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Transmission of Nematic Liquid Crystals in Electric Fields

ANTONIO E. COSTA PEREIRA
Instituto Nacional de Pesquisas Espaciais (INPE) — São José dos Campos, S.P.

ABEL ROSATO*
Instituto Nacional de Pesquisas Espaciais (INPE) — São José dos Campos, SP
Instituto Tecnológico de Aeronautica (ITA) — São José dos Campos, SP

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Measurements to observe the change in the transmission of nematic liquid crystal with respect to the applied voltage were performed. A theoretical model to explain the oscillatory behavior of the transmission for voltages between 2 and 5 volts and for a cell of 20 μm thick is presented.

Foram feitas medidas para observar a variação da transmissão de cristais líquidos nemáticos em relação à voltagem aplicada. E apresentado um modelo teórico para explicar o comportamento oscilatório da transmissão para voltagens entre 2 e 5 volts e para celas de espessura de 20 pm.

1. Experimental Arrangement

We placed a nematic liquid crystal cell (i.e. a liquid crystal layer sandwiched in two transparent electrodes) between two crossed polarizers, similar to an experiment previously reported[1]. The cell had a homeotropic alignment, achieved by the way reported by Kahn[2]. This arrangement stops light when there is no voltage applied between the two transparent electrodes. When an externa1 voltage is applied, a beam of white light is partially filtered in such a way that it is projected as a colour spot on a screen, after crossing the two polarizers and the cell. The colour of the spot changes when the voltage changes. This effect is observed more easily when the voltage is kept between 2V and 5V for a 20 μm thick cell. For a monocromatic beam of light, the transmission of the arrangement changes with voltage, showing maxima and minima, like in Fig. 1.

******Conselho Nacional de Pesquisas* (CNPq) researcher

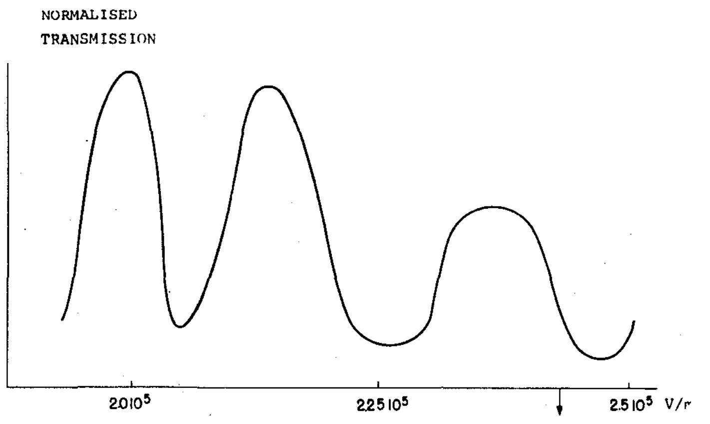

Fig. 1 — Normalized transmission of a nematic liquid crystal cell with homeotropic alignment *vs* applied electric field for a monocromatic beam of light ($\lambda = 6000$ Å).

2. Theoretical Considerations

Looking for an explanation of the experimental behavior of the above mentioned arrangement, let us determine first the way the index of refraction varies inside the LC and then we can calculate the intensity of the transmitted light. Let us suppose that the direction of the two principal dielectric axes of the liquid crystal rotates as we go from *A* to *B* or from *C* to *B,* according to Fig. 2. Let us suppose further that this rotation is $\theta(Z) = (2\pi/p)Z$, being the distance to the nearest plate and p the distance from x_1, x_2 plane for a 2π rd rotation.

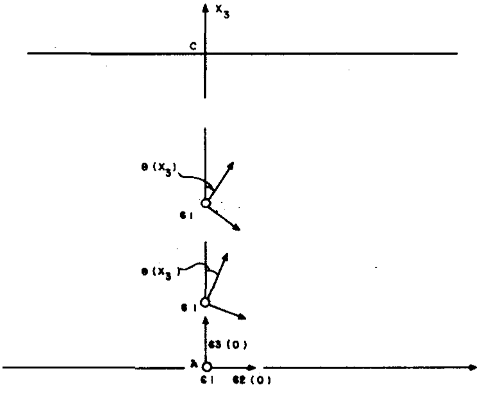

Fig. 2 — Theoretical model for the principal dielectric axes of the liquid crystal; x_1 is perpendicular to the paper and the distance from *A* to C is equal to *2h.*

The relation between the displacement vector D and the electric field vector E, in the rotated coordinate system caracterized by ε_1, ε_2, ε_3 is given by

$$D'_2 = \varepsilon_2 \; E; \qquad (2\text{-}1)$$
$$D'_3 = \varepsilon_3 \; E'_3 \qquad (2\text{-}2)$$

The components of D and E in the rotated system are related to the components in a fixed system of coordinates x_1, x_2, x_3 by the relations:

$$D'_2 = \quad D_2 \; \cos\theta + D_3 \; \sin\theta \qquad (2\text{-}3)$$
$$D'_3 = -D_2 \; \sin\theta + D_3 \; \cos\theta \qquad (2\text{-}4)$$
$$E'_2 = E_2 \; \cos\theta + E_3 \; \sin\theta \qquad (2\text{-}5)$$
$$E'_3 = -E_2 \; \sin\theta + E_3 \; \cos\theta \qquad (2\text{-}6)$$

From equations 2.1 to 2.6 we get:

$$D_2 = (\varepsilon_2 \cos^2\theta + \varepsilon_3 \sin^2\theta) \; E_2 - (\varepsilon_3 - \varepsilon_2) \sin 0 \; \cos 8 \; E_3, \qquad (2\text{-}7)$$
$$D_3 = -(\varepsilon_3 - \varepsilon_2) \sin\theta \; \cos\theta \; E_2 + (\varepsilon_3 \cos^2\theta + \varepsilon_2 \sin^2\theta) \; E_3. \qquad (2\text{-}8)$$

Equations 2.7 and 2.8 give the components of the dielectric dyadic:

$$\varepsilon_{31} = \varepsilon_{21} = 0, \qquad (2\text{-}9)$$
$$\varepsilon_{22} = \varepsilon_2 \; \cos^2\theta + \varepsilon_3 \; \sin^2 8, \qquad (2\text{-}10)$$
$$\varepsilon_{23} = \varepsilon_{32} = -(\varepsilon_3 - \varepsilon_2) \sin 8 \; \cos 8, \qquad (2\text{-}11)$$
$$\varepsilon_{33} = \varepsilon_3 \; \cos^2\theta + \varepsilon_2 \; \sin^2 8. \qquad (2\text{-}12)$$

Let's admit that: the vectors E, D, B, H are of the form E = $\mathbf{E}_0$ exp $i\omega[(n(Z)/c)$ **r.s** $- t]$, the direction of propagation **s** is parallel to the Z direction and the refraction index n varies with Z. Then, using Maxwell's equations we get:

$$n\mathbf{k} \times \mathbf{E} - Z \frac{dn}{dZ} \; (E_2 \; \mathbf{i} - E_1 \; \mathbf{j}) = \mu \; \mathbf{H}_0 \qquad (2\text{-}13)$$

$$n\mathbf{k} \times \mathbf{H} - Z \frac{dn}{dZ} \; (H_2 \; \mathbf{i} - H_1 \; \mathbf{j}) = -\mathbf{D}_0 \qquad (2\text{-}14)$$

Taking the components of these expressions and bearing in mind that $D_k = \sum_j \varepsilon_{kj} \; E_j$ and that $\varepsilon_{13} = \varepsilon_{31} = \varepsilon_{21} = \varepsilon_{12} = 0$ we get:

$$\varepsilon_{32} \; E_2 + \varepsilon_{33} \; E_3 = 0, \qquad (2\text{-}15)$$

$$\left[\varepsilon_{22} - \frac{1}{\mu}\left(n + Z\frac{dn}{dZ}\right)^2\right] E_2 + \varepsilon_{23} \; E_3 = 0, \qquad (2\text{-}16)$$

$$\left[\varepsilon_1 - \frac{1}{\mu}\left(n + Z\frac{dn}{dZ}\right)^2\right] E_1 = 0. \tag{2-17}$$

The determinant of the coefficients of E_1, E_2, E_3 should be zero in order to get a non-trivial solution. From this condition we get the equations:

$$\varepsilon_1 - \frac{1}{\mu}\left(n + Z\frac{dn}{dZ}\right)^2 = 0, \tag{2-18}$$

$$\varepsilon_{33}\left[\varepsilon_{22} - \frac{1}{\mu}\left(n + Z\frac{dn}{dZ}\right)^2\right] - \varepsilon_{23}^2 = 0. \tag{2-19}$$

The solution of the first equation, compatible with the boundary condition ($n = n_0$ for $Z = 0$), is $n = \sqrt{\mu\ \varepsilon_1}$ which corresponds to the ordinary wave.

For the second one, using Bernouille's method of solution and considering that $n = n_0$ for $Z = 0$, we get:

$$n(Z) = \frac{1}{Z}\int_0^Z \left[\frac{1}{n_0^2}\cos^2\theta(Z') + \frac{1}{n_e^2}\sin^2\theta(Z')\right]^{-1/2} dZ' \tag{2-20}$$

To get this expression for n, we used equations (2-10) to (2-12), and the fact that $\eta_0^2 = \mu\varepsilon_1 = \mu\varepsilon_2$ and $n_e^2 = \mu\varepsilon_3$. For $\theta(Z)$ = constant, these solutions will give us a well known result[3].

With this we can get the phase difference δ_1 between the ordinary wave and the extraordinary wave in the analyser. For $\theta = (2\pi/p)Z$, the phase difference is

$$\delta(4\pi/p) = \frac{4\pi}{\lambda}\int_0^h [n(Z) - n_0]\, dZ, \tag{2-21}$$

where h is half of the width of the cell.

We are now ready to find the intensity of the transmitted wave at the analyser. It is given by

$$I = E_0^2 \sin^2 2\phi \sin^2\left|\frac{1}{2}\,\delta\left(\frac{4\pi}{p}\right)\right| \tag{2-22}$$

and it is a function of $(4\pi/p)$. The transmission, that is, the above expression divided by E_0^2, was calculated numerically for $4\pi/p$ varying from 0 to 3 x 10^5. The range of $4\pi/p$ is determined by the fact that the thickness of the cell is $2h = 20\,\mu m$ and p cannot be smaller than $4h$. The numerical result for the transmission is represented in Figure 3. Comparing this theoretical result with the experimental one, represented in Fig. 1, we see that they agree with respect to the oscillations. Physical considerations allow us to assume that there is a connectiori between the applied electric field and $4\pi/p$. In the case that p is smaller than $4h$ we can consider, along the Z direction, three regions in the crystal liquid layer for calculating the phase delay: one central region, which behaves like a solid crystal, and others two, limited by the plates and the central region, which follows the model presented above.

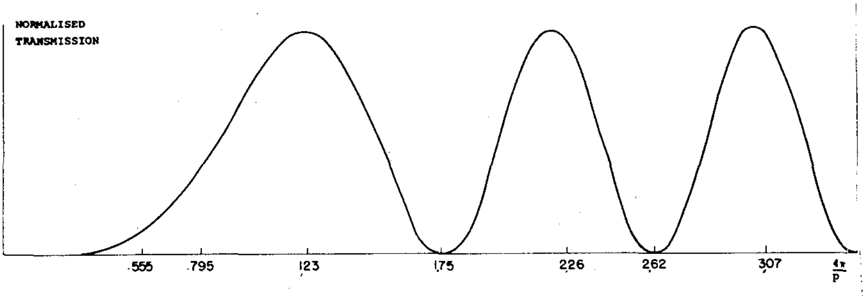

Fig. 3 — Normalized **transmission of nematic LC versus** $4\pi/p$, **as explained in the** text.

References

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2. J. Frederic Kahn, Appl. Phys. Letters, 22, 386 (1973).
3. M. Born and Wolf E., *Principles of Optics,* Pergamon Press, New York, 1959.