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Heavy Particle Transfer Interpretation for Anomalous Scattering of a-Particles from Light 4n-Nuclei*

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We show, by direct calculation, that the Heavy-Particle-Transfer (HPT) model of backward elastic a-particle scattering, together with the assumption of α-cluster structure for the ***4n*** light nuclei involved in the process, is capable of explaining the anomalous large backward peak in elastic a-scattering from ***4n*** target nuclei. Discussions are made concerning possible higher order contributions.

Mostramos, por cálculo direto, que o modelo de Transferência de Partículas Pesadas, para o espalhamento elástico de alfas a grandes ângulos, junto com a hipótese de que os núcleos $4n$, envolvidos no processo, sejam conglomerados de alfas, é capaz de explicar o pico anômalo, elástico, a grandes ângulos. Discutem-se possíveis contribuições, de ordem superior, ao processo.

1. Introduction

In recent years, a number of authors[1,2,11,12] have investigated the so-called anomalous α-particle backscattering peak, from $Z = N =$ even nuclei (light and *medium*), at various incident energies. However, the major criticism in most of these works refers to a lack of a more microscopic basis. A paper by Noble and Coelho[2], basically, gave the first idea for this problem, suggesting a Heavy-Particle-Transfer (HPT) interpretation in a more microscopic approach. In the last couple of years, two calculations more, by Agassi and Wall (knock-on exchange interpretation) and Boridy[12] (one-particle exchange interpretation), similar in spirit to the work by Noble and Coelho[2], attempted to consider this problem. While Coelho and Noble applied their model to a light nucleus (O^{16}), the two other papers dealt with a heavier nucleus (Ca^{40}). The central idea of the present work is mainly based on the generalization of the papers by Noble and Coelho[2] and Coelho[3]. Noble and Coelho[2] showed, by direct calculation, that the Heavy-Particle-Transfer (HPT) mechanism of backward and forward elastic

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a-particle scattering, together with the assumption of a-cluster structure of O^{16} and C^{12}, are capable of explaining the larger backward peak and the forward angular distributions of the differential cross section, in elastic a scattering from O^{16}. Our effort, here, is to try to generalize that theoretical approach in such a way that it would also be applicable to other light even-even $4n$ – nuclei, besides O^{16}. It has been observed experimentally[19,12] that the large backward peak shows strong isotopic dependence. While it appears for $4n$ nuclei, it is completely absent for other nuclei. This approach gives a microscopic insight into the anomalous backscattering phenomenon.

In Sec. 2, we describe our formalism, while, in Sec. **3,** we give our conclusions and discuss the differences from previous work.

2. Theory

Heavy particle transfer, for backward scattering, can be represented mostly by pole diagrams[2-4]. The interest in pole diagrams (instead of higher order diagrams, representing, for example, nucleon exchange between the incident a and the target nucleus) comes from the fact that they are simpler and because their singularities (as functions of the complex momentum transfer) lie somewhat nearer the physical region.

Nuclei, in the lower half of the sd shell, are particularly suitable targets for a cluster configurations[5]. Hence, one can consider, the light even-even $4n$ nuclei, as composed of n elementary α-particles. This composite system can represent a target which will scatter an extra a-particle. If $V_{\alpha\alpha}(r)$ is the a – a potential, it can be shown[6] that the plane-wave Born approximation (PWBA) amplitude, for a boson to be scattered from a normalized bound state of n identical bosons of the same type, may be expressed as

$$M_{\mathrm{fi}} = M_{\mathrm{d}} + n\, M_{\mathrm{ex}}, \tag{1}$$

where

$$\begin{aligned} M_{\mathrm{d}} = (2\pi)^{-3} & \int d\mathbf{r}' \exp\left[i(\mathbf{k}-\mathbf{k}') \cdot (\mathbf{r}' - \mathbf{R}_0)\right] \\ & \times \int d\mathbf{r}_1 \ldots d\mathbf{r}_n \; \psi_{\mathrm{f}}^{*}\, \psi_{\mathrm{i}}\, (\mathbf{r}_1 - \mathbf{R}_0, \ldots, \mathbf{r}_n - \mathbf{R}_0) \times \\ & \times \sum_{\mu=1}^{n} V_{\alpha\alpha}(|\mathbf{r}' - \mathbf{r}_\mu|)\, \delta\left(\mathbf{R}_0 - \frac{1}{n} \sum_{\nu=1}^{n} \mathbf{r}_\nu\right) \end{aligned} \tag{2}$$

and

$$M_{\mathrm{ex}} = (2\pi)^{-3} \int d\mathbf{r}'\, d\mathbf{r}_1 \ldots d\mathbf{r}_n \; \exp\,[i\mathbf{k}\cdot(\mathbf{r}' - \mathbf{R}_0)]$$

$$\times \; \exp\,[-i\mathbf{k}'\cdot(\mathbf{r}_1 - \mathbf{R}')] \; \psi_{\mathrm{f}}^{*}(\mathbf{r}' - \mathbf{R}', \; \mathbf{r}_2 - \mathbf{R}', \ldots, \mathbf{r}_n - \mathbf{r}')$$

$$\times \; \psi_{\mathrm{i}}(\mathbf{r}_1 - \mathbf{R}_0, \ldots, \mathbf{r}_n - \mathbf{R}_0) \left[\sum_{\mu=1}^{n} V_{\alpha\alpha}\left(|\mathbf{r}' - \mathbf{r}_\mu|\right) \delta\!\left(\mathbf{R}_0 - \frac{1}{n} \sum_{\nu=1}^{n} \mathbf{r}_\nu \right) \right], \quad (3)$$

where $\mathbf{r}_\nu$ is the νth α coordinate, $\mathbf{R}_0$ is the CM coordinate for the target nucleus and, $\mathbf{r}'$, the coordinate of the incident α-particle. The incident and outgoing wave vectors are represented by $\mathbf{k}$ and $\mathbf{k}'$, respectively, and $\mathbf{R}'$ is defined as

$$\mathbf{R}' = \mathbf{R}_0 + \frac{1}{n}(\mathbf{r}' - \mathbf{r}_1). \qquad (4)$$

The target wave functions ψ_{f} and ψ_{i} are normalized and fully symmetric in the n α-particle position coordinates. Eq. (3) represents the fact that the incident α is exchanged (with equal probability) with any of the n α-particles of the target. On the other hand, Eq. (2) corresponds to no exchange at all.

The following, suitable, change of variables will simplify Eqs. (2) and (3):

$$\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \\ \vdots \\ \\ \mathbf{r}_n \end{bmatrix} = \begin{bmatrix} \frac{n-1}{n} & 0 & 0 & \cdots & 0 & 1 \\ -\frac{1}{n} & \frac{n-2}{n-1} & 0 & \cdots & 0 & 1 \\ -\frac{1}{n} & -\frac{1}{n-1} & \frac{n-3}{n-2} & \cdots & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\frac{1}{n} & -\frac{1}{n-1} & -\frac{1}{n-2} & \cdots & +\frac{1}{2} & 1 \\ -\frac{1}{n} & -\frac{1}{n-1} & -\frac{1}{n-2} & \cdots & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_1 \\ \boldsymbol{\eta}_2 \\ \boldsymbol{\eta}_3 \\ \vdots \\ \boldsymbol{\eta}_{n-1} \\ \boldsymbol{\eta}_n = \mathbf{R} \end{bmatrix} \qquad (5)$$

where $n = 2, 3 \ldots$. The set $\{\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \mathbf{R}\}$ constitutes a Jacobi system of coordinates (the Jacobian of the transformation is unity).

If one uses the new system of coordinates given by Eq. (5), the definition shown by Eq. (4), a subsequent new transformation of variables $\mathbf{r}' \longrightarrow \mathbf{r}' + \mathbf{R}_0 - \frac{1}{n}\mathbf{r}$ (which becomes obvious afterwards) and finally the use of the symmetry in the variables $\mathbf{r}_2, \mathbf{r}_3, \ldots, \mathbf{r}_n$, one obtains

$$
\begin{aligned}
n M_{\mathrm{ex}} = (2\pi)^{-3} \int & d\mathbf{r}'\, d\boldsymbol{\eta}_1 \ldots d\boldsymbol{\eta}_{n-1} \exp\left[-i\boldsymbol{\eta}_1 \cdot \mathbf{Q}\right. \\
& \left. + i\mathbf{r}' \cdot \mathbf{Q}'\right] \rho_{\mathrm{fi}}(\mathbf{r}', \boldsymbol{\eta}_1 \mid \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1}) \times \\
& \times \left[n V_{\alpha\alpha}(|\mathbf{r}' - \boldsymbol{\eta}_1|) + (n-1) V_{\alpha\alpha}\left(\left|\mathbf{r}' - \frac{n-2}{n-1}\boldsymbol{\eta}_1\right|\right)\right],
\end{aligned} \tag{6}
$$

where the following definitions were used:

$$
\mathbf{Q} = \frac{1}{n}\mathbf{k} + \mathbf{k}', \quad \mathbf{Q}' = \mathbf{k} + \frac{1}{n}\mathbf{k}', \tag{7}
$$

and

$$
\begin{aligned}
\rho_{\mathrm{fi}}(\mathbf{r}', \boldsymbol{\eta}_1 \mid \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1}) = \psi_{\mathrm{f}}^{*}\left(\frac{n-1}{n}\mathbf{r}', -\frac{1}{n}\mathbf{r}' + \frac{n-2}{n-1}\boldsymbol{\eta}_2, \ldots\right) \times \\
\times\, \psi_{\mathrm{i}}\left(\frac{n-1}{n}\boldsymbol{\eta}_1, -\frac{1}{n}\boldsymbol{\eta}_1 + \frac{n-2}{n-1}\boldsymbol{\eta}_2, \ldots\right).
\end{aligned} \tag{8}
$$

Employing a similar reduction of variables, we find (here $\mathbf{r}' \longrightarrow \mathbf{r}' + \mathbf{R}_0$):

$$
\begin{aligned}
M_{\mathrm{d}} = (2\pi)^{-3} \int & d\mathbf{r}'\, d\boldsymbol{\eta}_1 \ldots d\boldsymbol{\eta}_{n-1} \exp(i\mathbf{q} \cdot \mathbf{r}') \times \\
& \times \rho_{\mathrm{fi}}(\boldsymbol{\eta}_1, \boldsymbol{\eta}_1 \mid \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1})\, n\, V_{\alpha\alpha}\left(\left|\mathbf{r}' - \frac{n-1}{n}\boldsymbol{\eta}_1\right|\right),
\end{aligned} \tag{9}
$$

where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$.

Combining the first term in Eq. (6) with Eq. (9), we have an amplitude M_{D} which represents the scattering of the incident boson from each of the bosons in the target (symmetrized) in PWBA. The second term, in Eq. (6), corresponds to HPT and we will denote it by M_{HPT}. Consequently,

$$
M_{\mathrm{fi}} = M_{\mathrm{D}} + M_{\mathrm{HPT}}, \tag{10}
$$

where

$$M_{\mathrm{HPT}} = n(2\pi)^{-3} \int d\mathbf{r}'\, d\boldsymbol{\eta}_1 \ldots d\boldsymbol{\eta}_{n-1} \exp(-i\mathbf{Q}\cdot\boldsymbol{\eta}_1 + $$

$$+ i\mathbf{Q}'\cdot\mathbf{r}')\, \rho_{\mathrm{fi}}(\mathbf{r}', \boldsymbol{\eta}_1 \,|\, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1})\,(n-1)\, V_{\alpha\alpha}\left(\left|\mathbf{r}' - \frac{n-2}{n-1}\boldsymbol{\eta}_1\right|\right), \quad (11)$$

and

$$M_{\mathrm{D}} = (2\pi)^{-3} \int d\mathbf{r}'\, d\boldsymbol{\eta}_1 \ldots d\boldsymbol{\eta}_{n-1} \exp\left[i\mathbf{q}\cdot\left(\mathbf{r}' + \frac{n-1}{n}\boldsymbol{\eta}_1\right)\right] \times$$

$$\times\, n\, V_{\alpha\alpha}(\mathbf{r}')\, \rho_{\mathrm{fi}}(\boldsymbol{\eta}_1, \boldsymbol{\eta}_1 \,|\, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1})$$

$$+ (2\pi)^{-3} \int d\mathbf{r}'\, d\boldsymbol{\eta}_1 \ldots d\boldsymbol{\eta}_{n-1} \exp(-i\mathbf{Q}\cdot\boldsymbol{\eta}_1 + i\mathbf{Q}'\cdot\mathbf{r}') \times$$

$$\times\, n\, V_{\alpha\alpha}(|\mathbf{r}' - \boldsymbol{\eta}_1|)\, \rho_{\mathrm{fi}}(\mathbf{r}', \boldsymbol{\eta}_1 \,|\, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1}). \quad (12)$$

If we expand the wave functions ψ_{f} and ψ_{i} in terms of a product of a complete orthonormal set of (n – *1)* α states $\Phi(\boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1})$, and the wave functions $\phi(\boldsymbol{\eta}_1)$, which describe the relative motion of a single α with respect to the (n – *1)* α-core, we can write

$$\psi_\lambda(\boldsymbol{\eta}_1, \ldots, \boldsymbol{\eta}_{n-1}) = \sum_{vJM} C_{vJ}^{(\lambda)}\, \phi_{vJ}(\eta_1) Y_{JM}(\hat{\eta}_1) \times$$

$$\times\, \Phi_{vJ,\,-M}(\boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1})\, \langle 00 \,|\, JJ, M, -M\rangle. \quad (13)$$

We have assumed that the target states have spin-parity 0^+, which is certainly true for the ground state. It is worth mentioning that the states $\Phi(\boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1})$ are eigenstates of the (n – 1) α Hamiltonian. These wave functions are identified as the states of the physical nucleus containing (4n – 4) nucleons. The $C_{vJ}^{(\lambda)}$'*s* appearing in **Eq.** (13), are the spectroscopic coefficients.

Substituting the expansion given by **Eq.** *(13)* into **Eq.** *(11)* and noticing that

$$\int d\mathbf{r}'\, d\boldsymbol{\eta}_1 \ldots d\boldsymbol{\eta}_{n-1} \exp(i\mathbf{Q}'\cdot r')\, \psi_0^*(\mathbf{r}', \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1}) \times$$

$$\times\, \psi_0(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1})\,(n-1)\, V_{\alpha\alpha}\left(\left|\mathbf{r}' - \frac{n-2}{n-1}\boldsymbol{\eta}_1\right|\right)$$

$$= -\left[E_{vJ} + \frac{\hbar^2}{2m_\alpha}\frac{n}{n-1}Q'^2\right] \int d\mathbf{r}'\, d\boldsymbol{\eta}_1 \ldots d\boldsymbol{\eta}_{n-1} \times$$

$$\times \exp(i\mathbf{Q}'\cdot\mathbf{r}')\, \psi_0^*(\mathbf{r}', \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1})\, \psi_0(\boldsymbol{\eta}_1, \boldsymbol{\eta}_2, \ldots, \boldsymbol{\eta}_{n-1}), \quad (14)$$

where

$$E_{vJ} = (-m_{n\alpha} + m_\alpha + m_{(n-1)\alpha})\, c^2 + E^*_{(n-1)\alpha}(v,\ J),$$

($E^*_{(n-1)\alpha}$ represents the excited levels of the (n—1) α-core system), we obtain

$$M_{\mathrm{HPT}} = -\, n(2\pi)^{-3}\left[E_{vJ} + \frac{\hbar^2}{2m_\alpha}\,\frac{n}{n-1}\,Q'^2\right] \times$$

$$\times \int d\mathbf{r}'\, d\boldsymbol{\eta}_1\ \exp\,(-i\mathbf{Q}\,.\,\boldsymbol{\eta}_1 + i\mathbf{Q}'\,.\,\mathbf{r}') \times$$

$$\times \sum_{\substack{v'J'M' \\ vJ\,M}} C^{(0)*}_{v'J'}\, C^{(0)}_{vJ}\ \langle 00\,|\,J'J'\ M',\ -M'\rangle \times$$

$$\times\ \langle 00\,|\,JJ\ M,\ -M\rangle\ \phi^*_{v'J'}(r')\ Y^*_{J'M'}(\hat{\mathbf{r}}') \times$$

$$\times\ \phi_{vJ}(\eta_1)\ Y_{JM}(\eta_1)\int d\boldsymbol{\eta}_2 \ldots d\boldsymbol{\eta}_{n-1}\ \Phi^*_{v'J',-M'}\,\Phi_{vJ,-M}(\boldsymbol{\eta}_2,\ldots,\boldsymbol{\eta}_{n-1}). \qquad (15)$$

The last integrals give the conditions $v' = v$, $J' = J$, $M' = M$. If we now use, in Eq. (15), the plane wave expansion,

$$\exp\,(i\mathbf{k}\,.\,\mathbf{r}) = 4\pi \sum_{l=0}^{\infty}\ \sum_{m=-l}^{l}\ i^l\, j_l(kr)\ Y^*_{lm}(\hat{\mathbf{k}})\ Y_{lm}(\hat{\mathbf{r}}),$$

and the addition theorem[15], we obtain a much simpler expression for M_{HPT}:

$$M_{\mathrm{HPT}}\,(\theta,\ k) = -\ \frac{\hbar^2}{2m_\alpha}\cdot\frac{n^2}{n-1}\cdot\frac{1}{2\pi^2}\sum_{vJ}|\,C^{(0)}_{vJ}\,|^2 \times$$

$$\times\ (Q^2 + \kappa^2_{vJ})\ P_J\,(\hat{\mathbf{Q}}'\,.\,\hat{\mathbf{Q}})\,|\,I_{vJ}(Q)\,|^2, \qquad (16)$$

where only elastic channels $|\,\mathbf{Q}\,| = |\,\mathbf{Q}'\,|$ were considered (on the energy shell). Observe that $\kappa^2_{vJ} = (2m_\alpha/\hbar^2)\,(n - 1/n)\,E_{vJ}$ and $I_{vJ}(Q)$ is defined as

$$I_{vJ}(Q) = \int_0^\infty d\boldsymbol{\eta}_1\ \boldsymbol{\eta}_1^2\ j_J(Q\eta_1)\ \phi_{vJ}(\eta_1). \qquad (17)$$

The result given by Eq. (16) could also be derived by application of Feynman rules[8] to the pole diagram shown in Fig. 1. That diagram is, obviously, important for backward scattering.

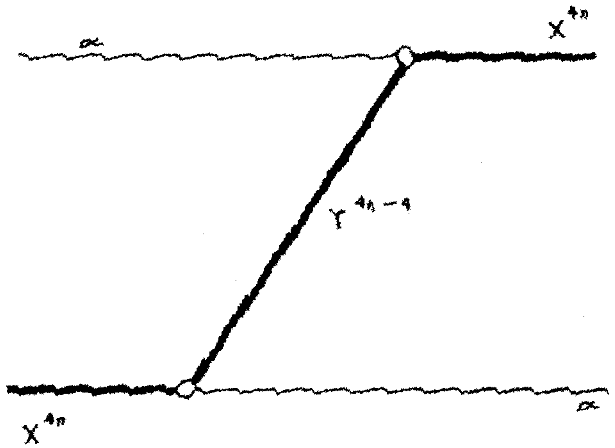


Fig 1 — Diagrammatic representation of the HPT (pole) amplitude with Y^{4n-4} transferred ($n = 3, 4, \ldots$) in its state with spin J, parity $(-)^J$, excitation energy $E^*_{(n-1)\alpha}$.

An analogous expression, for Eq. (16), could easily be written for inelastic channels ($|\mathbf{Q}'| \neq |\mathbf{Q}|$) and λ, in Eq. (13), could assume any other allowed value besides zero).

At this point, we should emphasize that we are assuming that the predominant amplitude, at backward angles, arises from HPT, but no corrections for inelasticity (strong absorption and rescattering by the direct forward-peaked amplitude M_D) were made. Certainly this is very important in our case. These corrections may be obtained by subtracting, from M_{HPT}, its lowest partial waves. This is essentially the *Blair sharpcutoff model*[9]. *That is*, we write the backward amplitude as

$$T_\lambda(\theta) \simeq \sum_{l=L}^{\infty} (2l+1)\, P_l(\cos\theta)\; T_l(k), \tag{18}$$

where

$$T_l(k) = \frac{1}{2} \int_{-1}^{+1} ds\; P_l(s)\, M_{HPT}(s,\, k),$$

and M_{HPT} ($\cos\theta$, E) is given by Eq. (16). The angular momentum, L, is left as a parameter which is, approximately, given by kR, where R is the dimension of the target nucleus.

The effect of Coulomb interaction (though small for backscattering) should also be included in the final scattering amplitude. We represent, by $T_C(\theta)$, the Coulomb scattering amplitude, which is given by[15]

$$T_C(\theta) = (ne^2/2\bar{\pi}^2)[k \sin\theta/2]^{-2} \exp[2i(\sigma_0 - \xi \log \sin\theta/2], \tag{19}$$

where

$$\sigma_l = \arg \Gamma(l + 1 + i\xi) \text{ and } \xi = \text{nm}_\alpha e^2/(n+1)\hbar^2 k.$$

Finally, the differential cross section is calculated by the expression[6]

$$\frac{d\sigma}{d\Omega} = \left[\frac{4\pi^2 nm_\alpha}{(n+1)\hbar^2}\right]^2 |T_k(\theta) + T_C(\theta)|^2. \tag{20}$$

3. Discussion and Results

The calculation of the differential scattering cross section given by Eq. (20) requires, basically, the knowledge of $\phi_{vJ}(\eta_i)$ (see Eq. (17)) and the spectroscopic coefficients, $|C_{vJ}^{(0)}|^2$ (see Eq. (16)). To obtain $\phi_{vJ}(\eta_1)$ (we recall that, it represents the relative motion wavefunction of a single α with respect to the core) is quite an interesting problem. But, so far, only phenomenological approaches have been considered[2,7,16]. It is a reasonable approximation[2,16] (and also convenient, because the integration given by Eq. (17) is straightforward) to take the Eckart wave function[7]

$$\phi_{vJ}(\eta_1) = N_{vJ}[1 - \exp(-\eta_1/R)]^{m+1} . \exp(-k_{vJ}\eta_1)/\eta_1, \tag{21}$$

where N_{vJ} is the normalization constant, R and m are free parameters and k_{vJ} is related to the separation energy, E_s. Certainly, the choice of $\phi_{vJ}(\eta_i)$ is very important concerning the final results. The calculation of the $|C_{vJ}^{(0)}|^2$ could be done indirectly, as shown in Ref. (2), (essentially by fitting $d\sigma/d\Omega$, given by Eq. (20), to the experimental ones) or, more microscopically, using nuclear structure models (shell model)[4,10].

We should be aware of the fact that the final expression for the cross section is based on three assumptions: (a) α-cluster structure for the target nuclei; (b) the heavy ion transferred; (c) the Blair strong absorption model to incorporate distortion effects of the strong direct (optical) potential. The angular momentum, L(Eq. 18), is left as a free parameter which is approximately given by kR.

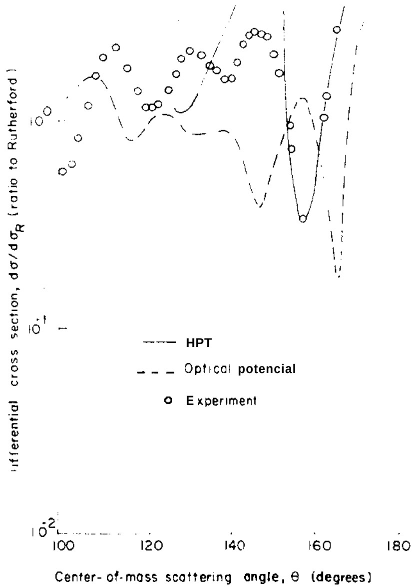
The idea of the HPT mechanism, plus α-cluster structure for light $4n$ nuclei has also been used successfully by Noble[14] to explain some features of the heavy ion reaction $C^{12}(C^{12}, \alpha)Ne^{20}$.

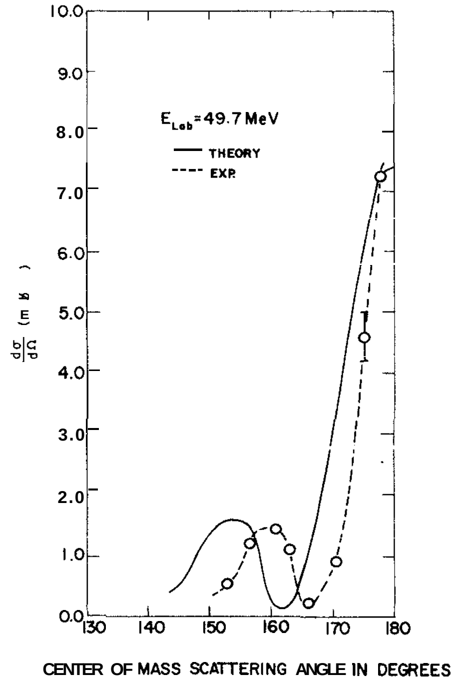
It is worth mentioning that it is impossible to use an optical potential in our problem for reasons explained in Ref. (2). This argument is also strengthened in Ref. (17).

Calculations were done for C^{12}, 0^{16} and Ca^{40} targets. Other $4n$ nuclei were not considered since no reliable experimental data were found in the literature. Our feeling is that the above nuclei represent the two major categories: light and medium $4n$ nuclei.

For the C^{12} case, we have taken[16] the parameter values $R = 1.56$ fm and $m = 4$ (they were obtained from rms radius and electric form factors calculations). Laboratory energy considered for the α-particle was 41 MeV. The spectroscopic coefficients, $|C_{vJ}^{(0)}|^2$, were obtained from Ref. (4): $|C_{0^+}^{(0)}|^2 = 0.70$, $|C_{2^+}^{(0)}|^2 = 0.29$ and $|C_{4^+}^{(0)}|^2 = 0.01$ (at 0., 2.9 and 11.4 MeV, respectively). An attempt was also made to obtain the $|C_{vJ}^{(0)}|^2$ by fitting the theoretical cross section, given by Eq. (20), to the experimental data. However, we obtained, for the spectroscopic factors, values not so different from those given in Ref. (4). The best fitting for the differential cross section was obtained for $L = 5$. Fig. 2 shows the angular distributions (theoretical and experimental) at $E_{lab} =$ 41 MeV, where the experimental points and the optical potential were obtained from Ref. (17). The agreement of our simple model with experimental data is encouraging.

The results for O^{16} are again reproduced from Ref. (2). In that paper, $R = 2.8$ fm, $m = 4$, $E_{lab} = 41$ and 49.7 MeV. It is important to notice two points in the calculation for O^{16}: i) the spectroscopic coefficients were obtained from fitting the theoretical cross sections to the experimental ones; their values are $|C_{2^+}^{(0)}|^2 = 0.375$, $|C_{3^-}^{(0)}|^2 = 0.075$, $|C_{4^+}^{(0)}|^2 = 0.15$ (at 4.44, 9.64 and 14.08 MeV, respectively); ii) the 0^+ state of Be^8 is the most important intermediate state for C^{12} target nucleus, in contrast with the predominant 2^+ intermediate state, of C^{12}, for O^{16} as the target nucleus. Those results are also in agreement with References (2) and (4). These results are understood by examining $|I_{vJ}(Q)|^2$ as a function of Q^2 (Eq. 17). For example, for the energy domain chosen, $|I_{vJ}(Q)|^2$, for the 0^+ state (C^{12} intermediate state), is a rapidly decreasing function of Q, whose maximum value occurs at $Q = 0$. A different situation occurs for the 0^+ intermediate state of Be^8, where $|I_{vJ}(Q)|^2$





Fip. 2 – Angular distributions (theoretical and experimental) at $E_{lab} = 41$ MeV. (Experimental points and optical potential from Ref. (17)).

Fig. 3 – Angular distributions (theoretical and experimental) at $E_{lab} = 49.7$ MeV. (Experimental points from Ref. (2)).

is large for the energy domain chosen. Fig. 3 shows an angular distribution for the O^{16} target nucleus.

The last nucleus considered was Ca^{40}, for which experimental data are available[19]. It is a medium double closed nucleus and quite good to check how far one can go with our HPT interpretation of anomalous backscattering. We have taken $E_{lab} = 24$ MeV and $R = 4.69$ fm, $m = 7$, and $R = 3.78$ fm, $m = 8$ (both set of values obtained by fitting the rms radius of Ca^{40}). We have let the cut off angular momentum, L, to vary free around $L = 8$ (obtained from $L = kR$ estimate). We were not able, in this case, to reproduce the angular distribution for backscattering. This result is not so surprising and has also been suggested by Austern[13]. This shows that, for heavier nuclei, the HPT amplitude is probably much smaller than other higher order processes, such as nucleon exchange between the incident a-particle and the target[11,12]. The latter process is more likely, in medium *4n* nuclei, than in light *4n* nuclei, which seems plausible from the fact that it is easier to remove a nucleon from a medium 4n nucleus than from a lighter *4n* nucleus. On the other hand, we are not completely sure that the choice for $\phi_{\nu J}$, given by Eq. (21), is so reasonable for heavier nuclei.

For lighter nuclei, we must say that, definitely, the HPT mechanism is the dominant one to explain the anomalous backward shape of the scattering. However, this agreement is not so good for an energy region below about 30 MeV, since, for this region, contributions from giant compound resonances at ligh excitation occur[18,17] and, possibly, our model does not work there.

It was gratifying to find that for the C^{12} case, $\sum |C_{\nu J}^{(0)}|^2 < 1$ (for the O^{16} case, this was also found in Ref. (2)). They were also energy independent.

It is interesting to observe that the PWBA HPT amplitude alone was large enough, so that the Blair model when incorporated into the model improved only the fine features of the angular distribution.

Recent papers, by Agassi and Wall[11] and Boridy[12], similar in spirit to the present work, attempt to explain the large-angle scattering of a particles from 4n nuclei. In both works only Ca^{40} nucleus is taken as the target necleus. Basically, they calculate the scattering cross section by considering one-necleon exchange between the incident a-particle and the target nucleus. In both papers agreement with experiment is still far from perfect. However it seems that for heavier nuclei the so far neglected one-nucleon exchange amplitude, could be an important contribution to the evaluation of the scattering amplitude[11,12]. Referentes (11) and (12), however, can not rule out the HPT mechanism used in this work. Our feeling is that, for lighter *4n* nuclei, one-nucleon exchange or any other higher order process do not give the main contribution for the total elastic scattering amplitude. This argument has also been discussed partly in Ref. (3). See Fig. 4.

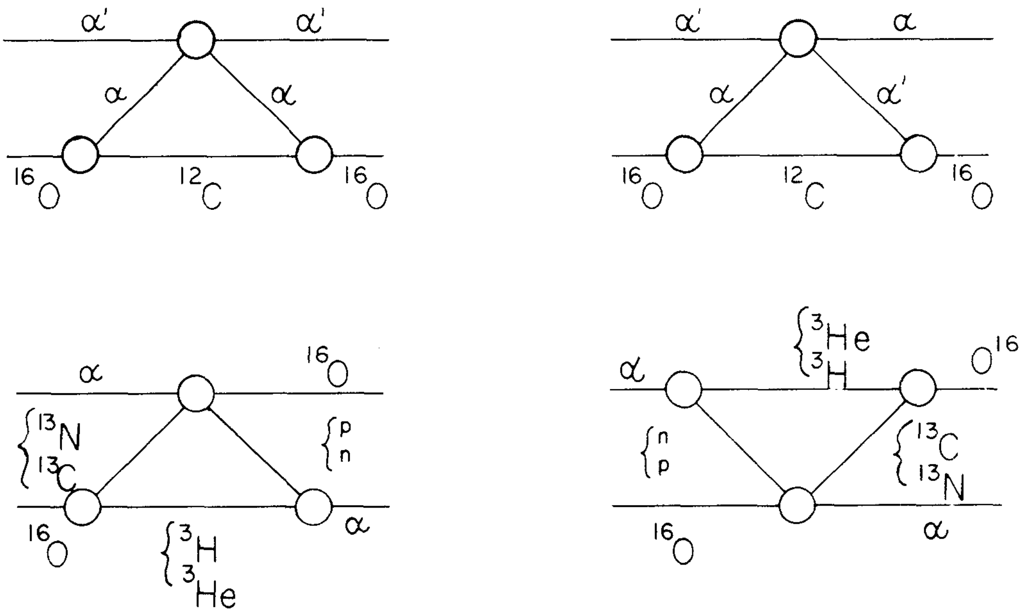


Fig. 4 – Possible higher ordei diagrams for O^{16} as a target nucleus.

We are perfectly aware that our model is the first attempt to consider, more microscopically, the interpretation of the backward peak arising in elastic scattering of α-particles by *4n* light nuclei. Because of that, we had no intention to make other refinements in the theory. Hence, the set of values found for the spectroscopic factors are not, probably, the best ones we could obtain.

Polarization effects, among α particles, and three-body effects (here, only two-body $\alpha-\alpha$ interaction was assumed), are possibly some of the improvements for the model, even though, they should count very little.

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