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Point-like Structure in a Relativistic Dynamical Quark Model

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Deep inelastic electron scattering is discussed in a resonance model derived from a two, spinless, massive, quark-antiquark bound state, in the framework of a relativistic dynamics. The conventional (structureless quark) — electromagnetic vertex does not preserve scaling in the deep inelastic region.

Discute-se o espalhamento inelástico de eletrons, na região inelástica profunda, em um modelo ressonante derivado de um estado ligado de quark-antiquark pesados, sem spin, em uma dinâmica relativística. O vértice (quark sem estrutura) — foton, usual, não preserva a invariância de escala na região inelástica profunda.

1. Introduction

The data from CEA and SPEAR[1], on e^+e^- annihilation into hadrons, surely pose a serious difficulty to our present formulation of the quark mode1[2]. The ratio,

$$R(s) = \sigma(e^+e^- \longrightarrow \text{hadrons})/\sigma(e^+e^- \longrightarrow \mu^+\mu^-),$$

in parton-quark models, is a constant, being therefore independent of the square of the center of mass energy, s (Ref. 3). The constancy of $R(s)$ is equivalent to the scaling phenomenon in deep inelastic electron scattering. The data, however, indicate that $R(s)$ rises roughly linearly with s, from 2, at $s \sim 4(\text{GeV})^2$, to a value 6, at $s \sim 30(\text{GeV})^2$, which shows a breakdown of scaling in the time-like region.

The same difficulty one also faces in a relativistic, dynamic, massive quark model, with resonance excitation, suggesting duality between "current quarks" and constituent quarks for e^+e^- annihilation[4].

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A way out of this difficulty, in the sense of preserving the good features of quark and parton-model results, and incorporating a rising $R(s)$ behaviour, is to provide the quarks themselves with a structure, in spite of the fact that quarks themselves have never been detected.

The idea of a quark structure has been proposed by Chanowitz and Drell[5]. To form hadrons, the quarks must bind together with interactions between them which should renormalize the fundamental quark vertices and change the conventional (structureless quark) — electromagnetic vertex. The quark thus acquires an anomalous magnetic moment and a size. West[6] has shown that, in the parton model, $R(s)$ rises with s if one attributes an anomalous magnetic moment to partons.

In the context of a relativistic, dynamic, massive, quark model, with resonance excitation, the importance of attributing an anomalous magnetic moment to quarks, in order to obtain a rising $R(s)$ behaviour, was discussed by Bohm, Joos, Krammer[4], and Bellandi[7]. A problem in this direction is to see what happens in the deep inelastic region if one assumes structureless quarks.

In a relativistic, dynamic, massive, quark model, the hadrons are described by means of a Bethe-Salpeter (B-S) equation[8]. For mesons, considerable progress has been achieved along these lines[4]. In the case of baryons, however, the solution of the dynamical problem is a formidable task and has not yet been accomplished[9].

In this paper, we investigate deep inelastic electron scattering in a resonance model derived from two, spinless, massive, quark-antiquark bound state, in the framework of a relativistic dynamics[4]. We show that the conventional (structureless quark) — electromagnetic vertex does not preserve scaling, in the deep inelastic region. In Section 2, we write down the B-S equation and its solutions. The structure function, in the Bjorken limit, is discussed in Section 3.

2. The Bethe-Salpeter Equation

Quark-antiquark bound states are described by B-S amplitudes

$$\Phi(k,\ p) = (2\pi)^{3/2} \int d^4x \exp\,(-ik.x)\,(0\,|\,T\phi_q(x/2)\,\phi_{\bar{q}}(-x/2)\,|\,{}^{M}_{p}\rangle. \tag{2-1}$$

These amplitudes are solutions of a B-S equation[S] which, in momenturn space, reads:

$$\left[\left(\frac{p}{2}+k\right)^2-m^2\right]\left[\left(\frac{p}{2}-k\right)^2-m^2\right]\Phi(k,p)=-\int d^4k'\,V(k-k')\,\Phi(k',p),$$

where k is the relative four-momentum of the quarks, m the quarlc mass, and M the bound state mass. As shown by Bohm, Joos and Krammer[4], for a smooth interaction kernel, $V(k)$, and $M^2 \ll m^2$, $(k^2) \ll m^2$, it is possible to expand $V(k)$, and approximate the B-S equation, (2-2), by a four-dimensional harmonic oscillator equation, after performing the Wick rotation, $k_0 \longrightarrow ik_4$:

$$\left(k^2+\frac{m^2}{2}-\frac{M^2}{4}\right)\Phi(k,p)=-(\alpha-\beta\,\Box_k)\,\Phi(k,p). \qquad (2\text{-}3)$$

This equation is exactly solvable and, in the center-of-mass, has the following solution:

$$\Phi(k)=C(2\pi)^2\,\beta^{-1/4}\left(\frac{2}{3\beta}\right)^{1/2}\left[\frac{r!}{(n+r+1)!}\right]^{1/2}\exp\left[-(1/2)\,k_E^2\,\beta^{-1/2}\right]\times$$

$$\times\left[k_E^2\,\beta^{-1/2}\right]^{n/2}L_r^{n+1}\left(k_E^2\,\beta^{-1/2}\right)\mathscr{Y}^n_{l,l_3}(\theta,\beta,\varphi). \qquad (2\text{-}4)$$

In Eq. 4, the coordinates k_E, 0, β, φ are the usual polar coordinates in the Euclidian four-dimensional space:

$$k_4=k_E\cos\beta,\;\; k_3=k_E\sin\beta\cos\theta,\;\; k_1=k_E\sin\beta\sin\theta\cos\varphi,$$

$$k_2=k_E\sin\beta\sin\theta\sin\varphi,\;\; k_E^2=k_4^2+\mathbf{k}^2>0,$$

where $0\le\theta\le\varphi,\; 0\le\beta\le\pi,\; 0\le\varphi\le 2\pi$.

The hyperspherical harmonics, $\mathscr{Y}^n_{l,l_3}(\theta,\beta,\varphi)$, have the form

$$\mathscr{Y}^n_{l,l_3}(\theta,\beta,\varphi)=\mathscr{C}^n_l(\cos\beta)\,Y_{l,l_3}(\theta,\varphi),$$

where

$$\mathscr{C}^n_l(\cos\beta)=l!\left[\frac{2^{2l+1}}{\pi}\;\frac{(n+1)\,(n-l)!}{(n+l+1)!}\right]^{1/2}(\sin\beta)^l\,C^{l+1}_{n-l}(\cos\beta).$$

The $Y_{l,l_3}(\theta,\varphi)$ are the usual spherical harmonics and, C^{l+1}_{n-l}, Gegenbauer polynomials[10].

The eigenvalue problem, for Eq. (2-3), yields the mass spectrum

$$|M|^2 = 4\left[\alpha + \frac{m^2}{2} + 2(n+2r+2)\,\beta^{1/2}\right] \tag{2-5}$$

The theoretical field normalization condition for the spinless case differs from the spinor case because, in the former, it explicitly depends on the quark mass. If one considers this scalar model as a substitute for the spinor case, we cannot attach any physical meaning to the normalization constant in **Eq.** (2-4).

3. Point-like Structure Function

The structure function is defined by

$$W = \sum_{n,r} \left\langle p \,|\, j(0) \,|\, \substack{M' \\ p'}\; n, r \right\rangle \left\langle n, r\; \substack{M' \\ p'} \,|\, j(0) \,|\, p \right\rangle \tag{3-1}$$
$$= \sum_{\text{Resonances}} |M_R|^2,$$

where $j(x)$ is the electromagnetic current with scalar photons.

As has been shown in Ref. 7, we can, in order to calculate a matrix element by means of a triangular graph technique[11], approximate the q $-$ $\bar{q}$ $-$ photon vertex by a Gaussian $-$ ground-state B-S amplitude with a variable width A, and take the limit A $\longrightarrow 0$ to obtain the point-like interaction. The triangular graph is shown in Fig. 1. The vertex function is

$$\Gamma(k,p) = \left[\left(\frac{p}{2} - k\right)^2 - m^2\right]\left[\left(\frac{p}{2} + k\right)^2 - m^2\right]\ \Phi(k, p), \tag{3-2}$$

and the kinematics of the q $-\bar{q}$ bound state vertex is exhibited in Fig. 2. We can, therefore, write

$$M_R = (2\pi)^{-9/2}\, i \int d^4k\, \Phi_3(k, p_3)\, \Phi_1(k - q/2,\ p_1)\, \Phi_2(k + p_1/2, q) \times$$

$$\times\ [(k - p_3/2)^2 - m^2]\ \ [(k + p_3/2)^2 - m^2]\ \ [(k - (p_1 - q)/2)^2 - m^2]. \tag{3-3}$$

If we take: for vertex 1, a ground-state B-S amplitude; for vertex 2, a Gaussian B-S amplitude of width Δ, and for vertex 3, an excited B-S amplitude, **Eq.** (2-3), then calculating (3-3), far from the quark mass-shell, we obtain

$$M_{\mathrm{R}} = \text{const.} \left[\frac{r!}{(n+r+1)!}\right]^{1/2} i^{-n} \frac{\Delta^r}{(\Delta+2)^{n+r+2}}$$
$$\exp\left\{\frac{1}{8(\Delta+2)}\left[q^2+\Delta(p_1^2+p_3^2)\right]\right\}\times$$
$$\times\left[Y_\Delta^2\right]^{n/2} L_r^{n+1}\left(-\frac{Y_\Delta^2}{\Delta(\Delta+2)}\right)\mathscr{Y}^n_{l,l_3}(\hat{Y}_\Delta), \tag{3-4}$$

where

$$Y_\Delta \equiv \frac{1}{2}\left[\Delta p_{0,1}-q_0,\ (-i)(\Delta \mathbf{p}_1-\mathbf{q})\right],\ \beta^{1/2}=1.$$

For a fixed total quantum number, $N=2r+n$, we have

$$W(q^2,N) = \text{const.}\ \exp\left\{\frac{1}{4(\Delta+2)}\left[q^2+\Delta(p_1^2+p_3)^2\right]\bullet\right.$$
$$\left.+\frac{1}{4(\Delta'+2)}\left[q^2+\Delta'(p_1^2+p_3^2)\right]\right\}\times$$
$$\times\left[(\Delta+2)(\Delta'+2)\right]^{-N-2}\sum_{2r+n=N}\frac{r!}{(n+r+1)!}\left[\Delta(\Delta+2)\Delta'(\Delta'+2)\right]^r\times$$
$$\times L_r^{n+1}\left(-\frac{Y_\Delta^2}{\Delta(\Delta+2)}\right)L_r^{n+1}\left(-\frac{Y_{\Delta'}^2}{\Delta'(\Delta'+2)}\right)\times$$
$$\times\sum_{l,l_3}\mathscr{Y}^{n*}_{l,l_3}(\hat{Y}_\Delta)\mathscr{Y}^n_{l,l_3}(\hat{Y}_{\Delta'})\left[Y_\Delta^2\right]^{n/2}\left[Y_{\Delta'}^2\right]^{n/2}. \tag{3-5}$$

We can now evaluate (3-5) with the help of the addition theorem for hyperspherical harmonics and use the fact that[10]

$$\lim_{\Delta\to 0}\left[\Delta(\Delta+2)\right]^r L_r^{n+1}\left(-\frac{Y_\Delta^2}{\Delta(\Delta+2)}\right)=\frac{1}{r!}\left[Y_{\Delta=0}^2\right]^r.$$

If we introduce the scale variable

$$\omega=\frac{2p_1\cdot q}{-q^2},\ Q^2=-q^2,$$

and $\beta^{1/2}$ again, we have, in the limit $A=\Delta' +0$,

$$W(q^2, N) \sim \frac{1}{N!} [Q^2/8\beta^{1/2}]^N \left[\frac{1+(\omega-1)^2}{2(\omega-1)}\right]^N \exp[-Q^2/8\beta^{1/2}]. \qquad (3\text{-}6)$$

The quantum number N is related to tlie scale variable through the mixing mass, P_3^2, given by Eq. (2-5):

$$N=(Q^2/8\beta^{1/2}) \,.\, (o-1).$$

For large N, i.e., in the Bjorken limit, the structure function behaves like

$$W(q^2, N) \sim (2\pi)^{-1/2}\frac{(\omega-1)^{1/2}}{N^{1/2}} \exp\,[(N/6)\,(\omega-2)^3], \qquad (3\text{-}7)$$

and, therefore, we do not have scaling for νW, in the Bjorken limit; $\nu=(p_1 \cdot q)/M$, M given by Eq. (2-5).

In the context of tlie Feynman-Kislinger-Ravndal[1,2] relativistic quark model, with a Wick rotation, we have the same result, namely, Eq. (3-12). It is interestiiig to point out that, in the framework of a nonrelativistic dynamics[13], the structure function is found to be given by

$$W_{NR} \sim (2\pi)^{-1/2}\frac{(\omega-1)^{1/2}}{N^{1/2}} \exp\left[-(N/2)\frac{(\omega-2)^2}{\omega-1}\right], \qquad (3\text{-}8)$$

and besides having νW scaling in the Bjorken limit, one has a quasi-elastic invariant peak. This result, however, depends strongly on dynamical considerations.

From the present calculations, it is plausible to conclude that if one wants to preserve scaling, in the deep inelastic region, in a relativistic, massive, quark model, it is necessary to change tlie conveiitional (structureless quark)-electromagnetic vertex, and provide tlie quarks themselves wiih a siructure similar to the one needed in e^+e^- annihilation, e.g., via generalized vector-meson dominance. Some progress along these lines has been achieved[7].

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