on which $\beta=0$. In the case of $SU(2)$, where $\Gamma_{6,a}^{(2N)}$ and $\Gamma_{7,a}^{(2N)}$ are linearly dependent, the isovector axial current is conserved on S_2. Thus, only for $SU(2)$ does S_2 have an intrinsic meaning.

If we apply $\tilde{D}$ to both sides of (2.8d), we will get

$$\eta\tilde{D}(1-\frac{e_2}{\eta}) - (\delta-1)(\alpha_2\delta_3 + \alpha_3\delta_6) = 0,$$

$$1 - 2\gamma - \frac{\tilde{D}\eta}{\eta} - (\delta-1)\eta = 0, \tag{2.12}$$

$$\alpha_2 \frac{\mathcal{D}\eta}{\eta} - \tilde{\mathcal{D}}\alpha_2 + 4\gamma\alpha_2 - (\delta-1)(\alpha_2\delta_4 + \alpha_3\delta_7) = 0 \ ,$$

$$\alpha_3 \frac{\tilde{\mathcal{D}}\eta}{\eta} - \tilde{\mathcal{D}}\alpha_3 + 4\gamma\alpha_3 - (\delta-1)(\alpha_2\delta_5 + \alpha_3\delta_8) = 0 \ .$$

Thus, if c_2 is chosen so that e_2/η depends only on g_2, and g_2 is fixed so that $\alpha_2\delta_3 + \alpha_3\delta_6 = 0$, it follows that

$$\beta_2(g_1, g_2(g_1, g_3), g_3) = 0 \ .$$

iv) The surfaces S_1 and S_2 intersect along a line L_1, passing through the origin, along which we have

$$\beta_i(g_1, g_2(g_1),\ g_3(g_1)) = 0 \ , \quad i = 1,2,3 \ , \tag{2.13}$$

and along which all axial currents are asymptotically conserved.

Now, $\beta_3 = 0$ follows from the last equation (2.11), which becomes

$$\beta_3 \frac{\partial\alpha_1}{\partial g_3} = 0 \ . \tag{2.14}$$

Conservation of the axial currents requires $\alpha_1 = \alpha_2 = \alpha_3 = 0$. From (2.12) , we have the homogeneous equations

$$\begin{aligned} &\alpha_2\left[4\gamma + \delta_4(1-\delta)\right] + \alpha_3\left[(1-\delta)\delta_7\right] = 0 \ , \\ &\alpha_2\left[(1-\delta)\delta_5\right] + \alpha_3\left[4\gamma + (1-\delta)\delta_8\right] = 0 \ . \end{aligned} \tag{2.15}$$

Mitter and Weisz[1] have verified that, in lowest order, a determinant similar to the one of Eq. (2.15) is nonvanishing, implying

$$\alpha_2 = \alpha_3 = 0 \ . \tag{2.16}$$

3. EQUATION OF MOTION IN TERMS OF CURRENTS

In this Section, we want to rewrite the equation of motion satisfied by the field ψ, described by the effective Lagrangian (2.1),

$$\langle 0|T\{(i\not\partial-m)\psi(x) - (g_1-c_1)N_{3/2}[\gamma_\mu\psi(\bar\psi\gamma^\mu\psi)](x) -$$

$$-(g_2-c_2)N_{3/2}[\lambda^a\gamma_\mu\psi(\bar\psi\gamma^\mu\lambda^a\psi)](x) -$$

$$-(g_3-c_3)N_{3/2}[\psi(\bar\psi\psi)](x)\} \prod_{i=1}^{N}\psi(x)\prod_{j=1}^{N}\bar\psi(y_j)|0\rangle$$

$$= \text{delta terms} \ , \qquad (3.1)$$

into one where the interaction is intrinsically parametrized in terms of limits of currents and the field $\psi(x)$. We will show that we obtain the naively expected equation only in the $m \to 0$ limit and on the curve L_1, along which all vector and axial vector currents are conserved i.e., in the situation when scale invariance holds. This reparametrization is essential, if one tries to solve the model exactly via the integration of Ward-Takahashi identities[3].

We will thus express the currents as limits of fields, when the separating distance vanishes[3]:

$$\begin{Bmatrix} j_\mu(x) \\ j_\mu^a(x) \end{Bmatrix} \equiv$$

$$\lim_{\varepsilon\to 0}\frac{1}{4}\{[\bar\psi(x+\varepsilon)\begin{Bmatrix}\gamma_\mu \\ \gamma_\mu\lambda^a\end{Bmatrix}\psi(x) - \begin{Bmatrix}\gamma_\mu \\ \gamma_\mu\lambda^a\end{Bmatrix}\psi(x+\varepsilon)\bar\psi(x)] + (\varepsilon\to\tilde\varepsilon)\}\begin{Bmatrix}Z_s^{-1} \\ Z_v^{-1}\end{Bmatrix} \ , \qquad (3.2)$$

where Z_s, Z_v are renormalization constants depending logarithmically on $\mu^2\varepsilon^2$. In terms of the Wilson expansions[9]

$$\bar\psi(x+\varepsilon)\gamma_\mu\psi(x)-\gamma_\mu\psi(x+\varepsilon)\bar\psi(x) = 2a_1^s N_1[\bar\psi\gamma_\mu\psi](x)+4a_2^s\frac{\varepsilon_\mu\varepsilon_\nu}{\varepsilon^2}N_1[\bar\psi\gamma^\nu\psi]+O(\varepsilon) \ ,$$

$$\bar{\psi}(x+\varepsilon)\gamma_\mu\lambda^a\psi(x) - \gamma_\mu\lambda^a\psi(x+\varepsilon)\bar{\psi}(x) = 2a_1^V N_1\left[\bar{\psi}\gamma_\mu\lambda^a\psi\right](x) +$$

$$+ 4a_2^V \frac{\varepsilon_\mu\varepsilon_\nu}{\varepsilon^2} N_1\left[\bar{\psi}\gamma^\nu\lambda^a\psi\right](x) + O(\varepsilon), \tag{3.3}$$

we have $Z_{S,V} = a_1^{S,V} + a_2^{S,V}$.

In order to rewrite (3.1), let us introduce the operators

$$A_1(x) = \lim_{\eta\to 0}\{N_1\left[\bar{\psi}\gamma_\mu\psi\right](x+\eta)\gamma^\mu\psi(x) - \gamma^\mu\psi(x+\eta)N_1\left[\bar{\psi}\gamma_\mu\psi\right](x)\},$$

$$A_2(x) = \lim_{\eta\to 0}\{N_1\left[\bar{\psi}\gamma_\mu\lambda^a\psi\right](x+\eta)\gamma^\mu\lambda^a\psi(x) - \gamma^\mu\lambda^a\psi(x+\eta)N_1\left[\bar{\psi}\gamma_\mu\lambda^a\psi\right](x)\},$$

$$A_3(x) = N_{3/2}\left[(\bar{\psi}\psi)\psi\right](x) . \tag{3.4}$$

They can be expanded in the following way:

$$A_1 = E_1 N_{3/2}\left[(\bar{\psi}\gamma_\mu\psi)\gamma^\mu\psi\right](x) + E_2 N_{3/2}\left[(\bar{\psi}\gamma_\mu\lambda^a\psi)\gamma^\mu\lambda^a\psi\right](x)$$

$$+ E_3 N_{3/2}\left[(\bar{\psi}\psi)\psi\right](x) + iE_4\not{\partial}\psi(x) + mE_5\psi(x) + O(\eta) ,$$

$$A_2 = F_1 N_{3/2}\left[(\bar{\psi}\gamma_\mu\psi\ \gamma^\mu\psi\right](x) + F_2 N_{3/2}\left[(\bar{\psi}\gamma_\mu\lambda^a\psi\ \gamma^\mu\lambda^a\psi\right](x)$$

$$+ F_3 N_{3/2}\left[(\bar{\psi}\psi)\psi\right](x) + iF_4\not{\partial}\psi(x) + mF_5\psi(x) + O(\eta) \tag{3.5}$$

The coefficients E_i and F_i are m-independent due to our subtraction scheme, (2.2) and (2.3). In order to show the absence of direction-dependent terms of the form

$$N_{3/2}\left[(\bar{\psi}\gamma^\alpha\lambda^a\psi)(\psi\gamma^\beta\lambda^b\psi)\right](x)\frac{\varepsilon_\alpha\varepsilon_\beta}{\varepsilon^2} , \qquad \gamma^2\partial^\beta\psi(x)\frac{\varepsilon_\alpha\varepsilon_\beta}{\varepsilon^2} ,$$

we proceed as follows. Due to our subtractíon scheme, the limit $m \to 0$ certainly exists (the problem is to establish an equation of motion like (2.8), below).

We now use the current conservation laws, which are the essential tool in establishing our result. In this limit, and along the curve found in the previous Section, where $\beta_i = 0$, $i = 1,2,3$, all vector and axial-vector currents are conserved. We can thus integrate the Ward-Takahashi identities with suitable boundary conditions, and find the current-current Green's functions with an arbitrary number of ψ and $\bar{\psi}$. Note that this is a trivial step and it is of courses unnecessary to exactly solve the model to do so. This fact will be used over and over agaín, below.

Now, we calculate

$$\lim_{\eta \to 0} N_1 \left[\bar{\psi} \left\{ \begin{matrix} \gamma_\mu \\ \gamma_\mu \gamma_\lambda a \end{matrix} \right\} \psi \right] (x+\eta)\, \psi(x) \; . \tag{3.6}$$

There do not appear any direction-dependent terms in (3.6), and hence limits defining $A_1(x)$ and $A_2(x)$, Eqs. (2.4), are direction-independent. Since the operators on the r.h.s. of Eq. (2.5) have direction-independent Green's functions, the same is true for the coefficients E_i and F_i.

We thus obtain the following zero mass limit, for (3.1):

$$\begin{aligned} <0|T\{ & [E_1F_2 - E_2F_1 - (g_1 - c_1)(E_2F_4 - E_4F_2) - (g_2 - c_2)(E_4F_1 - E_1F_4)] i\partial\!\!\!/ \psi(x) \\ & - [(g_1 - c_1) - (g_2 - c_2)F_1] A_1 + [(g_2 - c_2)E_1 - (g_1 - c_1)E_2] A_2 + \\ & + [(g_3 - c_3)(E_1F_2 - E_2F_1) + (g_1 - c_1)(E_2F_3 - E_3F_2) + (g_2 - c_2)(E_3F_1 - E_1F_3)] A_3 \\ & - (\text{delta terms})\} \prod_{i=1}^{N} \psi(x_i) \prod_{j=1}^{N} \bar{\psi}(y_i) |0> = 0, \end{aligned}$$

i.e.,

$$<0|T\{i\partial\!\!\!/\psi(x) - \sum_{i=1}^{3} d_i A_i + d_4 (\text{delta terms})\} X |0> = 0 .$$

Now we want to show that the coefficients d_i i = 1,2,3, are finite , using again the conservation of all currents along our curve L, . The coefficient d_4 will be shown to be logarithmically divergent, in every order, but actually exponentiates to zero.

With the. equation of motion (3.81, we calculate

$$- i\partial^\mu\{ \lim_{\varepsilon\to 0} [Z_s^{-1} : \bar{\psi}(x+\varepsilon)\gamma_\mu\gamma^5\psi(x):]\}$$

$$= \lim_{\varepsilon\to 0} \{d_3 Z_s^{-1} [:\bar{\psi}(x+\varepsilon)\gamma^5 A_3(x) + \bar{A}_3(x+\varepsilon)\gamma^5\psi(x):]\} \quad . \tag{3.9}$$

The terms containing A_1 and A can easily be shown to vanish identically. For the r.h.s. of (3.91, we make a Wilson expansion

$$Z_s^{-1} [:\bar{\psi}(x+\varepsilon)\gamma^5 A_3(x) + \bar{A}_3(x+\varepsilon)\gamma^5\psi(x):] = i(r_1 g_{\alpha\beta} + r_2 \frac{\varepsilon_\alpha\varepsilon_\beta}{\varepsilon^2}) \; .$$

$$. \; N_2[\partial^\alpha(\bar{\psi}\gamma^\beta\gamma^5\psi)](x) + r_3 N_1[\bar{\psi}\gamma^5\psi](x) + r_4 N_2[(\bar{\psi}\psi)(\bar{\psi}\gamma^5\psi)](x)$$

$$+ r_5 \frac{\varepsilon_\alpha\varepsilon_\beta}{\varepsilon^2} N [(\bar{\psi}\gamma^\alpha\gamma^5\lambda^a\psi)(\bar{\psi}\gamma^\beta\lambda^a\psi)](x) + O(\varepsilon) \; . \tag{3.10}$$

Inserting (3.10) into Eq.(3.9) we obtain

$$-i\partial^\mu\{ \lim_{\varepsilon\to 0} [Z_s^{-1}: \bar{\psi}(x+\varepsilon)\gamma_\mu\gamma^5\psi(x):]\} = id_3 r_1 \partial^\mu N_1 [\bar{\psi}\gamma_\mu\gamma_5\psi](x)$$

$$+ d_3\{ \frac{\varepsilon_\alpha\varepsilon_\beta}{\varepsilon^2} (r_2 N_2 [\partial^\alpha(\bar\psi\gamma^\beta\gamma^5\psi)](x) + r_5 N_2 [(\bar\psi\gamma^\alpha\gamma^5\lambda^a\psi)(\bar\psi\gamma^\beta\lambda^a\psi)](x)$$

$$+ r_3 N_1 [\bar\psi\gamma^5\psi](x) + r_4 N_2 [(\bar\psi\psi)(\bar\psi\gamma^5\psi)](x)\} . \tag{3.11}$$

But since the isoscalar axial current is conserved, we get

$$d_3 r_2 = d_3 r_5 = d_3 r_3 = d_3 r_4 = 0 . \tag{3.12}$$

Since $r = 1$, in zeroth order, we obtain

$$d_3 = 0 \tag{3.13}$$

which means

$$g_3 = f(g_1, g_2) , \tag{3.14}$$

with $f(g_1, g_2)$ determined from (3.13) as a power series in g_2 and g_3. Next, we apply the same procedure to the isovector axial current:

$$- i\partial^\mu\{\lim_{\varepsilon\to 0} [Z_V^{-1}: \bar\psi(x+\varepsilon)\gamma_\mu\gamma^5\lambda^a\psi(x):]\} = \sum_{i=2,3} Z_V^{-1} d_i .$$

$$. \{:\bar\psi(x+\varepsilon)\gamma^5\lambda^a A_i(x) + \bar A_i(x+\varepsilon)\gamma^5\lambda^a\psi(x):\} . \tag{3.15}$$

The term $i=1$ is absent, because of C-invariance. Now from

$$Z_V^{-1} \{:\bar\psi(x+\varepsilon)\gamma^5\lambda^a A_2(x) + \bar A_2(x+\varepsilon)\gamma^5\lambda^a\psi(x):\} = \lim_{\substack{\varepsilon\to 0\\ \eta\to 0}} \{ Z_V^{-1}$$

$$\cdot [N_1 [\bar\psi \gamma_\mu\lambda^b\psi] (x+\eta) : \bar\psi(x+\varepsilon)\gamma^5\gamma^\mu\lambda^a\lambda^b\psi(x):$$

$$- N_1[\bar{\psi}\gamma_\mu\lambda^b\psi](x) \;:\bar{\psi}(x+\varepsilon)\gamma^5\gamma^\mu\lambda^a\lambda^b\psi(x+\eta):$$

$$+ N_1[\bar{\psi}\gamma_\mu\lambda^b\psi](x+\eta+\varepsilon):\bar{\psi}(x+\varepsilon)\gamma^5\gamma^\mu\lambda^a\lambda^b\psi(x):$$

$$- N_1[\bar{\psi}\gamma_\mu\lambda^b\psi](x+\varepsilon) \;:\bar{\psi}(x+\eta+\varepsilon)\gamma^5\gamma^\mu\lambda^a\lambda^b\psi(x):]\} \tag{3.16}$$

We see that the ℓ.h.s. of *(3.16)* has no direction dependent terms, since, by explicit computation via Ward-Takahashi identities, the r.h.s. is direction independent. Consequently, we have the following Wilson expansion:

$$Z_v^{-1}\{:\bar{\psi}(x+\varepsilon)\gamma^5\lambda^a A_2(x) + \bar{A}_2(x+\varepsilon)\gamma^5\lambda^a\psi(x):]$$

$$= is_1^{(2)}\partial^\mu N_1[\bar{\psi}\gamma_\mu\gamma^5\lambda^a\psi](x) + s_2^{(2)}N_2[\bar{\psi}\gamma^5\lambda^a\psi](x)$$

$$+ s_3^{(2)}N_2[(\bar{\psi}\psi)(\bar{\psi}\gamma^5\lambda^a\psi)](x) + s_4^{(2)}N_2[(\bar{\psi}\lambda^a\psi)(\bar{\psi}\gamma^5\psi)](x) + O(\varepsilon)\;, \tag{3.17}$$

where C-invariance'was used to exclude a term proportional to

$$N_2[(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\gamma^5\lambda^a\psi)](x)\;.$$

There is no such argument for direction-independent available for A_3 :

$$Z_v^{-1}\{:\bar{\psi}(x+\varepsilon)\gamma^5\lambda^a A_3(x) + \bar{A}_3(x+\varepsilon)\gamma^5\lambda^a\psi(x):\}$$

$$= i(s_1^{(3)}g_{\alpha\beta} + s_7^{(3)}\frac{\varepsilon_\alpha\varepsilon_\beta}{\varepsilon^2})N_2[\partial^\alpha(\bar{\psi}\gamma^\beta\gamma^5\lambda^a\psi)](x) + s_2^{(3)}N_1[\bar{\psi}\gamma^5\lambda^a\psi](x)$$

$$+ s_3^{(3)}N_2[(\bar{\psi}\lambda^a\psi)(\bar{\psi}\gamma^5\psi)](x) + s_4^{(3)}N_2[(\bar{\psi}\psi)(\bar{\psi}\gamma^5\lambda^a\psi)](x)$$

$$+ if_{abc}(s_5^{(3)} g_{\alpha\beta} + s_6^{(3)} \frac{\varepsilon_\alpha \varepsilon_\beta}{\varepsilon^2}) N_2 [(\bar{\psi}\gamma^\alpha \lambda^b \psi)(\bar{\psi}\gamma^\beta \gamma^5 \lambda^c \psi)](x) + O(\varepsilon) \; . \qquad (3.18)$$

Inserting (3.17) and (3.18) into (3.16) we obtain

$$- i\partial^\mu \{ \lim_{\varepsilon \to 0} Z_v^{-1} : \bar{\psi}(x+\varepsilon)\gamma_\mu \gamma^5 \lambda^a \psi(x) : \} =$$

$$= \sum_{i=2}^{3} d_i \{ is_1^{(i)} \partial^\mu N_1 [\bar{\psi}\gamma_\mu \gamma^5 \lambda^a \psi](x) + s_2^{(i)} N_1 [\bar{\psi}\gamma^5 \lambda^a \psi](x)$$

$$+ s_3^{(i)} N_2 [(\bar{\psi}\psi)(\bar{\psi}\gamma^5 \lambda^a \psi)](x) + s_4^{(i)} N_2 [(\bar{\psi}\lambda^a \psi)(\bar{\psi}\gamma^5 \psi)](x) \}$$

$$+ id_3 \{ (s_5^{(3)} g_{\alpha\beta} + s_6^{(3)} \frac{\varepsilon_\alpha \varepsilon_\beta}{\varepsilon^2}) f_{abc} N_2 [(\bar{\psi}\gamma^\alpha \lambda^b \psi)(\bar{\psi}\gamma^\beta \gamma^5 \lambda^c \psi)](x)$$

$$+ s_7^{(3)} \frac{\varepsilon_\alpha \varepsilon_\beta}{\varepsilon^2} N_2 [\partial^\alpha (\bar{\psi}\gamma^\beta \gamma^5 \lambda^a \psi)](x) \} \; . \qquad (3.19)$$

Again from the conservation of the current above, we get

$$d_i s_2^{(i)} = d_i s_3^{(i)} = d_i s_4^{(i)} = 0 \; , \quad i = 2,3$$

$$d_3 s_7^{(3)} = d_3 s_5^{(3)} = d_3 s_6^{(3)} = 0 \; . \qquad (3.20)$$

Since, in zeroth order, we have

$$s_3^{(2)} = - s_4^{(2)} = - s_4^{(3)} = - \frac{1}{2} \; , \qquad (3.21)$$

we obtain, from (3.20),

$$d_2 = d_3 = 0 \; , \qquad (3.22)$$

in perturbation theory.

The field equation (3.8) now becomes

$$<0|T\{i\partial\!\!\!/\psi(x) - d_1 \lim_{\eta\to 0} [N_1[\bar\psi\gamma^\mu\psi](x+\eta)\gamma_\mu\psi(x)$$

$$- N_1[(\bar\psi\gamma^\mu\psi)(x-\eta)\gamma^\mu\psi(x)]\} X - i d_4 \sum_{k=1}^{N} \delta(x-x_k)\, X_k\, |0> = 0, \qquad (3.23)$$

where

$$X_k = \prod_{i=1}^{N} \psi(x_i)\,[\bar\psi(y_i)\ldots\bar\psi(y_{k-1})\,\bar\psi(y_{k+1})\ldots\bar\psi(y_N)]$$

Applying $\tilde{D}$, as given by Eq.(2.10), to (2.23), we obtain, using (2.7) and $\beta_i = 0$, ■ = 1,2,3 :

$$- 2N\gamma<0|T\{i\partial\!\!\!/\psi(x)X|0> - (d_1[-(2N-2)\gamma - (\delta-1)t_1] + \mu\frac{\partial}{\partial\mu} d_1)\ \cdot$$

$$\cdot (\lim_{\eta\to 0} <0|T\{N_1[\bar\psi\gamma_\mu\psi](x+\eta)\gamma^\mu\psi(x) - N_1[\bar\psi\gamma_\mu\psi](x-\eta)\gamma^\mu\psi(x)\}X|0> -$$

$$-i\,(d_4[-2N+1]\gamma + \mu\frac{\partial}{\partial\mu} d_4)\sum_{k=1}^{N} \delta(x-x_k)<0|TX_k|0>= 0\ . \qquad (3.24)$$

Since the isoscalar current scales canonically, we have that $(\delta-1)t_1=2\gamma$, so that from (3.24) follows

$$\mu\frac{\partial d_1}{\partial\mu} = 0\ , \quad \mu\frac{\partial d_4}{\partial\mu} + d_4\gamma = 0\ . \qquad (3.25)$$

Thus d_1 is an ε-independent function of g_1, whereas d satisfies

(3.26)

Since $\gamma > 0$, in the interacting case, we get

$$\lim_{\varepsilon\to 0} d_4 = 0 .$$

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6. We use the following conventions:

$$\gamma^{\mu}\gamma^{5} = \varepsilon^{\mu\nu}\gamma_{\nu} \;,\; \varepsilon_{\mu\nu} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \;,\; \varepsilon_{\mu\nu}\,\varepsilon^{\nu\rho} = g^{\rho}_{\mu}$$

$$\tilde{\partial}_{\mu} = \varepsilon_{\mu\nu}\partial^{\nu} \;,$$

$$\lambda^{a}\lambda^{b} = \frac{2}{n}\,\delta_{ab} + (d_{abc} + if_{abc})\,\lambda^{c} \;.$$

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