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$SU(3) \otimes SO_{I^c}(3)$ Three Triplet Model and the New Resonances

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The $SU(3) \otimes SO_{I^c}(3)$ three triplet model, first proposed by Tati, is examined in connection with the recently discovered narrow resonances, observed in the e^+e^- annihilation experiments.

O modelo a três tripletos, com simetria $SU(3) \otimes SO_{I^c}(3)$, proposto pela primeira vez por Tati, é examinado em face às recentemente descobertas ressonâncias estreitas, observadas em experiências de aniquilação e^+e^-.

1. Introduction

The recently discovered narrow resonances in the e^+e^- system[1] – the so called ψ's – have been discussed under several viewpoints, in order to account for their existence and basic properties.

Under the assumption that the new objects are massive vector mesons, a number of symmetry schemes have been proposed, corresponding to an enlargement of the ordinary $SU(3)$ unitary group of the strong interactions to groups such as $SU(4)$ – the charm scheme[2] – or $SU(3) \otimes SU^c(3)$ – the colour scheme[3] – and variants of them, in order to allow the introduction of a new quantum number, whose existence seems necessary to explain the metastability of the new states.

Although there is increasing evidence in favor of the charm, the complexity of the experimental situation which is emerging indicates that is relevant to study other conventional possibilities.

In this work, we study the classification of the narrow resonances in the framework of a model proposed by Tati[4], which is a three triplet

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model based on the group $G = SU(3) \otimes SO_{I^c}(3)$, where the $SU(3)$ stands for the ordinary unitary group of strong interactions and $SO_{I^c}(3)$ is a 3-dimensional rotation group associated with the colour iso-spin I^c.

One might remark that Tati's model is a particular case ($I^c = 1$) of a class defined by the group $G = SU(3) \otimes SU_{I^c}(2)$, where $I^c = 0, 1/2, 1, 3/2, \ldots$, corresponding, respectively, to models with 1, 2, 3, ..., $(2I^c + 1)$ triplets of quarks, which transform as the $3(2I^c + 1)$ dimensional irrep of $SU(3) \otimes SU_{I^c}(2)$.

The following features of the model are well-known:
i) it solves the problem of quark statistics in connection with the baryon states, described as singlets of the iso-spin colour I^c, in the (56,0) dimensional representation of $SU(6) \otimes SO_{I^c}(3)$. This was the original motivation of Tati, in proposing the model;
ii) secondly, it predicts for the e^+e^- annihilation ratio,

$$R = \frac{(e^+e^- \to \text{hadrons})}{(e^+e^- \to \mu^+\mu^-)},$$

in the asymptotic energy limit and for the parameter S of the $\pi^0 \to 2\gamma$ decay, the values[5] R = 8 and S = 1/2, respectively;
iii) Although it shares with the Han-Nambu model[6] a number of similar features, Tati's model has the distinct feature that the quarks are necessarily ***fractionally charged.***

This paper is organized as follows. In section 2, the basic features of the model are discussed and the mesonic states constructed. Section 3 discusses a possible assignment of the ψ particles (based on the properties of their leptonic widths) and the main features of the ψ decays, assuming that the group $SO_{I^c}(3)$ is an exact symmetry for strong interactions, the symmetry being broken by electromagnetic and weak interactions. Finally, in Section 4, some radiative decay widths of the ψ's are estimated by means of a prescription which takes into account the high momentum of the emitted photon in the calculation of the overlap integrals. Finally, Section 5 is devoted to a few final comments.

2. Basic features of the SU(3) ⊗ $SO_{I^c}(3)$ model

The nine quarks of the model are denoted by p_i, n_i, λ_i, with i = + 1, 0, − 1, where the subscript i corresponds to the possibles I_3^c values of the quark iso-spin colour $I^c = 1$, characteristic of the model, which allows no room for a coulour hypercharge, as in the Han-Nambu model. The electric charge is defined by

$$Q = Q' + I_3^c \tag{2-1}$$

where Q' corresponds to the ordinary $SU(3)$ charges. Hence, one has fractionally charged quarks, one with charge 5/3, three with charge 2/3, three with charge (– 1/3) and two with charge (– 4/3), in units of the electronic charge.

The quark quantum numbers are given in the Table. It follows from (2-1) that one has a uniquely defined electromagnetic current (we drop for simplicity the Lorentz indices):

$$\begin{aligned} J^{el} = & \frac{5}{3}\bar{p}_1 p_1 + \frac{2}{3}\bar{p}_0 p_0 - \frac{1}{3}\bar{p}_{-1} p_{-1} \\ & + \frac{2}{3}\bar{n}_1 n_1 - \frac{1}{3}\bar{n}_0 n_0 - \frac{4}{3}\bar{n}_{-1} n_{-1} \\ & + \frac{2}{3}\bar{\lambda}_1 \lambda_1 - \frac{1}{3}\bar{\lambda}_0 \lambda_0 - \frac{4}{3}\bar{\lambda}_{-1} \lambda_{-1} . \end{aligned} \tag{2-2}$$

	I_3'	Y'	Q'	I_3^c	Q
p_1	1/2	1/3	2/3	1	5/3
p_0	1/2	1/3	2/3	0	2/3
p_{-1}	1/2	1/3	2/3	– 1	– 1/3
n_1	– 1/2	1/3	– 1/3	1	2/3
n_0	– 1/2	1/3	– 1/3	0	– 1/3
n_{-1}	– 1/2	1/3	– 1/3	– 1	– 4/3
λ_1	0	– 2/3	– 1/3	1	2/3
λ_0	0	– 2/3	– 1/3	0	– 1/3
λ_{-1}	0	– 2/3	– 1/3	– 1	– 4/3

Table. Quark quantum numbers in the $SU(3) \otimes SO_{I^c}(3)$ model. The dashed quantities correspond to SU (3) and Q is defined by *(2-1).*

The mesons, as $\bar{q} - q$ structure, are obtained by the direct product

$$\overline{(3,1)} \otimes (3,1) = (1,0) \oplus (8,0) \oplus (1,1) \oplus (8,1) \oplus (1,2) \oplus (8,2), \tag{2-3}$$

in an obvious notation. Therefore, one gets the ordinary nonet $(1,0) \oplus (8,0)$ as $SO_{I^c}(3)$ coloured iso-spin singlets. The remaining states (2-3) are triplets ($I^c = 1$) and quintets ($I^c = 2$) in iso-spin colour[7].

The construction of the mesonic states is easily done by means of $SO(3)$ Clebsch-Gordan coefficients, with due attention to the phase of the antiquark states[8]. If we denote by $|v; I^c, I_3^c\rangle$ a meson state with colour iso-spin I^c, component I_3^c, corresponding to a $SU(3)$ state $|v\rangle$, one has

$$|v; I^c, I_3^c\rangle = N^{-1/2} \sum_{I_3^{c'} I_3^{c''}} \langle 1 I_3^{c'} 1 I_3^{c''} | I^c I_3^c \rangle (-1)^{1 - I_3^{c'}} |v\rangle_{-I_3^{c'} I_3^{c''}}, \tag{2-4}$$

where in $|v\rangle_{-I_3^{c'} I_3^{c''}}$ the subscripts are those respectively associated with antiquark and quark in (v), and N is a normalization constnt.

Among the 91 vector meson[9] states (2-4), we are interested in those *neutral* states having the same quantum numbers $J^{\pi c} = 1^{--}$ of the photon. The iso-spin colour singlets corresponding to the ρ^0, ω and ϕ (with *ideal* $\omega_1 - \omega_8$ mixing in $SU(3)$), from (2-4), are seen to be:

$$\begin{aligned}
|\rho^0\rangle &= \frac{1}{\sqrt{6}} |\bar{p}_1 p_1 + \bar{p}_0 p_0 + \bar{p}_{-1} p_{-1} - \bar{n}_1 n_1 - \bar{n}_0 n_0 - \bar{n}_{-1} n_{-1}\rangle, \\
|\omega\rangle &= \frac{1}{\sqrt{6}} |\bar{p}_1 p_1 + \bar{p}_0 p_0 + \bar{p}_{-1} p_{-1} + \bar{n}_1 n_1 + \bar{n}_0 n_0 + \bar{n}_{-1} n_{-1}\rangle, \\
|\phi\rangle &= -\frac{1}{\sqrt{3}} |\bar{\lambda}_1 \lambda_1 + \bar{\lambda}_0 \lambda_0 + \bar{\lambda}_{-1} \lambda_{-1}\rangle,
\end{aligned} \tag{2-5}$$

where we used, to short the notation, $|\rho^0\rangle = |\rho^0; 0, 0\rangle$, etc.

Assuming the *same ideal mixing angle* ($\sin\theta = (1/3)^{1/2}$) for the coloured ω and ϕ with $I^c = 1$ and $I^c = 2$, one also gets, from (2-4)[10],

$$\begin{aligned}
|\rho^0; 1, 0\rangle &= \frac{1}{\sqrt{4}} |-p_1 p_1 + \bar{p}_{-1} p_{-1} - \bar{n}_1 n_1 + \bar{n}_{-1} n_{-1}\rangle, \\
|\omega; 1, 0\rangle &= \frac{1}{\sqrt{4}} |-\bar{p}_1 p_1 + \bar{p}_{-1} p_{-1} + \bar{n}_1 n_1 - \bar{n}_{-1} n_{-1}\rangle,
\end{aligned} \tag{2-6}$$

$$|\phi;1,0\rangle = \frac{-1}{\sqrt{2}}|-\bar{\lambda}_1\lambda_1 + \bar{\lambda}_{-1}\lambda_{-1}\rangle,$$

$$|\rho^0;2,0\rangle = \frac{1}{\sqrt{12}}|\bar{p}_1p_1 - 2\bar{p}_0p_0 + \bar{p}_{-1}p_{-1} - \bar{n}_1n_1 + 2\bar{n}_0n_0 - \bar{n}_{-1}n_{-1}\rangle,$$

$$|\omega;2,0\rangle = \frac{1}{\sqrt{12}}|\bar{p}_1p_1 - 2\bar{p}_0p_0 + \bar{p}_{-1}p_{-1} + \bar{n}_1n_1 - 2\bar{n}_0n_0 + \bar{n}_{-1}n_{-1}\rangle,$$

$$|\phi;2,0\rangle = \frac{-1}{\sqrt{6}}|\bar{\lambda}_1\lambda_1 - 2\bar{\lambda}_0\lambda_0 + \bar{\lambda}_{-1}\lambda_{-1}\rangle. \qquad (2\text{-}6')$$

The point is now to know which of the states (2-6) and (2-6') could be excited in the e^+e^- experiment through their first order coupling with the photon. To this end, we calculate $\langle 0|J^{el}|v;I^c,I_3^c\rangle$, where $|0\rangle$ is the hadronic vacuum. Using (2-2) and (2-6), it is easy to see that the only non-vanishing matrix elements are:

$$\frac{1}{3}\langle 0|J^{el}|\rho^0\rangle = \langle 0|J^{el}|\omega\rangle = \frac{1}{\sqrt{2}}\langle 0|J^{el}|\phi\rangle = \frac{1}{\sqrt{6}},$$

$$-\frac{1}{2}\langle 0|J^{el}|\omega;1,0\rangle = \frac{1}{\sqrt{2}}\langle 0|J^{el}|\phi;1,0\rangle = 1. \qquad (2\text{-}7)$$

Therefore, besides the ρ^0, ω and ϕ ordinary states *(2-5),* we have the result that, under the assumption of ideal mixing for the coloured states, *only two* states" are coupled with the photon, namely, the ω and ϕ with $I^c = 1$.

3. Assignments of the ψ Particles

The first two resonances ψ and ψ' excited in the e^+e^- channel appear with masses of nearly 3.1 and 3.7 GeV and small total widths. Their leptonic decay modes are well established[2]:

$$\Gamma_{\psi\to e^+e^-} \approx \Gamma_{\psi\to\mu^+\mu^-} = 4.8 \pm 0.6\,\text{keV},$$
$$\Gamma_{\psi'\to e^+e^-} \approx \Gamma_{\psi'\to\mu^+\mu^-} = 2.2 \pm 0.5\,\text{keV}. \qquad (3\text{-}1)$$

In view of the results of Section 2, it seems natural, in this model, to make the following assignments:

$$|\psi\rangle \equiv |\psi(3.1)\rangle = |\omega;1,0\rangle,$$
$$|\psi'\rangle \equiv |\psi(3.7)\rangle = |\phi;1,0\rangle, \qquad (3\text{-}2)$$

corresponding to an $SU(3)$ G-parity of minus one.

It follows that, if other higher mass coloured vector mesons do exist, they should presumably be considered as radial (or orbital) excitations of the $\psi(3.1)$ and $\psi(3.7)$ states. If the broad enhancement, at 4.1 GeV, is such a state, we tentatively assign it as the *first* radial excitation of the $\psi(3.1)$:

$$|\psi''\rangle \equiv |\psi(4.1)\rangle = |\omega; 1, 0\rangle'. \tag{3-3}$$

From (3-2) and (3-3), calling $g_V^2 = \langle 0\,|J^{el}|\,V; I^c, 0\rangle^2$, one then gets from, (2-7), the following characteristic ratios:

$$g_{\rho^0}^2 : g_\omega^2 : g_\phi^2 : g_\psi^2 : g_{\psi'}^2 : g_{\psi''}^2 = 9 : 2 : 1 : 24 : 12 : 24. \tag{3-4}$$

The leptonic decay widths are given by[12]

$$\Gamma_{V \to l^+ l^-} = \frac{16\pi}{3}\alpha^2 g_V^2 \frac{|\psi(0)|^2}{M_V^2} \tag{3-5}$$

where $\alpha = 1/137$ and $\psi(\mathbf{r})$ is the wave-function of the decaying vector meson. It follows from (3-2), (2-7) and (3-5), under the assumption that $\psi(3.1)$ and $\psi(3.7)$ are both in the same state $\psi(\mathbf{r})$, that

$$\Gamma_{\psi' \to e^+e^-} = \frac{1}{2}\left(\frac{M_\psi}{M_{\psi'}}\right)^2 \Gamma_{\psi \to e^+e^-} = 1.68\ \text{keV}. \tag{3-6}$$

If we further assume that $\psi(\mathbf{r})$ is the ground-state wave-function of an harmonic-oscillator potential,

$$\psi(\mathbf{r}) = (\pi R_c^2)^{-3/4} \exp(-r^2/2R_c^2), \tag{3-7}$$

we get from (3-5) and (3-1) the value

$$R_c = 2.4\ \text{GeV}^{-1}. \tag{3-8}$$

Similarly, one gets from (2-7) and the experimental value $\Gamma_{\rho^0 \to e^+e^-} = 6.5$ keV, the value

$$R_0 = 4\ \text{GeV}^{-1}. \tag{3-9}$$

For the $\psi(4.1)$, one gets, from (3-3) and (3-5), taking into account the formula

$$|\psi_r(0)|^2 = \frac{2\Gamma(r + \frac{3}{2})}{\pi^2 r!} R^{-3}, \tag{3-10}$$

valid for radial excitations of the ground state of the harmonic oscillator, the result

$$\Gamma_{\psi'' \to e^+e^-} = \frac{3}{2}\left(\frac{M_\psi}{M_{\psi''}}\right)^2 \Gamma_{\psi \to e^+e^-} = 4.12\ \text{keV}. \tag{3-11}$$

It is clear, from (3-8) and (3-9), that the ψ-particles are described by a radius R, smaller than that of the ordinary vector mesons R_0. Further, the result (3-6) is in good agreement with the experimental value (3-1). From (3-6) and (3-11) comes out the interesting feature that $\Gamma_{\psi''} > \Gamma_{\psi'}$, in agreement with the data[13], although the result is strongly dependent on the use of the harmonic oscillator potential[14].

It is reasonable to assume that the $SO_{I^c}(3)$ group is an exact symmetry for strong interactions. In other words, the colour iso-spin I^c is conserved in strong interactions processes. We further assume that $SO_{I^c}(3)$ is broken by the electromagnetic and weak interactions.

As a consequence, the strong decays of the ψ's into normal hadrons are forbidden as they would imply I^c violating $1 \to 0$ transitions. On the other hand, the allowed decay $\psi(3.7) \to \psi(3.1) +$ hadrons is, due to the assignment (3-2), strongly suppressed by the Zweig-Iizuka[15] rule.

Of course, the process will occur in so far as the $SU(3)$ mixing differs from the ideal one assumed in (2-6).

It is clear from the above discussion that Tati's model, as a colour scheme, shares the same general properties (and difficulties) of the Han-Nambu model. However, the two models clearly differ by their specifíc and distinct predictions.

4. Radiative Decays of the ψ 's

In this Section, we discuss the radiative decays of $\psi(3.1)$ and $\psi(3.7)$ and estimate their partial decay widths corresponding to the decays

$$\begin{aligned} \psi, \psi' \to\ & \eta + \gamma, \\ & \eta' + \gamma. \end{aligned} \tag{4-1}$$

We note that the decays into π^0 are forbidden. In the following, we denote generically the decays (4-1) by $V \to P + \gamma$.

The decays (4-1) correspond to M1 transitions, which can be treated using $SU(6) \otimes SO_{I^c}(3)$ wave functions, easily derived from (2-6).

A characteristic feature of those decays is the high momentum of the emitted photon. Therefore, in the calculation of their widths, it is essential to take into account the overlap integral, which for the radiative decay of the normal particles, is reasonably taken to be 1, due to the smaller momentum of the final photon.

In fact, for the very energetic decays like (4-1), it seems physically reasonable to adopt the following prescription, namely, that of calculating the overlap integral[16]

$$\mathscr{I}(\mathbf{k}) = \int d^3r\, \psi_P^*(\mathbf{r}) \exp\left(\frac{i}{2} \mathrm{k} \cdot \mathrm{r}\right) \psi_V(\mathbf{r})$$

in the Breit system of velocities, taking also into account Lorentz contraction effects in the meson wave-functions. This prescription, when $SU(6)$ type wave-functions are used, is a sensible one, since in the Breit-velocity frame, the velocities of mesons involved in the decay are smaller than in any other frame[17].

In the Breit velocity frame,

$$\mathbf{v}_V + \mathbf{v}_P = 0, \tag{4-2}$$

one gets, for the momentum k of the emitted photon in the Breit-velocity frame,

$$\mathbf{k}^2 = \mathbf{k}_{CM}^2 \frac{M_V}{M_P} \tag{4-3}$$

and, for the Lorentz contraction factor, the expression

$$\gamma = \left[1 + \frac{(M_V - M_P)^2}{4\, M_V M_P}\right]^{1/2} \tag{4-4}$$

The effect of the Lorentz contraction of the meson wave-functions can be taken into account by the simple substitution law[16]

$$\mathscr{I}(\mathbf{k}) \to (1/\gamma)\, \mathscr{I}(\mathbf{k}/\gamma). \tag{4-5}$$

If we describe the mesons by harmonic oscillator wave functions (3-7), with the radius parameter R, given by (3-8), we get

$$(1/\gamma^2)\, \mathscr{I}^2(\mathbf{k}/\gamma) = (1/\gamma^2) \exp\left[- (R_c^2/4)(k^2/\gamma^2)\right]. \tag{4-6}$$

The electromagnetic M1 transition operator is

$$M^{e'} = \mu \sum_{i=1}^{2} (e_i/e)\, \boldsymbol{\sigma}_i \cdot (\mathrm{k} \times \boldsymbol{\varepsilon}^\lambda) \exp\left[(i/2)\, \mathrm{k} \cdot \mathbf{r}\right]. \tag{4-7}$$

where μ is a scale parameter for the quark magnetic moment. After summing over the polarization of the final photon and averaging over the spin direction of the decaying vector meson, we get the average matrix element squared

$$|M|^2_{\mathrm{AV}} = \frac{1}{3}\sum_{\lambda}\sum_{S_3=-1}^{+1} |\langle \mathrm{P}|\, M^{\mathrm{el}}\, |\mathrm{V}, S_3\rangle|^2 (1/\gamma^2)\, \mathscr{I}^2(\mathbf{k}/\gamma). \tag{4-8}$$

From (4-8), we obtain for the decays (4-1)

$$|M_{\mathrm{VP}}|^2 = \frac{2}{3}\mu^2 \mathbf{k}^2 (1/\gamma^2)\, \mathscr{I}^2(\mathbf{k}/\gamma)\, C^2_{\mathrm{VP}}. \tag{4-9}$$

where[18]

$$\begin{aligned} C^2_{\psi\pi^0} &= C^2_{\psi'\pi^0} = 0 \\ 2C^2_{\psi\eta} &= C^2_{\psi\eta'} = C^2_{\psi'\eta} = 2C^2_{\psi'\eta'} = \frac{16}{9}. \end{aligned} \tag{4-10}$$

Finally, for the radiative decay widths in the center of mass frame, we have, with help of (4-3), (4-6) and (4-9), the expression

$$\begin{aligned} \Gamma_{\mathrm{V}\to\mathrm{P}\gamma} = \frac{4}{3}\, \frac{\mu^2}{4\pi}\, k^3_{\mathrm{CM}}(M_{\mathrm{V}}/M_{\mathrm{P}})(1/\gamma^2) \times \\ \times \exp\left[-\,(R_c^2/4)(k^2_{\mathrm{CM}}/\gamma^2)(M_{\mathrm{V}}/M_{\mathrm{P}})\right] C^2_{\mathrm{VP}}. \end{aligned} \tag{4-11}$$

It is clear from this result that the partial radiative widths of the ψ's are strongly depressed due to the exponential factor in (4-11). However, if we take for μ the $SU(6)$ value usually used in the calculation of the radiative decay of ordinary vector mesons[12], namely,

$$\mu = 2.79(e/2M_{\mathrm{proton}}), \tag{4-12}$$

we still obtain, from (4-11), large values for the radiative widths.

However, it is conceivable that, for the decay of the ψ's, μ should be taken different from the value (4-12). This fact has been stressed by Kuroda and Yamaguchi[19] in their analysis of the problem within the Han-Nambu colour scheme. If, following them, we take for the magnetic moment of quarks in the ψ's, the value

$$\mu_c = \mu\frac{M_\omega}{M_\psi} \simeq \mu\frac{M_\phi}{M_{\psi'}}, \tag{4-13}$$

corresponding to the assignments (3-2), we get a magnetic moment smaller by a factor of $\sim$ 4 than that given by (4-12). The values for the widths, computed from (4-11) and (4-13), are then the following:

$$\begin{aligned} \Gamma_{\psi\to\eta\gamma} &= 1.1\,\mathrm{keV}, & \Gamma_{\psi'\to\eta\gamma} &= 0.04\,\mathrm{keV}, \\ \Gamma_{\psi\to\eta'\gamma} &= 22\,\mathrm{keV}, & \Gamma_{\psi'\to\eta'\gamma} &= 0.6\,\mathrm{keV}. \end{aligned} \tag{4-14}$$

We notice that the $\Gamma_{\psi\to\eta'\gamma}$ is unreasonably large, compared with data.

It interesting to note that the widths in Tati's model are systematically higher than those obtained from the Han-Nambu scheme, taken the corresponding assignments[19]. Hence, the experimental indication of very small values, for the radiative widths of the decays (4-1), tends definitelly to favour the Han-Nambu model with respect to Tati's.

5. Final Remarks

The $SU(3) \otimes SO_{I^c}(3)$ model has many aspects in common with the Han-Nambu model although each one gives rise to distinct quantitative predictions.

Even though the pattern which is emerging from the e^+e^- annihilation data seems to favour the charmonium picture[2], it is perhaps premature to discard alternative possibilities. It is clear that, due to the complexity of the experimental pattern, a much more detailed analysis of the ψ's is necessary as well as more detailed data.

The present analysis was based on the harmonic oscillator wave functions with a radius R, = 2.4 GeV^{-1}, obtained from the fitting of the ψ leptonic decay. With this value for R,, the radiative decay widths, calculated using the "Breit velocity frame plus Lorentz contraction" prescription, have values reasonably small only if the quark magnetic moments in the ψ's are given by (4-13). This result is similar to the one obtained in the Han-Nambu model[19]. However, the widths predicted by Tati's model are systematically higher than those predicted by the Han-Nambu model, a fact which definitely seems to favour the latter model.

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7. Clearly, the model predicts mesons with charges in the range *3, . . . , − 3.*
8. We recall that, if a particle state transforms under $SO(3)$ as the spherical harmonics Y^l_m, the corresponding antiparticle transforms as $(-)^{l-m}\, Y^{l*}_{-m}$. Since we are coupling q with $\bar{q}$, via $SO(3)$ Clebsch-Gordan coefficients, it is essential to take this phase into account.
9. We note that, given a $SU(3)$ state $|v\rangle$ of charge Q, the state $|v; I^c, I'_{,}\rangle$ of G has, according to (2-1), the charge $Q + I'_{,}$.
10. A similar assumption was made, in the Han-Nambu colour scheme, by M. Krammer, D. Schildknecht and F. Steiner, DESY *74/64,* December *1974.*
11. Instead, in the Han-Nambu colour scheme, this number is *2* or *4,* depending on the color iso-spin breaking. See Ref. 2.
12. See, for example, J. J. J. Kokkedee, The ***Quark Model,*** W. ***A.*** Benjamin, Inc., New York *(1969).*
13. It is amusing to note that the experimental value for $\Gamma_{\psi''\to e^+e^-}$ is − 4 KeV, see Ref. 2.
14. For a linear potential, since the value of $|\psi_r(0)|^2$, for S-wave radial excitations, is independent of *r,* it follows from our assignment *(3-3)* that $\Gamma_{\psi'} < \Gamma_{\psi'}$.
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16. This prescription has been proposed by A. Itô, Prog. Theoret. Phys. 47, *228* (1972), in the problem of strong decays of mesonic resonances. According to the prescription, in the overlap integral $\mathcal{I}(\mathbf{k})$, k is the photon momentum in the Breit-velocity frame. Notice that the integration variable r in $\mathcal{I}(\mathbf{k})$ is the relative coordinate of the constituents. Thus, we have a factor 1/2 in the exponent.
17. The use of the Breit velocity frame has also been advocated by A. Le Yaounc et *al.,* Nucl. Phys. **B29,** *204* (1971), in another context.
18. We took for the $\eta_1 - \eta_8$ angle, of $SU(3)$ mixing, the value zero; that is $|\eta'\rangle \simeq |\eta_1\rangle$ and $|\eta\rangle \simeq |\eta_8\rangle$.
19. M. Kuroda and Y. Yamaguchi, University of Tokyo preprint *UT-248,* April *1975.* See also the report by Y. Yamaguchi in the ***Conference on High Energy Physics,*** Palermo, June *1975.*
20. In fact, for the corresponding assignments $\psi \equiv (\omega, \omega^c_8)$ and $\psi' \equiv (\phi, \omega^c_8)$, in the $SU(3) \otimes SU(3)$ Han-Nambu model, we obtain the values $2C^2_{\psi\eta} = C^2_{\psi\eta'} = 4C^2_{\psi'\eta} = 8\, C^2_{\psi'\eta'} = 8/27$, to be compared with *(4-10).*