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Particle Production on Deuteron near Threshold

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We have studied, in the context of impulse approximation, the effects of Fermi motion and threshold energy constraint on the incoherent particle production on deuterons. In this paper, we present a simple analytical model, and its application to the reaction $K^+d \to K^0\pi^+p\,(n)$, with the neutron as a spectator. It is shown that, while the effect of Fermi motion reduces the production rate, in qualitative agreement with experimental data, the threshold energy constraint has a negligible effect.

Estuda-se, no contexto da aproximação de impulso, os efeitos do movimento de Fermi e do vínculo imposto pelo limiar sobre a produção incoerente de partículas em dêuterons. Apresenta-se, aqui, um modelo analítico simples e sua aplicação à reação $K^+d \to K^0\pi^+p(n)$, onde o neutron é tratado como espectador. Mostra-se, ademais, que, se por um lado, o efeito do movimento de Fermi faz baixar a taxa de produção, o que está em concordância qualitativa com os dados experimentais, o vínculo correspondente ao limiar da reação tem efeito desprezível.

1. INTRODUCTION

The phenomenological study of particle-deuteron interactions is important from at least three different points of view: (i) to test the validity of theories and models for the three-body problem, and for scattering by composite targets; (ii) to extract data on the particle - neutron interaction; (iii) to obtain information on the interaction of short lived particles, and resonances with nucleons. Following any of these goals, a great deal of experimental and theoretical work has been done in the last few years.

The coherent differential cross section, at higt momentum transfer, where the double scattering dominates, is the most useful source of information concerning objective (iii) (Ref.1). For objective (ii), however, the incoherent differential and integrated cross sections are generally used, because the measurement of the final nucleon momenta, after the deuteron break up, gives us a criterion to define the spectator nucleon. Conceptually, the spectator is the nucleon which did not interact with the incident particle; operationally, it is simply the slower one.

In the spectator model, the production rate on deuteron, $\bar{a}_p$, defined in the half phase space where the neutron is a spectator, can be written as an integral of the production rate on the proton, a_p, over the deuteron momentum spectra, with some kinematical factors. If we neglect the dependence of the proton cross section on the interna1 nucleon momentum, we have the simple result $\bar{\sigma}_p = a_p$. Vice-versa, changing proton by neutron, we can do a direct measurement of the neutron cross section, when we go to the half phase space where the proton is a spectator.

From the point of view (i), mentioned above, it is worthwhile to check the spectator model. The comparison of $\bar{\sigma}_p$ with the value of a_p, measured using a hydrogen target, shows that $\sigma_p > \bar{\sigma}_p$ by 20% to 30% (Refs. 2, 3). The same percentage is used to correct the neutron cross section extracted from deuterium[3]. This disagreement has been qualitatively attributed to deuteron effects, such as Fermi motion, off mass shell and multiple scattering. In a recent paper, we have taken into account double scattering and final state interaction, using Glauber theory and closure approximation to calculate the pion production rate deuteron[4]. That model has reduced, to within the experimental errors, the discrepancy between the value of the $K^+p \to K^0\pi^+p$ cross section in hydrogen, and the same quantity extracted from the reaction $k^+d \to K^0\pi^+p(n)$ with the neutron as a spectator, for incident momenta higher than 1. up to 4.6 (Refs.3,6).

At lower momenta, however, very close to the production threshold, the validity of Glauber's theory is not quite good, and it would be interesting to explore an alternative explanation to account for the experimental results.

In this paper, we are interested in studying, in the context of the spectator model, possible threshold effects on particle production in deuteron break up by incident mesons. We divide these effects in two kinds: Fermi motion and threshold energy. The first effect arises from the dependence of the elementary particle-nucleon amplitude on the nucleon momentum. This dependence is negligible when the amplitude variation is small in comparison with the width of the deuteron momentum spectra. Near a resonance, for instance, this cannot be true: the Fermi motion has been shown to be important in πd scattering in a resonance region[7]. Above threshold, the production amplitude, starting from zero, varies strongly. This fact could also make the Fermi motion contribution important to the deuteron cross section. The second effect is closely related to the first one, and its triviality comes from the condition that the energy of the particle-nucleon system should be greater than the reaction threshold energy. When we consider Fermi motion, the two body energy depends on the nucleon momentum, which can provide more energy (say if the projectile and the nucleon momenta are anti-parallel) or less energy (if the two momenta are parallel, say) than in the static target approximation. Moreover, as the scattering is virtually inside the deuteron, the energy is not well-defined, and depends on our choice of an effective nucleon mass. This kind of threshold constraint modifies the limits of the phase space integration, and it has been shown by Atwood and West[8] to be important in the photoproduction on deuterons. Therefore, we have taken this into consideration in the present study. As shown in the following discussion, while the Fermi motion actually contributes to reduce $\bar{\sigma}_p$ in relation to σ_p, the deuteron momentum spectra makes the threshold energy constraint irrelevant for the reaction $K^+d \to K^0\pi^+p(n)$.

2. THE SPECTATOR MODEL WITH FERMI MOTION

The main features of the spectator model, that is, the single scattering impulse approximation in deuteron break up, are reported in reference[9]. We use here a rather simplified version of it, in which we neglect spin, isospin and deuteron D-wave. Their inclusion is straightforward, and we intend to take them into account in a forthcoming paper on the single scattering analysis of the general process $xd \to YNN$. They are, however, not relevant for our present purposes, since the relative effects we intend to discuss here are quite independent from spin and isospin effects. The differential cross section of a deuteron break up reaction, in the laboratory frame, is given, in the single scattering approximation[9], by .

$$\frac{d\sigma}{d\Omega} = \int d^3p \,|f_1(s_1,q)\ \gamma(-\vec{p})\psi(-\vec{p}) +$$

$$+ f_2(s_2,q)\ \gamma(-\vec{q}+\vec{p})\psi(-\vec{p}+\vec{q})\,|^2 \ . \tag{1}$$

The functions in Eq. 1 are: the elementary amplitude, $f_i(s_i,q)$, for production on nucleon i; the ratio, γ, between nucleon and deuteron flux factors; ψ, the deuteron wave function. The kinematical variables are: the square of the center of mass energy, s_i, of the projectile-i nucleon system, and the momentum transfer q. We can interpret $\vec{p}$ and $\vec{q} - \vec{p}$ as the final momenta of nucleons 2 and 1, respectively. As the deuteron is at rest, $\vec{p}_1 = -\vec{p}$ is the initial momentum of nucleon 1, when 2 is the spectator, and $\vec{p}_2 = -\vec{q} + \vec{p}$ is the initial momentum of 2, when 1 is the spectator (Fig.1).

We can write, in good approximation,

$$\gamma(\vec{p}_i) = 1 - (\ E_k/k^2 m_N\ )\ \vec{k}\ .\ \vec{p}_i\ , \tag{2}$$

where $\vec{k}$ and E_k are momentum and energy of the projectile, and m_N the nucleon mass. The two body CM energy is a function of the projectile and nucleon momenta: $s_i = s(k,p_i)$. If we take $p_1 = 0$ in f_i and in γ, we have

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega_1} + \frac{d\sigma}{d\Omega_2} + 2\rho(\vec{q})\,\mathrm{Re}\, f_1^* f_2, \qquad (3)$$

where $\rho(\vec{q}) = \int d^3p\psi(-\vec{p})\psi(-\vec{q}+\vec{p})$ is the deuteron form factor.

In Eq.3, the energy argument of the amplitudes f_i is $s_0 = s(k,0)$. It is easy to see that $d\sigma_1/d\Omega$, in (3), arises from the term $|f_1|^2\int d^3p|\psi(-\vec{p})|^2$. As $\psi(-\vec{p})$ is concentrated near $p=0$, $d\sigma_1/d\Omega$ comes from the phase space region where the final momentum of nucleon 2 (spectator of the interactionwith 1) is small ($p\simeq 0$). Analogously, $d\sigma_2/d\Omega$ comes from the region where the final momentum of 1 (spectator) is small ($|\vec{q}-\vec{p}| \simeq 0$). The interference term is controlled by the form factor $\rho(\vec{q})$, whose maximum value occurs at $q=0$, and it decreases for $q\neq 0$. Therefore, this term is only important in the boundary region where both final nucleons have almost equal momenta ($\vec{p} \simeq \vec{q} - \vec{p}$). If we neglect the interference term , we can select the events occuring in one of the nucleons. In the spectator approximation, the differential cross section for the nucleon, for instance, is given by $d\sigma_1/d\Omega = d\sigma/d\Omega|_{p<|\vec{q}-\vec{p}|}$. After integrating in Ω , we have

$$\sigma_1 \equiv \sigma \,\Big|_{p<|\vec{q}-\vec{p}|} = \sigma_1(s_0). \qquad (4)$$

Let us now consider the Fermi motion, that is the p-dependence of f_1 and γ in (1). Instead of (4), we have:

$$\bar{\sigma}_1 = \int d^3p\ \sigma_1(s_1)\gamma(-\vec{p})\,|\psi(-\vec{p})|^2 \quad , \qquad (5)$$

with

$$s_1 = s(k,\vec{p}_1) = s_0 + \alpha p_1^2 - 2\vec{k}\cdot\vec{p}_1 \ , \qquad (6a)$$

and $\vec{p}_1 = -\vec{p}$. For a physical nucleon a = E_k/m_N, but if we take the interacting nucleon off mass shell, assuming the spectator on mass shell[10], then

$$\alpha = -\frac{E_k + m_D}{m_N}$$

where m_D is the deuteron mass.

3. PRODUCTION NEAR THRESHOLD

Using (2), and making a linear approximation to a, $(s_1) = \sigma_1(s_0) + \sigma_1'(s_0)(\alpha p^2 + 2kp_{||})$, where σ_1' is the first derivative of σ_1, and $p_{||}$ is the component of $\vec{p}$ parallel to $\vec{k}$, we arrive at .

$$\bar{\sigma}_1 = \sigma_1(s_0) + \sigma_1'(s_0) \int d^3p|\psi(p)|^2(\alpha p^2 + 2akp_{||}), \qquad (6b)$$

with $a = E_k/km_N$. We have taken ψ as an even function, and therefore excluded the odd integrais in $\vec{p}$. It is further convenient to use a single Gaussian deuteron wave function [9]

$$\psi(p) = Ne^{-bp^2}$$

with N = $(2b/\pi)^{3/4}$ and b = $66(\mathrm{GeV/c})^2$. In this case,

$$\bar{\sigma}_1 = \sigma_1(s_0) + \sigma_1'(s_0)\frac{1}{b}\left(\frac{3\alpha}{2} + ka\right). \qquad (7)$$

Near the threshold, σ_1 increases, and so $\sigma_1' > 0$. To explain the behaviour $\bar{\sigma}_1 < \sigma_1(s_0)$, it is enough to note that the term inside parentheses is negative. From (6b), we can see that

$$\frac{3\alpha}{2} + ka = -\frac{E_k}{2M_N} - 3$$

is always negative, as we naed it. Moreover, the linear approximation to σ_1 is very naive, and we have to check this prediction in a more realistic case.

We, then, assume that σ_1 can be parametrized, above threshold, by the smooth function of s_1, namely,

$$\sigma_1(s_1) \approx A\{1 - \exp[-\beta(s_1 - s_T)]\}, \qquad (8)$$

which describes the rise of the cross section from zero, at the threshold s_T, up to a value A. This hypothesis allows us to perform, analytically, the integration in (5):

$$\bar{\sigma}_1 = 2\pi A N^2 \int_0^\infty p\,dp \int_{-p}^{p} e^{-2bp^2}(1+ap_\parallel)\{1-\exp[-\beta(s_0-s_T+\alpha p^2+2kp_\parallel)]\}, \qquad (9)$$

which gives

$$\bar{\sigma}_1 = A\ \{1 - \delta\exp[-\beta(s_0-s_T)]\}\ , \qquad (10)$$

with

$$\delta = \left(\frac{2b}{B}\right)^{3/2} \left(1 - \frac{a\beta k}{B}\right) \exp(\beta^2 k^2/B)\ . \qquad (11)$$

This result shows that, for $\delta=1$, we go back to $\bar{\sigma}_1 = a_1(s_0)$, and for $\delta > 1$ we have the agreement with $\bar{\sigma}_1 < \sigma_1(s_0)$.

As we hope to have, in general, $\beta \ll b$, because the deuteron wave function is very sharp, we take $\delta>1$ in all practical cases.

Before applying our formula to a physical situation, it is better to examine the threshold energy condition, $s_1>s_T$. From this condition and (6a), we see that the cosine of the angle, between projectile and nucleon momenta, has to obey the relation

$$\cos\theta_1 < \frac{s_0 - s_T + \alpha p_1^2}{2kp_1} \equiv \chi(p_1)\ .$$

If we change the variables $-\vec{p} = \vec{p}_1$ in (5), the above constraint means that we have a physical upper limit to $\cos\theta_1$ when we integrate in $\vec{p}_1$. In (9), this gives a lower limit to the integration in $p_\parallel$. The presence of the deuteron wave function as a factor in the integrand, however, makes this new restriction irrelevant. The point is thaf (11) is effective only for $\chi < 1$ and this, in general, happens at high values of p_1. The role of $\psi(p_1)$ is just to drop the high momenta contribution to the integrand. We can picture the situation clearly in a concrete case.

4. APPLICATION AND CONCLUSIONS

As an application, we have considered the reaction $K^+ d \rightarrow K^0\pi^+ p(n)$. The fitting of the $k^+ p \rightarrow K^0\pi^+ p$ production rate, measured for a hydrogen

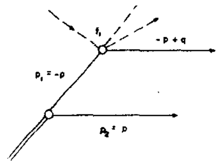

Fig. 1 - The impulse approximation description of deuteron break up

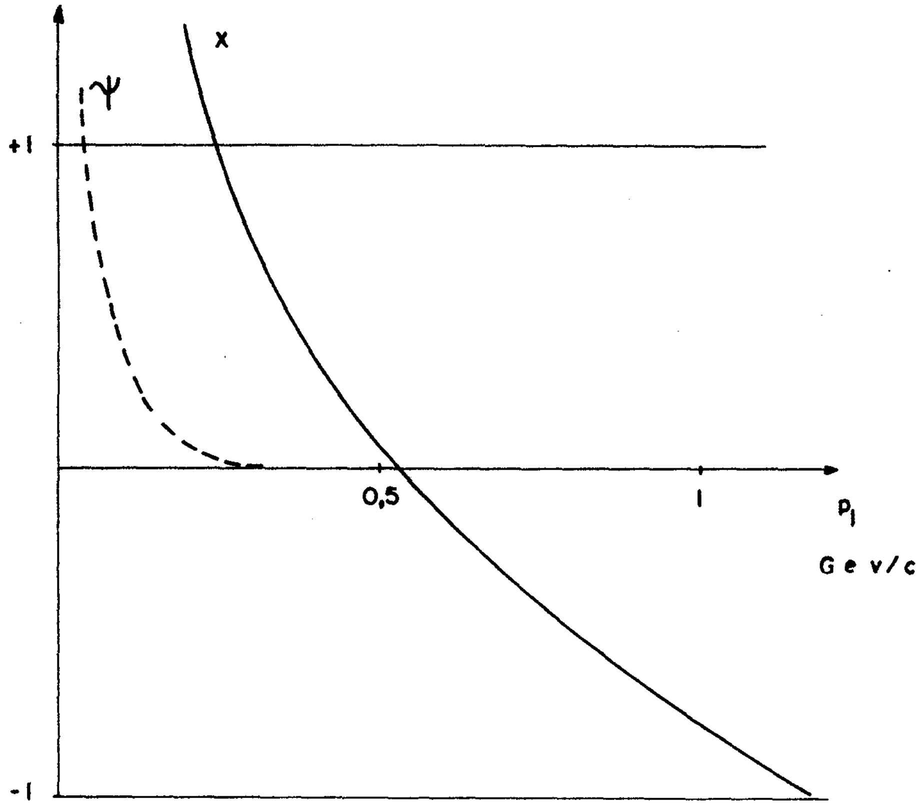

Fig.3 - Upper limit for χ as a function of the nucleon momentum p for the incídent momentum 0.9 Gev/c, applying to the threshold energy constraint, and the deuteron wave function.

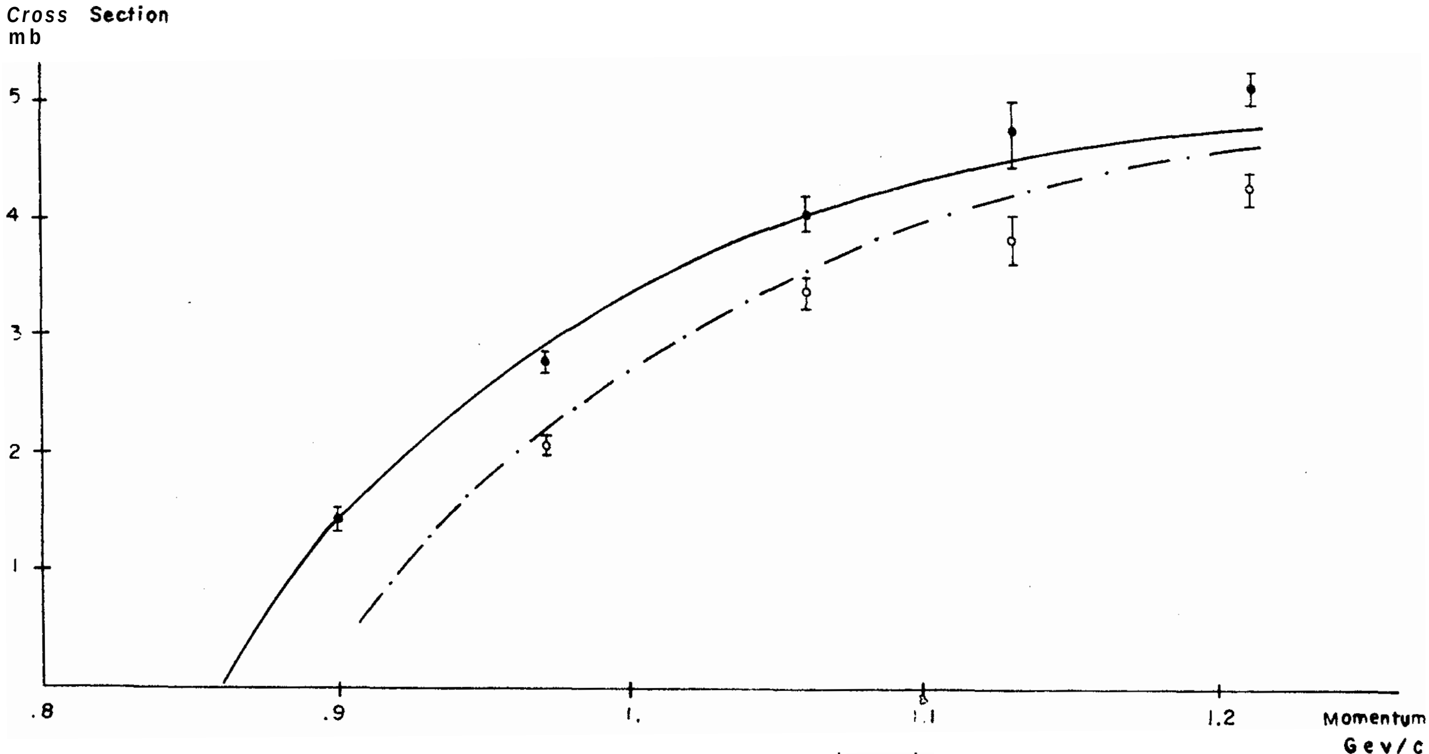

Fig.2 - Theoretical curves are: ——— parameterization of the $K^+p \to K^0\pi^+p$ cross section; ----- calculation ot the $K^+d \to K^0\pi^+p(n)$ cross section using the spectator model with Fermi motion. Experimental points refer to ● → K^+p, ○ → K^+d.

target (Fig.2) determines the parameters of our formula (8): $A = 5.12$ mb, $\beta = 4.8$ Gev^{-2}.

Instead of the actual threshold energy, s_T, we have used an effective value for it, which has been empirically obtained from the point where the experimental production rate practically vanishes. This empirical value is different from the theoretical one because the cross section in the neighbourhood of the threshold cannot be described by a function as simple as (8). The effective s_T corresponds to an incident momentum $k_T = 0.86$ Gev/c.

We have calculated the deuteron cross section using formula (10) with α given by (6b). The result is shown in Fig.2 in comparison with the experimental data. We can say that, in this simple model, the Fermi motion contributes to make $\bar{\sigma}_1 < \sigma_1(s_0)$ near the threshold. It is, however, important to make some comments on the significance of our calculations. First of all, in the measurement of, there were cuts in the phase space: events with spectator momenta higher than 0.25 Gev /c are rejected because they are in disagreement with deuteron wave function prediction [3]. We have not included explicitly such a cut in our analytical calculation, in which we integrated (9) over all phase space. However, the presence of the deuteron wave function in the integrand (5) practically cuts all contribution from higher momenta. This damping of the wave function makes also irrelevant the, threshold energy condition (11)

In Fig.3, we see the behaviour of the upper limit χ of $\cos\theta_1$, at incident momentum 0.9 Gev/c. As χ is less than 1 only for nucleon momenta greater than 0.25 Gev/c, where $\psi \to 0$, we can disregard the threshold energy constraint in the phase space integration.

In conclusion, we should like to call attention to the fact that although the Fermi motion seems to contribute actually to the $K^+ d \to K^0 \pi^+ p(n)$ cross section, it is not relevant enough to lead to a quantitative agreement with the experimental data. Besides, at higher momenta, not very far from threshold, the spectator model fails completely to explain the experimental values of $\sigma_p/\bar{\sigma}_p$, indicating that other effects, such as interference and multiple step interactions have to be considered [5].

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