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Geometrical and kinematical Reconstruction of a Zero Prong Event in a Bubble Chamber*

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This paper introduces a simplified routine for the geometrical reconstruction and the kinematical calculations of an individual track produced in a bubble chamber, to be used in small computers. it was applied to the neutral reactions n-D, as described in another paper to appear shortly. it can be applied also to events with severa1 tracks if the constraint equations associated to the energy and momentum, conservation are introduced. Since this routine is to be applied to zero prong events, one has to be careful with the energy calculation of the particle. The events with an error in the energy larger than a certain value ,must be remeasured and reanalyzed, with the error limit depending on the particular reaction being studied.

Introduz-se uma rotina simplificada para a reconstrução geométrica ca e os cálculos cinemáticos de um traço individual produzido em uma câmara de bolhas, para ser usada em computadores de pequeno porte. Ela foi aplicada as reações neutras ?r-D, como descrito em outro artigo a aparecer brevemente. Pode também ser aplicado a eventos com muitos traços se forem introduzidas as equações de vínculo associadas i conservação de energia e quantidade de movimento . Como a rotina é para ser aplicada a eventos com um só traço, deve-se ter cuidado com o cálculo da energia da partícula. Eventos com um erro na energia maior que um certo valor devem ser remedidos e reanalizados, com o erro 1imite dependendo da reação particular que esteja sendo estudada.

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1. INTRODUCTION

For definiteness, we consider a cylindrical bubble chamber with glasses in the bases. Its positioning relative to the photographic system lenses is shown in Fig. 1.

In this figure, AA' is the bubble chamber axis and B and B' the parallel glass surfaces. Light from a flash in A is focused in A'.

The reference for the coordinate system is made through several fiducial marks graved in the internal surfaces B and B'. Three of these marks, F_A, F_B and F_C are situated in the vertices of an equilateral triangle in B'. The lenses from three photographic cameras are placed in the vertices C_I, C_{II} and C_{III} of another equilateral triangle parallel to the glass. This system is then aligned in such a way that the centers of the two triangles lie on the line AA' and the direction $C_I C_{II}$ is parallele to $F_A F_B$.

2. MEASUREMENTS

To measure one event we project on a measuring table one of the views of that event. After the plane coordinate measurements are completed, we project another view of the same event, taken by another camera, and so on.

Although one generally disposes of three or more different photographies for each event, two only are sufficient for the spatial reconstruction, the others being used in more complicated events. In this paper only two views are considered, those we called I and II taken by cameras C_I and C_{II} respectively.

The measurements in each view are made over the fiducial marks F_A and F_B, in this order, and over four points along the track, starting from the interaction point (vertex) and dividing the track in three approximated equal intervals.

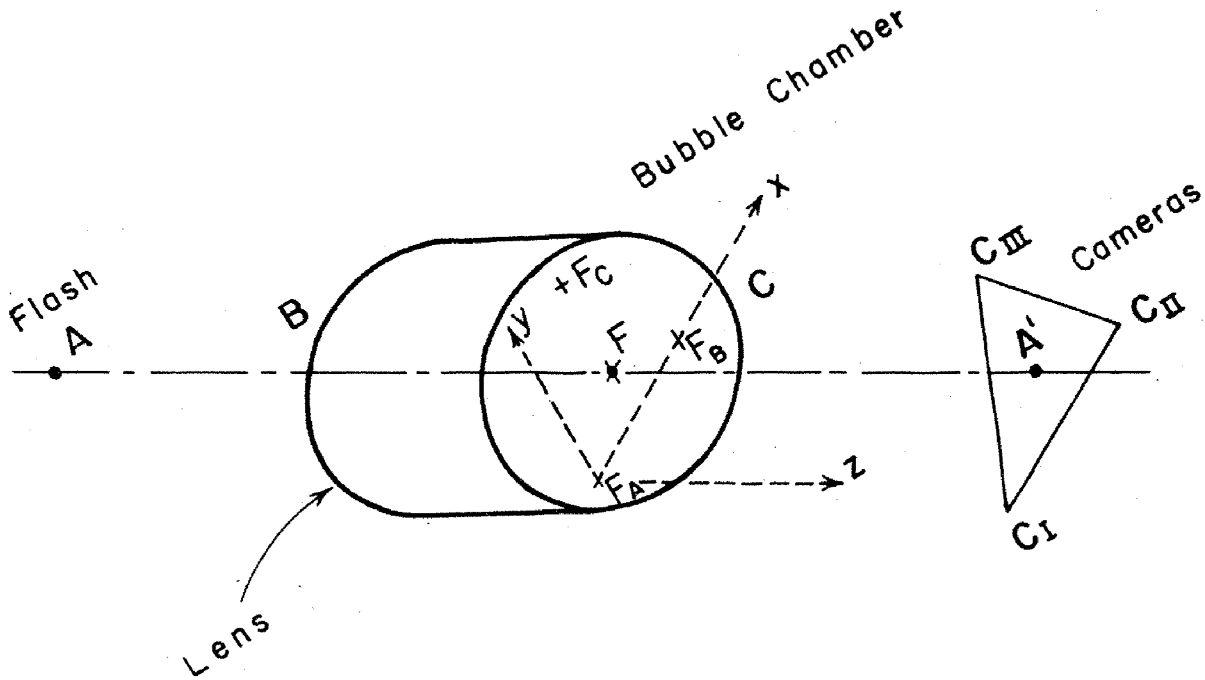
Flash
A
B
Bubble Chamber
x
+F_C
C
y
F
F_B
F_A
z
Lens
C_III
Cameras
C_II
A'
C_I

Fig. 1

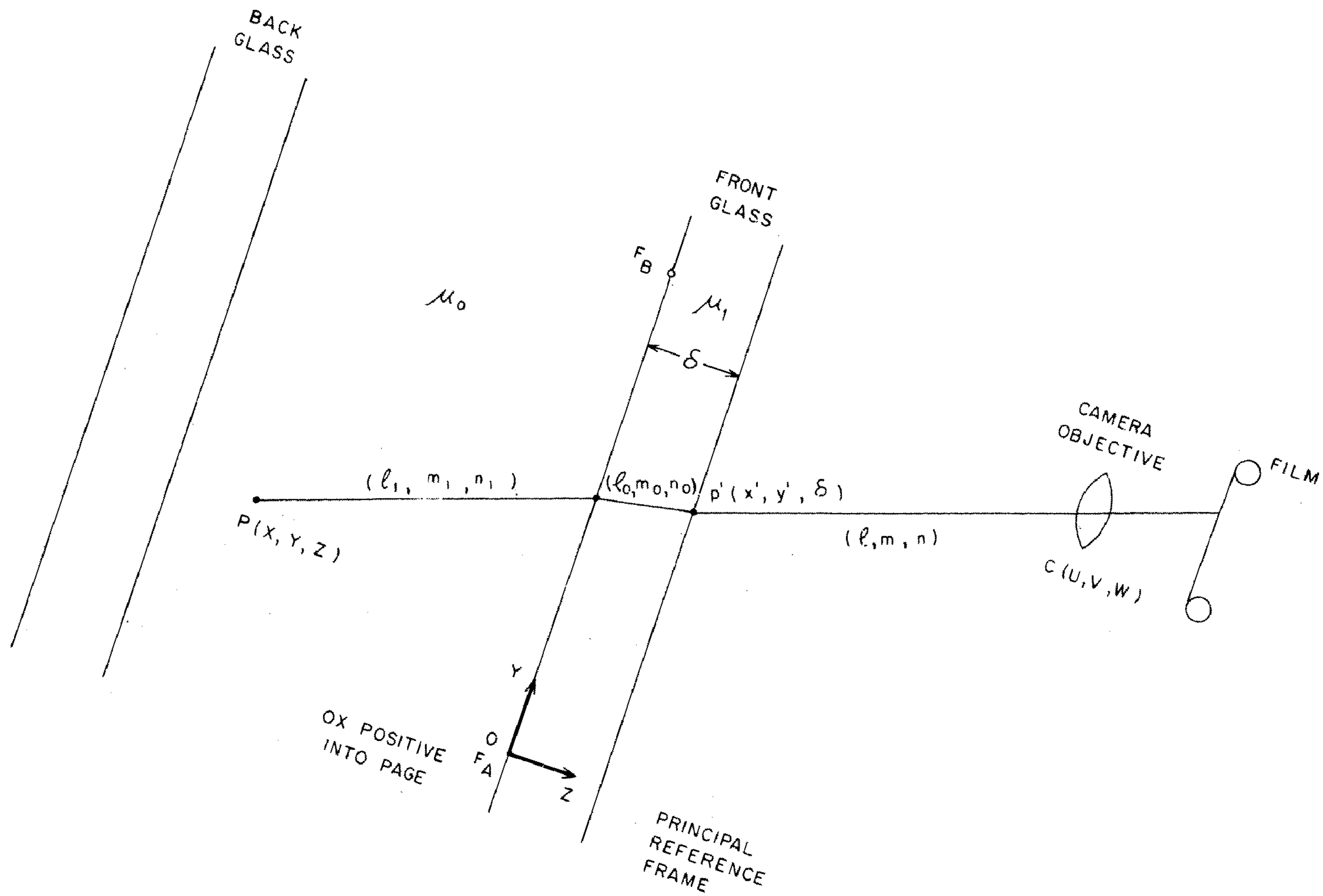
BACK GLASS
FRONT GLASS
F_B
μ_0
μ_1
δ
CAMERA OBJECTIVE
FILM
(ℓ_1, m_1, n_1)
(ℓ_0, m_0, n_0)
$p'(x', y', \delta)$
$P(X, Y, Z)$
(ℓ, m, n)
$C(U, V, W)$
Y
O
F_A
Z
OX POSITIVE INTO PAGE
PRINCIPAL REFERENCE FRAME

Fig. 2

The masurement over each point gives the X and Y coordinares of that point relative to the measuring table coordinate system.

3. REFERENCE FRAMES

Three reference systems are used in the analysis. *The principal* **three** *dimensional reference frame* (OXYZ) is defined such that its origin coincides with the fiducial mark F_A, the X and Y axes being parallel and perpendicular to the direction $F_A F_B$, respectively, and the positive Z--direction points from F_A to the cameras. *The fiducial two dimensional reference frame* is defined for each view projected on'the masuring table, in such a way that its origin coincides with the fiducial mark image F_A, the X and Y-directions being parallel and perpendicular to the projection on the table of the line $F_A F_B$, respectively. *The measuring table two-dimensional reference frame* is the one fixed on the measuring table, whose coordinates are given by the two encoders.

4. RECONSTRUCTION EOUATIONS

Fig. 2 shows the principal frame OXYZ, the two fiducial marks F_A and F_B and the position of one photographic camera relative to the bubble chamber. The camera lens optical nodal point is at $C(U,V,W)$ and the optical axis is parallel to Z. μ_0, μ_1 and 1 are the refractive indices of the liquid inside the bubble chamber, the frontal glass and the air between the frontal glass and the camera, respectively. 6 is the frontal glass thickness[1,2].

The reconstruction line shown in Fig. 2 connecting the point $P(X,Y,Z)$ inside the bubble chamber and the film, represents a liyht ray path crossing the external glass surface at the point $P'(X', Y', 6)$. The relations among the coordinates of a point $P(XYZ)$ inside the bubble chamber and its image $P'(X', Y', \delta)$ in the plane $Z=\delta$ are

(1.a)

$$Y' = \frac{m_1}{n_1} \delta + Y - \frac{m_0}{n_0} Z \quad , \tag{1.b}$$

where (l_1, m_1, n_1) and (l_0, m_0, n_0) are the direction cosines of the light ray inside the bubble chamber and inside the frontal glass, respectively.

Calling (l,m,n) the direction cosine of the line $P'C$, the above relations can be written as

$$X' = X - (U-X') \frac{Z}{\nu_0} + (U-X') \frac{6}{\nu_1} \quad , \tag{2.a}$$

$$Y' = Y - (V-Y') \frac{Z}{\nu_0} + (V-Y') \frac{\delta}{\nu_1} \quad , \tag{2.b}$$

where

$$\nu_0 = \{(\mu_0^2-1)\left[(U-X')^2 + (V-Y')^2\right] + \mu_0^2 (W-\delta)^2\}^{1/2} \quad , \tag{3.a}$$

$$\nu_1 = \{(\mu_1^2-1)\left[(U-X')^2 + (V-Y')^2\right] + \mu_1^2 (W-\delta)^2\}^{1/2} \quad . \tag{3.b}$$

The plane coordinates furnished by the measuring table of the points P_{ij} (i= 1,2,3,4 is the point measured on the track and j=1,2 is the view) are transformed to the fiducial reference frame, after which we take care of the magnification factors. Due to refraction in the frontal glass, we have to transform all the coordinates to the principal frame, as required by equations 2a. and 2b. For this purpose, we must determine the positions of the fiducial mark images on the plane $Z = 6$.

Let us call $(F_i, G_i, 0)$ the coordinates of the i^{th} fiducial mark relative to the principal frame and $(\alpha_{ij}, \beta_{ij}, \delta)$ the coordinates of the intersection with the plane $Z=\delta$ of the light ray that goes frorn that fiducial mark to the camera j. Omitting for simplicity the indices, equations 2.a and 2.b give

$$\alpha = F + (U - \alpha)\frac{\delta}{\nu_1} \quad , \qquad (4.a)$$

$$\beta = G + (V - \beta)\frac{\delta}{\nu_1} \quad . \qquad (4.b)$$

The parameter ν_1 given by equation (3.b) is a function of a and B and so the system of equations (4) must be solved by iteration. Knowing (a, β) for each wiew, we can transform the coordinates of the points measu-red on the track from the fiducial frame to the principal frame of reference.

5. CORRESPONDENT POINTS

The system of equations *(2)* can be applied only to coordinates in the two views that correspond to the same point inside the bubble chamber. This condition, however, is in principle satisfied by the fiducial marks and the vertex only. In consequence, one must find over one of the projections of the track the other three points correspondent tothe three points measured in the other view. This determination is not so complicated because each point in the bubble chamber has the same Y coordinate in the two views. This results from the choice of the fiducial system, whose X-axis is parallel to the line $C_I C_{II}$. Using the interpolation method cailed spline we fit a curve connecting the four points of the largest image of the track. Over this curve, we determine the three correspondent points measured on the shortest image of the track. According to the spline method, the curve is described by a set of different third degree polynomials, each one describing the curve between two known points. The polynomials are adjusted by imposing that the 1^{st} and 2^{nd} derivatives rnust be continuous at each point. Performing two integrations, one determines the polynomial for each interval. The two integration constants are obtained by using the condition that the polynomial passes by the two known points[3,4].

The interpolation in the set of values $(X_1, Y_1), \ldots, (X_m, Y_m)$ consists in determining Y between the values Y_{k+1} and Y_k and then in calculating X by the formula

$$X = C_{1,k}(Y_{k+1} - 1) + C_{2,k}(Y-Y_k) + C_{3,k}(Y_{k+1} - Y) + C_{4,k}(Y-Y_k),$$

where the constants $C_{i,k}$ are previously calculated and given by

$$C_{1,k} = \frac{D_k}{6d_k} \quad , \quad C_{2,k} = \frac{D_{k+1}}{6\ d_k} \quad ,$$

$$C_{3,k} = \frac{X_k}{d_k} - \frac{D_k d_k}{6} \quad , \quad C_{4,k} = \frac{X_{k+1}}{d_k} - \frac{D_{k+1}\ d_k}{6} \quad ,$$

with $d_k = X_{k+1} - X_k$,

where D_k and D_{k+1} are the values of the second derivatives of the curve at the points (X_k, Y_k) and (X_{k+1}, Y_{k+1}), respectively.

6. SOLUTION FOR THE RECONSTRUCTION SYSTEM OF EQUATIONS

Oncethe correspondent points are determined these values can be used to obtain two pairs of equations (2), one for each view. The system is over determined since the values of Y' are the same.

Each pair of equations (1) define one straight line inside the bubble chamber. These two straight lines intercept a certain plane Z = constant inside the chamber at two points. By calculating and minimizing the distance d_i between these two points we get the best solution for the system. We then define a χ_i^2 through

$$\chi_i^2 = \sum_j d_{ij}^2 = \sum_j \left[(X_i - X'_{ij} - G_{ij} Z_i - A_{ij})^2 + (Y_i - Y'_{ij} - G'_{ij} Z_i - A'_{ij})^2 \right] ,$$

where

$$G_{ij} = \frac{U_j - X'_{ij}}{(\nu_0)_{ij}} \quad , \qquad A_{ij} = \frac{(U_j - X'_{ij})\delta}{(\nu_1)_{ij}}$$

$$G'_{ij} = \frac{V_j - Y'_{ij}}{(\nu_0)_{ij}} \quad , \qquad A'_{ij} = \frac{(V_j - Y'_{ij})\delta}{(\nu_1)_{ij}}$$

We now write $\partial\chi_i/\partial X_i = \partial\chi_i/\partial Y_i = \partial\chi_i/\partial Z_i = 0$ to get the best solution , which is

$$\begin{pmatrix} 2 & 0 & -\sum_j G_{ij} \\ 0 & 2 & -\sum_j G'_{ij} \\ -\sum_j G_{ij} & -\sum_j G'_{ij} & \sum_j (G_{ij}+G'_{ij}) \end{pmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} = \begin{pmatrix} \sum_j (X''_{ij} + A_{ij}) \\ \sum_j (Y''_{ij} + A'_{ij}) \\ \sum_j (X''_{ij}+A_{ij})G_{ij} + \sum_j (Y''_{ij}+A'_{ij})G'_{ij} \end{pmatrix}$$

These equations give the values X_i, Y_i and Z_i we are looking for. Replacing these values in equation (5) we also determine χ^2 (not normalized) for each reconstructed point.

7. KINEMATICAL QUANTITIES

A particle with a kinetic energy T in the presence of a magnetic field B has a curvature whose radius is given by

$$r = \frac{P\cos\alpha}{0.3B} \quad ,$$

where P is the momentum given by $P = \left[T(T+2m)\right]^{1/2}$ in the system where $c=1$, and a is the angle between the direction of motion and a plane normal to the magnetic field. This expression has to be correcced, however, to take care of the energy loss of the particle along its motion in the liquid inside the bubble chamber. For this correction we adjust the known published curves for the energy of the particle, $-(1/\rho)(dT/ds)$, versus T in the medium under consideration, where s is the the track length and ρ the density of the medium, with an equation of the form $dT/ds = -K\rho T^{-n}$, for instance, in an energy interval containing the interaction energy of the reaction being studied[5]. By the least square method we determine the constants K and n. Integrating the equation we get

$$T = \left[T_s^{n+1} - (n+1)K\rho s \right]^{1/(n+1)} , \qquad (7)$$

where T_s is the particle kinetic energy at a distance s from the interaction point.

Simulation in a computer of the curve correspondent to a π^- meson with a kinetic energy interval from 150 to 280 MeV has shown that the radius of the best circle adjusted to the projection of the four measured points on a plane perpendicular to the magnetic field, was the curvature radius of the track at its medium point. Once this radius is known, it is possible to calculate the component P_{XY} of the momentum parallel to the (XY) plane at that point,

The angle a can approximately be given by

$$\alpha = \tan^{-1} \frac{Z_1 - Z_4}{l} ,$$

where l is the path length in the (X,Y) plane and Z_1 and Z_4 are the Z coordinates of the vertex and initial point $(i=4)$, respectively. Knowing a, the transversal momentum is given by $P_Z = P_{XY} \tan\alpha$. a , as given above, is not entirely correct, since there is a loss of energy associated to the component of the momentum in the direction of the magnetic field . This loss was not taken into consideration here because the length of the track in this direction is generally a very small fraction of the total length. The correspondent error is, however, taken into consideration later.on.

8. ERRORS

The calculation of the error propagation in the energy takes into consideration (a) the errors in the adjustment by a circle of the curve passing through the four points projected in a Z = constant plane, (b) the applied magnetic field indetermination, (c) the error in the adjustment of the straight lines connecting adjacent projected points in the Z = constant plane, which gives an error in the track inclination angle a.

As a consequence of these errors, it results an indetermination in the particle momentum at the middle of the path, given by

$$\Delta P = \Delta P_{XY} + \frac{ZP_{XY}}{s^2} (\Delta Z + Z\Delta s) ,$$

where the error ΔP_{XY} of the component of the momentum normal to the magnetic field $\vec{B}$ is given by

$$\Delta P_{XY} = 0.3 \, BR \left(\frac{\Delta B}{B} + \frac{\Delta R}{R} \right) ,$$

where R is the radius of the circle projected in the plane normal to $\vec{B}$, s is the track length projected in this plane and Z is the Z-coordinate of the spatial points.

The kinetic energy error in the middle of the track is finally givenby

$$\Delta T = \frac{\Delta P}{\left\{ 1 + \frac{m^2}{P^2} \right\}^{1/2}}$$

When $\Delta T/T$ is larger than a certain previously chosen value, the track must be remeasured.

9. CONCLUSION

It was possible to use a simplified routine for the geometrical reconstruction of an individual track produced in a bubble chamber, without introducing appreciable error in the calculation of the energy of the particle. The routine can be applied to several values of energy and magnetic field, if the particle energy loss is described by an equation adjusted for the appropriate energy interval.

The calculations were simplified by the introduction of the "spline" interpolation method to calculate the correspondent points between the tracks viewed by different cameras, which allows the use of small computers for the calculations without loss of precision. This kind of analysis can be applied to an event with several tracks if the constraint equations associated to the energy and momentum conservation are taken into account. Also some trivial modifications should be done for long tracks or for particle energy losses larger than those considered here; in particular, more points should be measured along the tracks.

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