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Grain-Gas Interaction in Envelopes of Red Giants

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In this work a model for the ejection of the dust shell of red giant stars through the action of the stellar radiation pressure is developed. Being rnomentum-coupled to the gas, the dust shell can drive an effective mass loss. On the other hand, the grain injection rate into the interstellar space can be estimated.

Neste trabalho desenvolve-se um modelo para a ejeção da camada de grãos de estrelas gigantes vermelhas através da ação da pressão da radiação estelar. Como a camada de grãos está dinâmicamente acoplada ao gás, ela pode levar a uma perda de massa. Por outro lado, a taxa de injeção de grãos no espaço interestelar pode ser estimada.

1. INTRODUCTION

In previous works[1,2] we have studied the motion of charged and uncharged silicate grains in the envelopes of red giants, giving emphasis to the socalled Mira variable stars. In those works, the stellar envelopes was assurned to be initially at rest, and the existence of a strong momentum-coupling between grains and gas was inferred, supporting thus Gilman's work[3]. Since a single "test" grain was considered, the total momentum transferred to the gas was negligible. In the present work , a sornewhat more detailed model is developed. We consider a cloud of spherical silicate grains expanding through a gaseous stellarenvelope which is also expanding. As in the referred works[1,2], the stellar radiation

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pressure is invoked as the driving mechanism for the ejection of the dust shell. On the other hand, further grain-gas and gas-gas collisions share the radiation-pressure imparted momentum, accelerating thus the gas shell. In this way, we say that the gas shell is indirectly accelerated by the stellar radiation pressure.

In the following section, we developed our model equations giving emphasis to the hydrodynamic aspect of the problem. The stellar-envelope-grain parameters are presented, and a rather detailed discussion on silicate efficiency factors for radiation pressure is given. In the last section, our main results are shown and the effect of grain sizes on escape velocities is discussed. Finally, the role of late giants in interstellar grain clouds enrichment is estimated.

2. THE MODEL

In a spherically symmetric steady-state expanding grain shell, the mass conservation equation can be written as

$$r^2 \rho_{gr}(r) V_{gr}(r) = \text{constant} \quad , \tag{1}$$

where $\rho_{gr}(r)$ is the mass density of grains and $V_{gr}(r)$ is the grain velocity relative to the star. Adopting a two-fluid approximation, the conservation of mass for the gaseous envelope if given by

$$r^2 \rho(r) V(r) = \text{constant} \quad , \tag{2}$$

where $\rho(r)$ is the mass density of the gaseous envelope, assumed to be a mixture of H and H_2, and $V(r)$ is its velocity relative to the star. In order to write the equation for the conservation of momentum for the "grain" fluid, a few assumptions are made:

- the ideal gas equation of state for the envelope is assumed to be valid, with a pressure $P_{tot} \simeq P_{gas}$;
- adiabatic expansion with a constant index $\gamma = C_p/C_v = 5/3$ is considered;
- the interaction between grains and gas is of the "diffuse" type. (Explicit formulae for the resistance to the motion of a grain through the gas particles are obtained from Baines *et al.*[4]).
- the stellar flux has an inverse square dependence on position;
- the interaction among the grains themselves is neglected.

In view of these assumptions, the motion of the grains is governed by (a) stellar gravitational attraction, (b) stellar radiation pressure and (c) drag force due to collisions with gas particles. In such conditions, the conservation of momentum for the grains can be written as

(i) $S \leq 1.4$

$$V_{gr} \frac{dV_{gr}}{dr} = \frac{\alpha}{r^2} - \rho^{2/3}\left[\beta\rho^{2/3}(V_{gr}-V) + \delta(V_{gr}-V)^3\right] \qquad (3a)$$

or

(ii) $S > 1.4$

$$V_{gr} \frac{dV_{gr}}{dr} = \frac{\alpha}{r^2} - \rho\left[\varepsilon(V_{gr} - V)^2 + \eta\rho^{2/3} + \theta\rho^{1/3}(V_{gr} - V)\right], \qquad (3b)$$

depending on the regime of the flow, as measured by the parameter

$$S \equiv \left[\frac{\mu m_H}{2kT(r)}\right]^{1/2} (V_{gr} - V) \qquad (4)$$

(for details the reader is referred to an earlier work[1]). In these equations we have used the following parameters:

$$\alpha \equiv - GM + \frac{3\pi hcR^2}{2aS_g} \int_0^{\infty} \frac{Q(a,\lambda,m)\,d\lambda}{\lambda^5 \left[\exp(hc/(\lambda kT_e)) - 1\right]} \quad , \tag{5}$$

$$\beta \equiv \frac{1 + \pi/8}{aS_g} \sigma \quad , \tag{6}$$

$$\delta \equiv \frac{4}{5\pi aS_g} \frac{1}{\sigma} \quad , \tag{7}$$

$$\varepsilon \equiv \frac{3}{4aS_g} \tag{8}$$

$$\eta \equiv \frac{3\pi}{16aS_g} \sigma^2 \quad , \tag{9}$$

$$\theta \equiv \frac{\pi}{8aS_g} \sigma \quad , \tag{10}$$

where

$$\sigma \equiv \left[\frac{8k}{\pi\mu m_H} \frac{T(R)}{\rho^{2/3}(R)} \right]^{-1/2} \tag{11}$$

Apart from well known constants we have

- μ: mean molecular weight of the envelope, $\simeq 1.4$;
- $T(r)$: gas temperature as a function of position. In the present model it is calculated at each step.
- M: stellar mass, assumed to be 1 $M_\odot$;
- R: stellar radius, calculated to be R = 3.75 × 10^{13} cm;
- a: grain radius, used as a free parameter. It takes the values 100 Å, 500 Å , 1000 Å ;
- S_g: grain inner density, = 3 g cm^{-3} ;

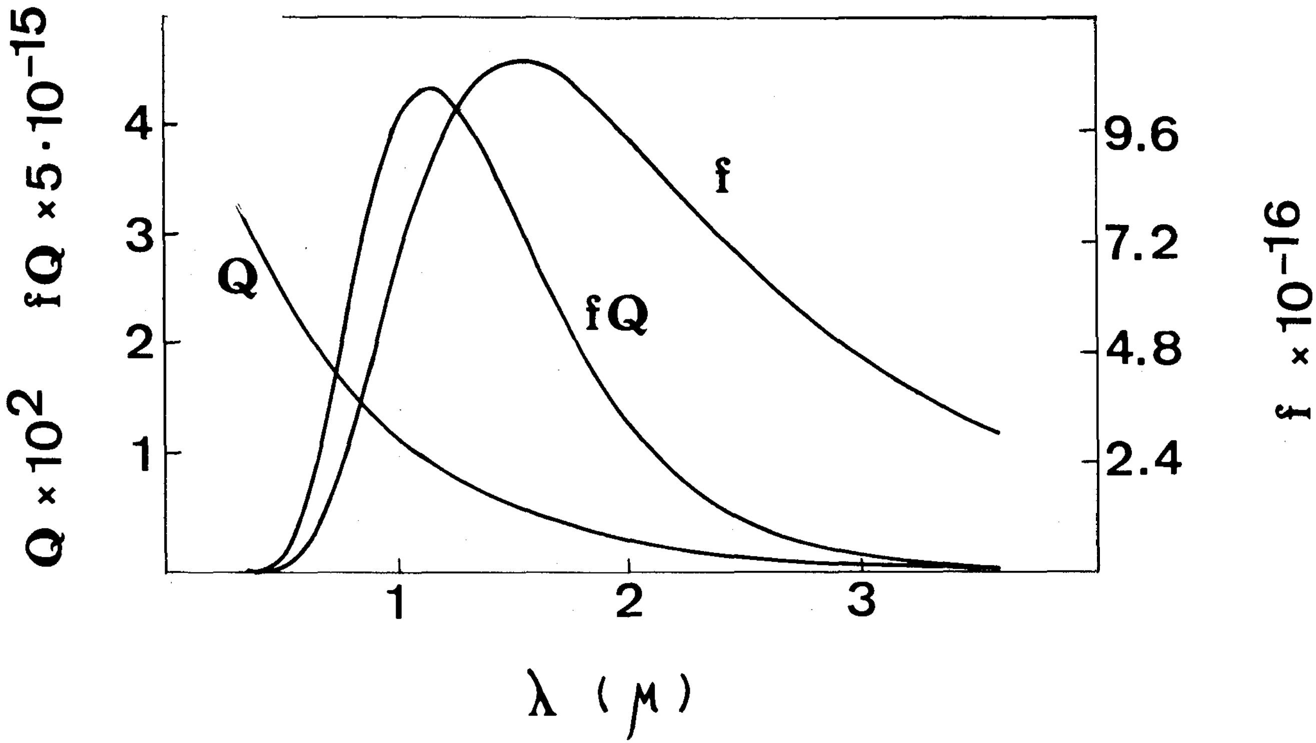

Figure 1 - Profiles of functions $f(\lambda)$, $Q(\lambda)$ and $fQ(\lambda)$ for a = 100 Å, T_e = 2000 K. Efficiency factors were taken from Dorschner[6] for silicates.

- T_e: star's effective temperature, taken as 2500 K. The stellar position on the HR diagram was selected in order to get the best conditions for grain formation (see for example the paper by Langer[5]);
- $Q(a,\lambda,m)$: efficiency factor for radiation pressure for silicate grains with radius a, complex refractive index m, at wavelength λ. The integral entering equation (5) was calculated for a = 100, 500, 1000 Å, using black body fluxes at several effective temperatures and with efficiency factors given by Dorschner[6] for silicates with m = 1.7 - 0.1 i. We have verified that the difference between the adopted values and those corresponding to m = 1.7 - 0.01 i was quite small. Figure 1 shows the obtained profiles $f(\lambda)$, $Q(\lambda)$ and the product fQ for the case a = 100 Å, T_e = 2000 K. $f(\lambda)$ is given by

$$f(\lambda) \equiv \frac{1}{\lambda^5\left[\exp(hc/(\lambda kT_e))-1\right]} \qquad (12)$$

The product fQ, integrated from 0 to ∞, is proportional to the parameter a, which ultimately reflects the ejection acceleration. It can be seen that the regions where Q is large have low f values, corresponding to a low product fQ. This limits the importance of silicates for the process studied, as was pointed out earlier[1].

The motion of the gas molecules is determined by (a) stellar gravitational attraction, (b) drag force due to interactions with grains and (c) interactions among the molecules themselves. In this model, we have not considered the action of radiation pressure on molecular bands, which was the subject of a separate work[7]. Depending on the values of parameter S we have

(i) $S \leq 1.4$

$$V\frac{dV}{dr} = -\xi\rho^{-1/3}\frac{d\rho}{dr} - \frac{GM}{r^2} + \rho_{gr}\left[\beta\rho^{1/3}(V_{gr}-V) + \delta\rho^{-1/3}(V_{gr}-V)^3\right], \qquad (13a)$$

(ii) S > 1.4

$$V \frac{dV}{dr} = - \xi\rho^{-1/3} \frac{d\rho}{dr} - \frac{GM}{r^2} + \rho_{gr} \left[\varepsilon(V_{gr}-V)^2 + \eta\rho^{2/3} + \theta\rho^{1/3}(V_{gr} - V) \right] \quad , \tag{13b}$$

where we have defined

$$\xi \equiv \frac{5\pi}{24} \sigma^2 \quad . \tag{14}$$

3. RESULTS AND DISCUSSION

Equations (1), (2), (3) and (13) form a system involving explicitly four variables (V_{gr}, V, ρ_{gr}, ρ) as functions of position. The gas temperature and pressure can be calculated using the equation of state and the adiabatic equation.

Our system was solved numerically as an initial value problem through a Runge-Kutta scheme. Five initial values are needed:

- $\rho(R)$ - The gas density at the base of the envelope ($r=R$) was estimated by $\rho(R) = \mu m_H N/R$, where $N - 10^{22} cm^{-2}$ is the observed H column dencity. We got $\rho(R) \simeq 6.2 \times 10^{-16}$ g cm^{-3} , which is in good agreement with Gilman[3].
- $T(R)$ - The gas temperature at $r=R$ was taken $T(R) \simeq T_e$. In fact, we have verified that a decrease in the initial temperature by a few hundreds of degrees Kelvin does not appreciably change the obtained profiles.
- $\rho_{gr}(R)$ - Assuming that essentially all the available silicon is condensed in grains, and taking $n_{Si}/n_H = 3.55 \times 10^{-5}$ (Keeley[8]), we get $\rho_{gr}(R) = 6.2 \times 10^{-19}$ g cm^{-3}.
- $V_{gr}(R), V(R)$ - Since we were essentially interested in the supersonic part of the flow, we assumed the envelope to be at an initial velocity $V(R) \simeq V_{gr}(R) \simeq V_{esc} = 26.6$ kms^{-1}, where V_{esc} is the escape velocity from the star. Apart from the drag due to grains, other mechanisms may account for the initial acceleration of the gas, such as radiation pressure on molecular bands[1,7] .

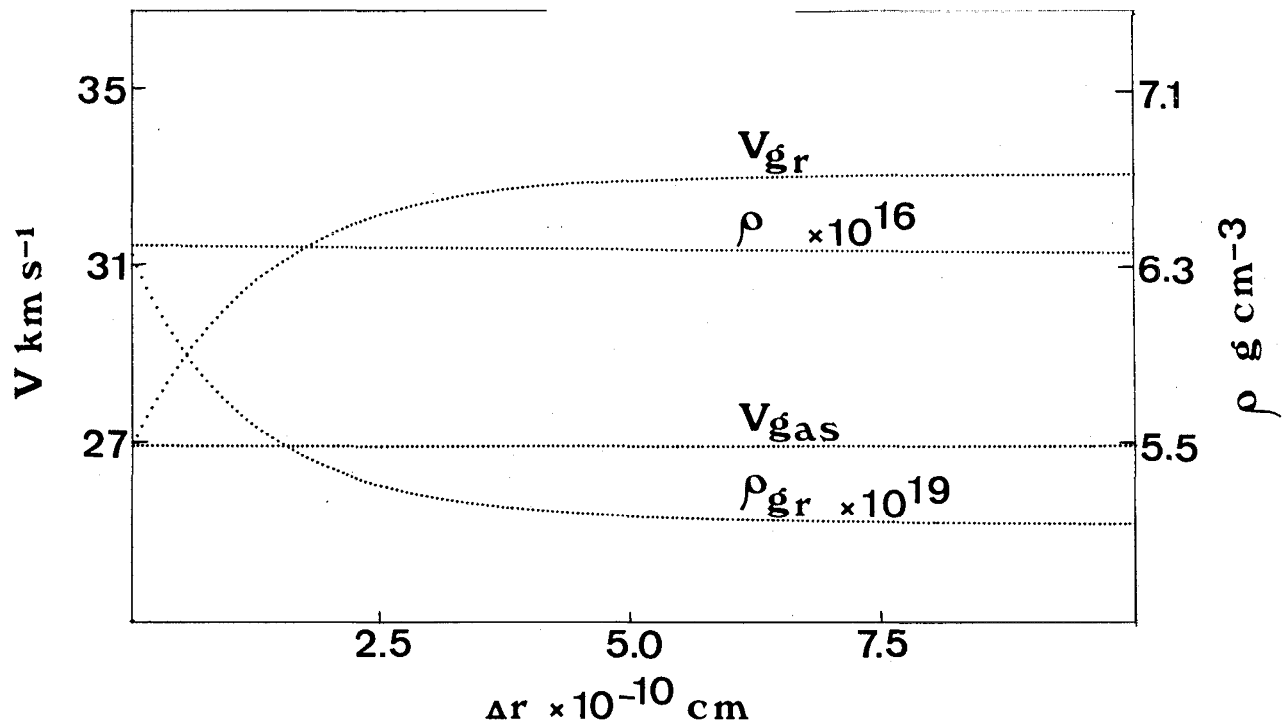

Figure 2 - Run of velocities and densities in the inner parts of the envelope for small grains (a=100 Å) as functions of $\Delta r = r - R$.

Table 1 shows the main results for grain sizes a = 100, 500 and 1000 Å. In all cases the terminal velocity (taken as the velocity at $r = 3R$) is reached quite close to the stellar surface, that is, at $r/R < 1.02$. For comparison, the results are given at two different points in the envelope, $r = 2R$ and $r = 3R$. We have verified that the obtained profilesare not very much sensitive to the adopted initial temperature, so that the existence of a dust shell can be warranted.

a (Å)	r/R	V_{gr} (km s^{-1})	V (km s^{-1})	$\rho_{gr} \times 10^{20}$ (g cm^{-3})	$\rho \times 10^{17}$ (g cm^{-3})	T (K)
100	2	35.9	28.1	11.5	14.8	958
100	3	39.6	29.6	4.63	6.24	539
500	2	48.5	27.0	8.49	15.4	985
500	3	51.0	27.4	3.59	6.73	568
1000	2	65.3	27.5	6.32	15.1	971
1000	3	65.9	27.7	2.80	6.71	565

Table 1 - Velocities, densities and temperature at two selected points in the stellar envelope for three grain sizes.

Figure 2 shows the run of the velocities and densities in the inner parts of the envelope for the smallest (a=100 Å) grains. In this figure, we have defined $\Delta r = r - R$. For grains having larger radii, the situation is qualitatively the same. From Table 1 and figure 2 we can see that the grains reach a large drift velocity with respect to the gas shortly after departure. However, the gas shell is being steadily accelerated by interaction with grains, which is readily seen from Table 1. Therefore, the mechanism of mass ejection by the indirect action of stellar radiation pressure through a circumstellar dust shell is verified to work for cool giant stars. The mass loss rate assumed by the model is $4.7 \times 10^{-7} M_{\odot}$ yr^{-1}, which lies within the observed range[9]. Although the dust shell reaches higher velocities than the gas shell, the mass lost by the star lies essentially on the gas molecules, owing to the low grain densities available.

The largest (a = 1000 Å) grains have a stronger interaction with the stellar radiation, as measured by the integrated efficiency factor Q. As a consequence, they reach the highest terminal velocity (~ 66 km s^{-1}), which permits them to travel a distance of about two stellar radii in a time shorter than the stellar period by a factor of ~ 3. In such conditions they can escape from the star and probably contribute to the observed grain densities in interstellar clouds. Forthe smallest grains (a = 100 Å), the terminal velocity is about 40 km s^{-1}, which means that they take six tenths of the stellar period to travel a distance of about 2 stellar radii. In all cases the terminal velocities are higher than those obtained previously[1,2]. However, since the present model is more complete than the previous ones, we believe the results shown in this paper to be more accurate. As a comparison, we have calculated the terminal velocity for large grains (a = 1000 Å) using the mean thermal velocity (~ 6 km s^{-1}) as the initial velocity. In this case, a smaller terminal velocity (~ 40 km s^{-1}) was obtained. However, such a velocity seems to be high enough to warrant the grain to escape from the star. Our calculated values are much lower than those reported by Wickramasinghe[10] (≳ 1000 km s^{-1}). As was shown by Gilman[11], circumstellar gas drag is not negligible, and escape velocities are more likely < 100 km s^{-1}.

The rate of grain injection into the interstellar space can be roughly estimated by

$$\frac{dn_{gr}}{dt} \simeq \frac{3\rho_{gr}(r_c)r_c^2 V_{gr}(r_c)}{a^3 S_g} N_* \quad . \tag{15}$$

where N_* is the number of stars per unit volume, which is ~ 10^{-7} pc^{-3} (see for example the review by Reimers[12]). Taking for the critical radius $r_c \simeq 3R_*$ the highest rate obtained is ~ 2.5 × 10^{-22} cm^{-3} yr^{-1}, for a = 100 Å. Using an upper limit $\tau \sim 10^9$ yr for the grain lifetime[13], we get an equilibrium density $\rho_{eq} \simeq 3 \times 10^{-30}$ g cm^{-3}, which is approximately the same for all grains, irrespective of sizes. Such a density is a few orders of magnitude lower than the observed cloud densities, so that the contribution of the stars considered to interstellar grain clouds is expected to be small.

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