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The Renormalization Group of Relativistic Quantum Field Theory as a Set of Generalized, Spontaneously Broken, Symmetry Transformations*

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The Renormalization Group Theory has a natural place in a general framework of svmmetries in Quantum Field Theories. Seen in this way, a "Renormalization Group" is a one-parametric subset of the direct product of dilatation and renormalization groups. This subset of spontaneously broken symmetry transformations connects the inequivalent solutions generated by a parameter-dependent regularization procedure, as occurs in Renormalized Perturbation Theory. By considering the global, rather than the infinitesimal, transformations, an expression for general vertices is directly obtained, which is the formal solution of exact Renormalization Group equations.

A Teoria do Grupo de Renormalização encontra seu lugar natural no quadro geral de simetrias, nas Teorias Quânticas de Campo. Visto dessa maneira, um "Grupo de Renormalização" é um sub-conjunto, a um parâmetro, do produto direto de grupos de dilatação e de renormalização. Esse sub-conjunto de transformações de simetria, espontaneamente quebradas, liga as soluções não equivalentes geradas por um método de regularização dependente de um parâmetro, como ocorre em Teoria de Perturbação Renormalizada. Considerando as transformações globais, ao invés das infinitesimais, obtem-se diretamente uma expressão para vértices gerais, que é a solução formal das equações exatas do Grupo de Renormalização.

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1. INJRODUCTION

In recent years, the original idea[1,2] of the Renormalization Group, in Relativistic Quantum Field Theory, has been generalized and deepened (see, for example, Refs. 3-14.).
At present, the basic formalisrn appears to be well understood.

One point which is not yet quite transparent is the position of the Renormalization Group is relation to the conventional symmetry concepts, and the question which one the actual symrnetry group exactly is. It is the purpose of the present paper to clarify these questions and related ones by showing that the Renormalization Group fits quite naturally in a framework of generalized symmetries in Quantum Field Theories[15].

From our viewpoint, a Renormalization Group is a special one-parametric subset contained in a certain group of generalized, spontaneously broken, symmetry transformations. The Renormalization Group differential equation represents the effect of an infinitesimal change of the continuous parameter characterising the transforrnations of the set. Because we work with the global, instead of the infinitesimal, transformations, we find directly the solution of the usual approxirnate Renormalization Group equations for general vertices, without needing this equation . Besides, an exact formal expression for these vertices is obtained.This possibility of avoiding the detour over the Renorrnalization Group differential equation, which occurs quite naturally in the present formulation, seems not to be generally known.

In the next section, the direct product of the passive dilatation and renormalization groups is discussed as an exarnple of a spontaneously broken symmetry group, and in the third section the connection with the Renormaiization Group equations is made. In the Concluding Remarks , some aspects of the relation of the present approach to other recent papers are discussed.

2. THE DILATATION-RENORMALIZATION TRANSFORMATIONS AS A SPONTANEOUSLY BROKEN SYMMETRY GROUP

Our considerations are made in a framework for generalized symmetries in Quantum Field Theories which has been given in Ref. 15. In the next paragraph we review its essential points, omitting all details.

The basic definition is the one of a Passive Symmetry Transformation of a theory as being an algebraic transformation, of the field operators, which carries any solution (often given as an irreducible representation) into another solution. Because each solution is supposed to describe the physical properties of the system, all uniquely measurable quantities must be invariant under any passive symmetry transformation. If such a transformation is represented by a unitary operator, it is a physical or "good" symmetry; otherwise, we call it, in a generalized sense, an "spontaneously broken"[16] symmetry. To draw in this case nontrivial physical conclusions, additional assumptions, replacing unitary equivalence, are necessary. If, for instance, spontaneously broken passive symmetries form a Lie Group and are, in each space-time plane, generated by a local (in general not conserved) current, one obtains a charge algebra with the structure coefficients of the passive symmetry group. If a generating current is even conserved, one has an spontaneously broken symmetry in the usual, more restricted, sense and Goldstone's Theorem[17,18] states that the symmetry must be a physical one, unless a boson or long range interaction occurs with the quantum numbers of the zero component of the current.

The occurrence of a passive symmetry in the basic theory of a physical system manifests itself by the existence of different descriptions of the same Physics, in each of which any uniquely measurable quantity[15] has the same value. Conversely, the availability of a continuous set of such descriptions indicates the possibility of formulating the theory in a manifestly passive-symmetric way. Two special cases of this situation are essential for our later considerations:

a) A change in the length unit of a closed physical theory changes the mathematical description, but not the physical quantities which are always dimensionless[19]. This is what one would expect in a theory with a passive dilatation symmetry, and in the following we shall assume that it is possible to formulate the theory under consideration so that it is passively dilatation invariant. In fact, it appears in some cases nearly unavoidable to do so. For example, In Quantum Electrodynamics the bare electron mass is the only parameter of mass dimension entering in the basic equations. In usual perturbation theory, one is compelled to take the bare mass infinite, whereas in a selfconsistent calculation the bare mass has to vanish[20,21]. Both of these mass values are dilatation invariant and consequently the basic theory does not select the electron mass, allowing for an arbitrary mass scale with the corresponding infinite set of mathematically non-equivalent solutions.

The passive dilatation symmetry is a typical case of a spontaneously broken symmetry, in the sense of the present section. If $\{\psi(x), A^{\mu}(x)\}$ is a solution (with "physical" electron mass λm), then $\{\psi'(x)=\lambda^{3/2}\psi(\lambda x),$ $A'^{\mu}(x) = \lambda A^{\mu}(\lambda x)\}$ is also a solution (with electron mass m).

It is interesting to note ttiat, for the case of more general conformal invariance, the above argument is not valid. In a general physical system, no simple change of description are known wtiich leave physical quantities invariant and which correspond to conformal transformations.

b) Dilatation invariance is a space-time symmetry and has, therefore, a clear classical meaning. A non-classical symmetry transformation,which occurs quite naturally in certain Quantum Field Theories, is the renormalization. The values of coupling constants, and the <u>a priori</u> not defined renormalization of a finite number of singular local operator products are considered as a part of the solution. Then on can, at least in renormalizable Quantum Field Theories, construct new solutions from a givem one, by multiplying field operators, coupling constants and the mentioned products by suitable related finite numbers. Under such transformations, all physical quantities remain unchanged.

A renormalization symmetric formulation of the theory cannot, of course, contain fixed canonical commutation rules, but these are in general anyhow untenable. As in the case of passive dilatation symmetry in

Q.E.D., which one only can break by brute force as by postulating a definite value for the electron mass, one can also oniy break renormalization symmetry rather artificially. One may do this, for instance, by demanding, as part of the theory, a definite normalization of certain Green's functions at prescribed momenta, in this way excluding the other solutions and with that destroying the passive symmetry.

In trying to give an explicit example of a renormalization invariant field theory, we are of course hampered by the fact that no clearly consistent Quantum Field Theory in four dimensions has yet been constructed. In Quantum Electrodynamics, one might, for instance, imagine the free Lagragian of the electron to be defined by[15]:

$$L_0(x) = \int_{V(\varepsilon)\to 0} \overline{\psi}(x + \tfrac{\varepsilon}{2}) S^{-1}(\varepsilon)\psi(x - \tfrac{\varepsilon}{2}) d^4\varepsilon, \tag{1}$$

where $S^{-1}(\varepsilon)$ (containing as a factor the equivalent of the conventional Z_2) is the inverse of the electron propagator $S(\varepsilon)$:

$$\int S(\varepsilon-\varepsilon') S^{-1}(\varepsilon') d\varepsilon' = \delta^4(\varepsilon). \tag{2}$$

An analogous form can be assumed for the free Maxwell Lagragian. In the interaction, the renormalization factor of $S^{-1}(\varepsilon)$ is also included, playing the role of $Z_1 (=Z_2)$, and the value of the coupling constant is assumed to belong to the solution of the theory, To guarantee that one has the usual strength of the electromagnetic interaction, one demands, as part of the theory, the condition:

$$\lim_{k^2\to 0} \frac{e^2}{4\pi} k^2 D_t(k^2) = \alpha, \tag{3}$$

with a being the usual fine structure constant and D_t the transversal photon propagator. As a result, the theory will have the desired passive symmetry: if $\{\psi(x), A^\mu(x), e\}$ is a solution, then $\{\psi'(x) = Z_2^{1/2}\psi(x), A'^\mu(x) = Z_3^{1/2} A^\mu(x), e' = Z_3^{-1/2} e\}$, with the Z_i's being arbitrary, finite, positive constants, is also a solution.

In the following, we shall assume that the theory under discussion can be formulated passively symmetric under renormalization transformations, and dilatations as well.

The direct product of both, commuting, abelian Lie groups will be called the "Dilatation-Renormalization" (D.R.) Group. As we have remarked, this symmetry is in agreement with the results of renormalized perturbation theory. Any passive invariance of a field theory is, by definition, inherited by equivalent equations derived from it, as, for instance, the equations for the Green's functions. As long as a non-trivial relativistic quantum field theory cannot be consistently formulated, one may instead start from these equations. The non-uniqueness of the solutions corresponding to the D.R. group is then generated be the non-uniqueness of spontaneously broken symmetry solutions[21] of Dyson - Schwinger equations or by the arbitrariness in the renorrnalization procedure. However, the general symmetry view (as many other considerations) becomes clearer, if one assumes that there exists some underlying field theory, which hopefully will once be exactly defined.

A transformation, characterized by the dilatation parameter h and the renormalization parameters Z_i acts on the field operators and coupling constants as:

$$\begin{aligned} \psi'_i(x) &= Z_i^{1/2}\,\lambda^{-d_i}\,\psi_i(\lambda^{-1}x), \\ e'_j &= Z'^{1/2}_j\,e_j, \end{aligned} \tag{4}$$

where the d's are canonical mass dimensions. If $\{\psi_i, e_j\}$ solves the theory, then $\{\psi'_i, e'_j\}$ is another solution.

For the vacuum expectation values, it follows that

$$\begin{aligned} &\langle\psi'_1(x_1)\ldots\psi'_1(x_{n_1})\psi'_2(Y_1)\ldots\psi'_2(Y_{n_2})\ldots\rangle = \\ &\prod_i \lambda^{-n_i d_i}\,Z_i^{1/2\,n_i}\,.\,\langle\psi_1(\lambda^{-1}x_1)\ldots\psi_1(\lambda^{-1}x_{n_1}). \\ &.\,\psi_2(\lambda^{-1}Y_1)\ldots\psi_2(\lambda^{-1}Y_{n_2})\ldots\rangle\,. \end{aligned} \tag{5}$$

Vertex functions, in momentum variables, transform as:

$$\Gamma'(p) = \lambda^{-d_\Gamma} Z_\Gamma \Gamma(\lambda p), \tag{6}$$

where d_Γ and Z_Γ are, respectively, the canonical mass dimension and renormalization constant of the general vertex Γ, calculated from the ones of the composing fields and inverse propagators, and p denotes all momentum arguments.

Let us suppose that one has found a solution of the theory defined by the complete infinite set of vertices. Then, a continuous manifold of sets of other solutions can be found by applying the transformation (6), for different values of Z_i and λ, to each vertex of the solution:

$$\Gamma_{Z_i,\lambda}(p) = \lambda^{-d_\Gamma} Z_\Gamma(Z_i)\Gamma_{Z_i=1,\lambda=1}(\lambda p), \tag{7}$$

$\Gamma_{11}(p)$ being the original solution. This statement means just the assumed passive symmetry of the theory. The parameters λ and Z_i, of the D.R. group, may in this way be used to characterize any of the solutions if one solution, $\Gamma_{11}(p)$, is given.

3. APPLICATIONS

As is true in general for passive symmetries, also in the case of the D.R.group, additional assumptions are necessary to draw physical conclusions.

a) Let us first make the strong assumption that the many-parameter D.R. transformations, Eq. (7), contain a one-parameter subgroup, $Z_i(\lambda)$, which is a "good" symmetry, being representable by a unitary operator with an invariant vacuum. For this subgroup, the coupling constants and the L.H.S. of Eq.(7) must be independent of λ (say, equal to $\Gamma_{11}(p)$). We can, therefore, write

$$0 = \frac{d}{d\lambda} \ln\left[\lambda^{-d_\Gamma} Z_\Gamma(\lambda)\Gamma(\lambda p)\right], \tag{8}$$

or

$$-d_\Gamma + \frac{d \ln Z_\Gamma(\lambda)}{d \ln \lambda} + \frac{d \ln \Gamma(\lambda p)}{d \ln(\lambda p)} = 0 .$$

As this equation should be valid for all values of p, one obtains

$$\frac{d \ln Z_\Gamma(\lambda)}{d \ln \lambda} = - a_\Gamma , \tag{9}$$

with a_Γ being constant. The solution with $Z(1) = 1$ is

$$Z_\Gamma = \lambda^{-a_\Gamma} , \tag{10}$$

and inserting Eq. (10) in Eq. (7) one has

$$\Gamma(\lambda p) = \lambda^{d_\Gamma + a_\Gamma} \Gamma(p) . \tag{11}$$

Applying the same analysis to a number of suitable vertices for the Z_i's, of which the Z_Γ's are composed, a power law is also found:

$$Z_i(\lambda) = \lambda^{-2a_i} \tag{12}$$

This type of conclusion, in which an assumption concerning the L.H.S. of Eq. (7) determines the trajectory $Z_i(\lambda)$, will in the following recur repeatedly.

The "good" one-parameter subgroup of the D.R. transformation (4) becomes, from Eq. (12),

$$\begin{aligned} \psi'_i(x) &= \lambda^{-(d_i + a_i)} \psi_i(\lambda^{-1} x) , \\ e'_j &= e_j . \end{aligned} \tag{13}$$

One might call this combination of a normal dilatation and a renormalization an "anomalous dilatation". Usually, the transformation (13) is considered as being a normal dilatation but with the "anomalous dimensions" a_i.

From the present point of view, it becomes understandable why, in theories with anomalous dimensions, many observables transform with their "normal" dimensions, which are obtained by assigning canonical dimensions to the composing field operators. Observables are most often so defined that they are renormalization invariant. Their representation, with respect to the pure Renormalization Group, is then the trivial one, and the effect of the D.R. group on these quantities reduces to pure dilatations which are defined to be the conventional ones. As an example, we may quote the electromagnetic current, which is renormalization invariant: because of Ward's Identity, and has therefore the normal dimension, $d = 3$.

We have shown that if the passive D.R. group contains a good subgroup, then this can only be an anomalous dilatation. However, in this case it follows from Eq. (11) that such a theory cannot have a discrete non-zero mass spectrum. For our additional condition to be applicable to a realistic theory, we have therefore to weaken it.

b) Let us suppose that all symmetries of the D.R. group are spontaneously broken, but that there is a subgroup which leaves all vertices asymptotically invariant for sufficiently large values of the momenta (or h). In this case, exactly the same reasoning as above can be applied, with the understanding that it is only valid for sufficiently large p-values. Expression (11) now determines only the asymptotic behaviour of the vertices. The anomalous dilatation, Eq. (13), "becomes a good symmetry" at sufficiently high momenta.

Although the present case admits of a discrete mass spectrum, its assumption is probably still too strong to be generally valid.

c) To obtain a realistic case, we shall add to relation (7) some information which Renormalized Perturbation Theory suggests, and which is supposed to be true in each order of the perturbation. Of course, the hope is that these relations are also valid for the exact solution. Proofs of these results of perturbation theory, so far as they exist, may be found in standard treatments[3,22].

Renormalized Perturbation Theory is an approximate method of solving the problem by means of a limiting procedure, which consists of a smooth retraction of a regularization. If the solution of a theory is not unique, then the particular solution one finds by any lirniting procedure will, in general, depend on the way in which the limit is reached. This is what happens in perturbation theory. By choosing different values for a parameter, which is kept fixed in the limiting procedure, one is able to generate a one-parameter set of solutions, cornparable to the one-parameter sets under a) and b) of this section. However, the relation between these solutions, following from perturbation theory, is still weaker than the asymptotic invariance condition.

Let us now consider a theory with severa1 physical nasses. Under the D. R. group, the mass ratios rernain invariant, and we can therefore define the scale of any solution by giving the physical rnass m_R of one specified particle. Usually, the limíts in perturbation theory are so performed that this "renormalized" rnass has, in all cases, the same fixed value. The parameter, to which different values are given, is the quantity μ^2 of the externa1 $(\text{momenta})^2$, at which a sufficient nurnber of simple vertices are normalized in order to fix all renorrnalization constants, Z_i. These constants, and consequently also the coupling constants e_j, which are associated with a part of the Z's, are therefore functions of μ.

Perturbation theory gives the vertices as a power series of the coupling constants. These may be written as

$$\Gamma(p, e\ (m_R, \mu)\ ,\ m_R, \mu), \tag{14}$$

where Γ abbreviates the infinite set of vertices of the solution under consideration, and p denotes the occurring mornenta, while e stands for the coupling constants e_j .

For the same momenta, corresponding vertices in two solutions (14), for different values of μ, differ only by a constant normalization factor , but this factor is rather indirectly given. The value of μ does not indicate directly the solution which is selected, but it defines the way in which this selection is performed. To simplify this situation, we choose an arbitrary but definite value μ_0 of μ. On a representation with $\mu = \lambda\mu_0$, we apply a (normal) passive dilatation transformation with parameter λ, and as a consequence of passive dilatation invariance we obtain again a solution. One has

$$\Gamma' = \lambda^{-d_\Gamma} \; \Gamma(\lambda p, e(m_R,\mu), m_R, \mu) \equiv$$

$$\equiv \Gamma(p, e(m_R/\lambda, \mu_0), m_R/\lambda, \mu_0), \qquad (15)$$

where the identity follows from dimensional analysis.

Instead of the set (14) we shall use the set (15), in which each member is characterized by a value of λ.

In a similar way, the set (15) could have been derived for the case of a dimensional regularization, where μ has then another meaning[16].

In fact, from our point of view, we could have more naturally obtained the set of solutions (15) directly from the renormalization procedure, by keeping the parameter μ and the renormalization prescription fixed, and instead varying the mass scale, selecting the particle mass to be equal to m_R/λ. The detourover expression (14) was taken to make the connection with the usual treatment.

As all our solutions (15) are related only be renormalizations and dilatations, one has from Eqs. (7) and (15),

$$\Gamma(\lambda p, e(m_R), m_R) = \lambda^{d_\Gamma} \; Z_\Gamma^{-1}(\lambda)\Gamma(p, e(m_R/\lambda), m_R/\lambda), \qquad (16)$$

where we have ommited the fixed parameter μ_0. In this way, the one-parameter set of perturbation solutions defines a one parametric trajectory $Z_i(\lambda)$, in the multi-parameter (λ, Z_i)-space of the D.R.group (as in a) and b) of this section). By varying the normalization conditions at the point μ_0, one may find a dense set of one-parameter trajectories .

The $Z_i(\lambda)$ values of any two trajectories are, however, related, by h-independent factors and it is therefore sufficient to discuss one of these trajectories.

By writing Eq.(16) for those vertices by which the coupling constants and renormalization constants are defined, differentiating the logarithm of these equations with respect to the logarithms of the external (momenta)^2, and taking these equal to μ_0^2, one finds, as usual, equationsof the type

$$\lambda \frac{de_j}{d\lambda} = \beta_j(e,m_R/\lambda), \tag{17}$$

$$\lambda \frac{d\ \ln Z_i}{d\lambda} = \gamma_i(e,m_R/\lambda). \tag{18}$$

The dependence on the fixed value μ_0 has again been dropped. Perturbation theory gives the functions β and γ as series expansions in the coupling constants. (Because these constants are directly related to some of the renormalization constants, the corresponding β's and γ's are not independent.) Eqs. (17) and (18) gives the changes of the coupling and renormalization constants along a trajectory $Z_i(\lambda)$, and may be compared with the much stronger Eq. (9).

The important assumption made by Gell-Mann and Low[2], which seems plausible and is confirmed by perturbation theory, is that for sufficiently large values of h, i.e., for sufficiently small values of m = m_R/λ in the set (15), under fixed normalization conditions, the explicit dependente on the scale of the particle masses vanishes. This means that, for high A,

$$\Gamma(p,e(m_R/\lambda),m_R/\lambda) \to \Gamma(p,e(m_R/\lambda),0) \ . \tag{19}$$

Inserting this limit into Eq.(16), one obtains, for large A,

$$\Gamma(\lambda p,e(m_R),m_R) \approx \lambda^{d_\Gamma} Z_\Gamma^{-1}(\lambda)\Gamma(p,e(m_R/\lambda)\ ,0\ ) \ . \tag{20}$$

If, in addition, also the limits

$$e_j(m_R/\lambda) \underset{\lambda\to\infty}{\to} e_j(0) \qquad (21)$$

would exist, one would have exactly the previous case b) of asymptotic invariance, leading to anomalous dimensions. This assumption, however, may be in general too strong. Instead, as in Eq. (19), only on the R.H.S. of Eqs. (17) and (18) the explicit dependences on the mass-scale seems, from perturbation theory, to vanish in the limit $m_R/\lambda\to 0$. One obtains, in this limit,

$$\lambda \frac{de_j}{d\lambda} = \beta_j(e,0) \quad , \qquad (22)$$

$$\lambda \frac{d\, \ln Z_i}{d\lambda} = \gamma_i(e,0) \quad , \qquad (23)$$

with the λ-dependence on the R.H.S. occurring only implicitly in the coupling constants.

Eqs. (20), (22) and (23) form the basis of most discussions on the Renormalization Group. In fact, the conventional Renormalization Group equation for general vertices is obtained by differentiating the logarithm of Eq. (20), with respect to ℓnλ, inserting Eqs. (22) and (23) and going back to the μ variable. We obtain the wellknown solution of this differential equation directly, substituting in Eq. (20) the coupling and renormalization constants by the conventional solutions of Eqs. (22) and (23). The reason why we save one integration is that the usual approach implicitly works with an infinitesimal version of the transformation (7) whereas we use the global transformation. From the present point of view, it is understandable why the Renormalization Group equations can be integrated, and the general form of the solution becomes transparent.

One might derive exact Renormalization Group equations by differentiating the logarithm of Eq. (16), with respect to ℓnλ, and using Eqs. (17)

and (18). However, this procedure is again a detour, as Eq.(16) is already the exact formal solution in which one is interested. Given sufficient information concerning the m_R/λ-dependences of the functions occurring in Eqs.(16), (17) and (18), corrections to the vanishing mass approximation, in the conventional Renormalization Group equations, would be calculable.

We remark that any solution of the Renormalization Group, Eq.(18), defines, by Eq.(4), a global symmetry transformation of the field operators, which is a generalization of the usual dilatation transformations or of the anomalous ones given in Eq.(13).

4. CONCLUDING REMARKS

The main point of the present paper is to show that Renormalization Group Theory has a common root with other symmetry theories, be they concerned with physical or spontaneously broken symmetries. The root is the natural occurrence of a passive symmetry in the basic theory. The transformations relevant to the Renormalization Group form a one-parameter subset of the direct product of the usual dilatation and renormalization passive symmetry groups. This subset is given by the set of solutions which it connects, i.e., in practice, by the selected renormalization procedure. Seen in that way, the only essential difference with the case of conventional symmetries is that the transformed representations are not unitary equivalent, but have a weaker relation, on which, for instance, perturbation theory may give information.

One may wonder how could we have avoided to even mentioning the infinities of Quantum Field Theories, which appear to be basic in many treatments of the Renormalization Group. From our viewpoint, the essential function of these infinities is to allow for the non-uniqueness of the solution which must occur in the theory, if it possesses the passive D.R. Symetry. After having postulated this symmetry, and the finiteness of the theory, and as long as we avoid explicit calculations, we do not need to discuss anymore how, in detail, the Renormalization Program masters the infinities.

Recently, 't Hooft and Weinberg[9,10] (see also Refs. 11,12) have found a variant of the usual way of handling perturbative renormalization, which leads to "New Renormalization Group Equations" corresponding to trajectories for which Eqs. (22) and (23) are exactly true for all values of A. The price one has to pay is that the arguments corresponding to m_R/λ of Γ, in Eq.(16), are now "effective" masses, which are functions of the coupling constants and A. The main advantage is that the New Renormalization Group Equation can be formally integrated,. before the vanishing mass approximations have been made. Therefore, the validity of the approximation (19) can be investigated and corrections to it calculated. The new solution is again obtained if one substitutes, in Eq. (7), the relevant expressions for the vertex and renormalization constants. As remarked earlier, Eq.(16) may also be considered as the formal solution of exact Renormalization Group equations.

Our point of view is quite different from the one taken in the imaginative work of Wilson[14], who discovered the remarkable analogy between Kadanoff's scaling properties of Many Body Systems,at critical points[23], and the Renormalization Group Equations of Relativistic Quantum Field Theory. Whereas in our treatment, the Renormalization Group, as any conventional symmetry group, is a set of transformations between solutions of one definite theory, in Wilson's theory it transforms different cutoff-interactions into each other. The connection between the two approaches is about the following. Our non-equivalent solutions (15) of one definite theory may be generated by renormalizing, differently, regularizeci theories. These regularized theories are related to each other as Wilson's different interactions on the "renormalized" trajectory in his space of interactions, namely, by a combined dilatation and renormalization.

Any symmetry of a theory is unavoidably linked to a non-uniqueness of the mathematical description, which therefore must contain quantities without a direct physical meaning. The anomalous dimension of a charged field, being gauge-dependent, is one example of such a quantity. This general symmetry property may explain why considerations on the relativistic Renormalization Group often do not follow the patterns of naive physical intuition.

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