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Some Comments on PCDC and an Alternative Assumption

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Some aspects of the PCDC hypothesis are discussed and criticized in a framework in which one considers a first order expansion in the parameters that break $SU(3)$ x $SU(3)$ chiral and dilational symmetries. It is shown that Eliezer's and Dutt's mass formula[2], for the σ_0 is valid for any dimension, d. It is also shown that PCDC, in the case of two poles, may have some difficulties if one makes some physical assumptions about the order of magnitude of $(f_\pi - f_K)$ and of the breaking symmetry term in the a_0-meson-meson coupling constant. An alternative assumption which retains the basic features of PCDC is presented: the hypothesis of the dominance of the o particle in the process which involve the scalar current u_0. A mass formula for the o particle is obtained for any value of d.

Discutem-se e criticam-se alguns aspectos da hipótese de PCDC em um ponto de vista que considera expansões, em 1.ª ordem, nos parâmetros que quebram as simetrias, $SU(3)$ x $SU(3)$ quiral, e dilatacional. Mostra-se que a fórmula de massa de Eliezer e Dutt[2], para o σ_0, vale para qualquer valor da dimensão, d. Mostra-se também que PCDC:, no caso de dois polos, pode apresentar algumas dificuldades se fazemos algumas hipóteses acerca da ordem de grandeza de $(f_\pi - f_K)$ e do termo de quebra na constante de acoplamento a_0-meson-meson. Apresenta-se uma hipótese alternativa que mantern as características da hipótese de PCDC. a saber. a dominância da partícula a nos processos que envolvam a corrente escalar u_0. Obtem-se, então, uma fórmula de massa, para a partícula a, em qualquer dimensão, d.

1. Introduction

In this paper, we wish to make some comments on the PCDC hypothesis. In our calculations, we follow the philosophy excellently expressed in de Alwis' article'. In Sec. 2, we shall mention only some of the important points of that paper, while giving close attention to some of the generalizations of the theory therein. In Sec. 3, this generalized theory is applied to the specific case of PCDC. We consider the two poles case, distinguishing two different situations: a) both poles are due to Goldstone bosons; b) only one is due to the Goldstone boson.

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We obtain, in case a), a unacceptable result, namely, d = 4, if we assume $(f_\pi - f_K) \sim O(\lambda, \varepsilon)$ and $F_{\pi\sigma_0} - F_{K\sigma_0} \sim O(\lambda, e)$, λ and e being breaking parameters. In case b), one does not have the above difficulty but, nevertheless, it is necessary to introduce, when $d \neq 2$, an explicit breaking term in the σ_8-meson-meson coupling. Finally, in Sec. 4, we present an alternative physical assumption which retains some of the good aspects of PCDC and, at the same time, gives rise to some improvements over PCDC. The assumption we make is the one of dorninance of the dilaton σ in the processes which involve the scalar current u_0, while u_8 couples with o without exclusiveness. We expect that there exist other intermediate states that couple with this current. We assume that their contribution is $SU(3)$ parametrizable. For d = 2. our hypothesis can be identified with case b), of Sec. 3, when assuming the existence of some effective field. Our results are, basically, identical to those obtained when using PCDC.

2. Generalities

In this paper, we assume, for the Hamiltonian density, the expression

$$\theta_{00} = \bar{\theta}_{00} + \lambda\delta + \varepsilon_0 u_0 + \varepsilon_8 u_8, \tag{2-1}$$

where $\bar{\theta}_{00}$ is invariant under $SU(3)$ x $SU(3)$ chiral transformations and dilations, as well . δ is a c-number, world scalar, invariant under $SU(3)$ x $SU(3)$, but breaking dilational symmetry; its dimension is zero. The u_i are scalar currents, world scalars, which break $SU(3)$ x $SU(3)$ chiral and dilational symmetries. They belong to the $(3 \times 3'') \oplus (3^* \times 3)$ representation of $SU(3) \times SU(3)$. It is assumed, for simplicity, that they have the same, integral, dimension $1 \leq d \leq 3$.

The Hamiltonian density (2-1) permits the realization of both $SU(3)$ x $SU(3)$ chiral and dilational symmetries in the Nambu-Goldstone way with a simultaneous symmetry limit. As a consequence, there appears a particle, a o dilaton, i.e., a Goldstone boson, associated with dilational symmetry.

Following Ref. 1, let us consider now the Ward identities:

$$- ik^2 T_{\lambda\mu}(k, p) = T_\mu(k, p) -$$

$$- i \int d^4x \; e^{ikx} \, \delta(x_0) \, \langle 0 \,|\, [\mathscr{D}_0(x), \, A_\mu(0)] \,|\, \pi(p) \rangle \tag{2-2}$$

and

$$- iq^{\mu}T_{\mu}(k,p) = T(k,p) -$$
$$- i \int d^4x \; e^{iqx} \; \delta(x_0) \; \langle 0 | [A_0(x), \; \theta(0)] | \pi(p) \rangle, \tag{2-3}$$

where

$$\mathscr{D}_{\mu} = x_{\nu} \, \theta'_{\mu} \tag{2-4}$$

is the dilational current and

$$T_{\lambda\mu}(k,p) = - \mathrm{i} \int d^4x \; e^{ikx} \; (0 \,|\, \mathscr{T}(\mathscr{D}_{\lambda}(x)A_{\mu}(0)) \,|\, \pi(p)\rangle, \tag{2-5}$$

$$T_{\mu}(k,p) = - i \int d^4x \; e^{ikx} \; (0 \,|\, \mathscr{T}(\theta(x)A_{\mu}(0)) \,|\, \pi(p)\rangle, \tag{2-6}$$

$$T(k,p) = - \mathrm{i} \int d^4x \; e^{ikx} \; (0 \,|\, \mathscr{T}(\theta(x)\partial^{\mu}A_{\mu}(0)) \,|\, \pi(p)\rangle, \tag{2-7}$$

where $\mathscr{T}$ is the time-ordering operator.

In the evaluation of expressions *(2-5)–(2-7),* there appear matrix elements such as

$$\langle 0 | \theta_{\mu\nu} | \sigma(\mathrm{k}) \rangle, \; \langle \pi(p) | \theta_{\mu\nu} | \pi(q) \rangle,$$
$$\langle \sigma(k) | A_{\mu} | \pi(p) \rangle, \quad \langle \sigma(k) | \partial^{\mu} A_{\mu}(0) | \pi(p) \rangle.$$

In Ref. 1, it is assumed that the pion pole and the pole from one *o* furnish important contributions, at low energies, to these matrix elements. This suggests to make explicit those pole contributions. We thus have:

$$\langle 0 | \theta_{\mu\nu}(0) | \sigma(k) \rangle = \frac{f_{\sigma}}{3} \; (k_{\mu}k_{\nu} - k^2 g_{\mu\nu}), \tag{2-8}$$

$$\langle \pi(p) | \theta(0) | \pi(q) \rangle = - \frac{f_{\sigma} \, m_{\sigma}^2 \, g_{\sigma\pi\pi}}{k^2 - m_{\sigma}^2} + \bar{\theta}_{\pi}(k^2)$$
$$= 2m_{\pi}^2 \; F_1(k^2) - \frac{k^2 f_{\sigma} \, g_{\sigma\pi\pi}}{k^2 - m_{\sigma}^2} - 3k^2 \; F_2^R(k^2), \tag{2-9}$$

$$\langle \sigma(k) | A_{\mu}(0) | \pi(p) \rangle = i[(k+p)_{\mu} \; F_{\pi\sigma}(q^2) + (k-p)_{\mu} \; G_{\pi\sigma}(q^2)], \tag{2-10}$$

where

$$G_{\pi\sigma}(q^2) = - \frac{f_{\pi} \, g_{\sigma\pi\pi}}{q^2 - m_{\pi}^2} + G^R(q^2), \tag{2-11}$$

$$\langle \sigma(k) | \partial^{\mu} A_{\mu}(0) | \pi(p) \rangle = \frac{f_{\pi} \, m_{\pi}^2 \, g_{\sigma\pi\pi}}{q^2 - m_{\pi}^2} + \bar{\partial}_{\pi\sigma}(q^2)$$
$$= - (m_{\sigma}^2 - m_{\pi}^2) F_{\pi\sigma}(q^2) - q^2 \left[- \frac{f_{\sigma} \, g_{\sigma\pi\pi}}{q^2 - m_{\pi}^2} + G^R(q^2) \right] . \tag{2-12}$$

In this way, quantities (2-5) – (2-7), basic ingredients of Eqs. (2-2) – (2-3), become calculable.

Next, a limiting procedure is applied to **Eq.** (2-2), according to the following prescriptions:

— during the limiting process, the relation $q = p - k$ always holds;

— first, it is performed the limit $q^2 \longrightarrow 0$;

next, one takes $k \longrightarrow 0$.

In this way, we obtain the relation

$$f_\pi = f_\sigma F_{\pi\sigma} + O(\lambda, \varepsilon). \tag{2-13}$$

In a similar way, a limiting procedure is applied to Eq. (2-3), following another set of prescriptions:

— during the limit process the relation $q = p - k$ always holds;
– one takes $q \longrightarrow 0$;
— p is kept on the mass shell, what implies $k^2 \longrightarrow p^2 = m_\pi^2$.

We obtain, in this way, the relation

$$f_\pi f_\sigma g_{\sigma\pi\pi} - f_\sigma m_\sigma^2 F_{\pi\sigma} = (d-2)\, m_\pi^2 f_\pi + O(\lambda^2, \lambda\varepsilon, \varepsilon^2). \tag{2-14}$$

By combining (2-13) and (2-14), the Kleinert-Weisz relation[5],

$$f_\sigma g_{\sigma\pi\pi} = m_\sigma^2 + (d-2) m_\pi^2 + O(\lambda^2, \lambda\varepsilon, \varepsilon^2), \tag{2-15}$$

is obtained. During the calculation, one can see that

f_π	$\sim$	$O(1)$,	$F_2^R(k^2)$	$\sim$	$O(\lambda, \varepsilon)$,
f_σ	$\sim$	$O(1)$,	$G^R(q^2)$	$\sim$	$O(\lambda, \varepsilon)$,
$g_{\sigma\pi\pi}$	$\sim$	$O(\lambda, \varepsilon)$,	$F_{\pi\sigma}(q^2)$	$\sim$	$O(1)$,
m_π^2	$\sim$	$O(\lambda, \varepsilon)$,	$\theta_\pi(k^2)$	$\sim$	$O(\lambda, \varepsilon)$,
m_σ^2	$\sim$	$O(\lambda, \varepsilon)$,	$\partial_{\pi\sigma 0}$	$\sim$	$O(\lambda, \varepsilon)$

Table 1.

Now, we shall consider the generalization of these results to the case of two σ poles. In the presence of two poles, we must distinguish two different situations: a) both particles are Goldstone bosons; b) one is a Goldstone boson, but the other is not. Let us fix our attention on the following particles:

a σ_0 $SU(3)$ singlet – Goldstone boson,

a σ_8, eighth component of an $SU(3)$ octet, which may be a Goldstone boson or not. To understand better this choice, we discuss briefly the relation of possible dilatons with the properties of vacuum.

One of the ways of formulating Goldstone's Theorem[3] is to assume that:

There exists a current $j^a_\mu(x)$, such that $\partial^\mu j^a_\mu(x) = 0$;

$Q^a = \int d^3x\, j^a_0(x)$ is the generator of some symmetry transformation;

the vacuum is not invariant under such symmetry transformation, i.e., $\langle 0 | [Q^a, \phi_i] | 0\rangle \neq 0$.

Then, the theorem asserts that ϕ_i is the field of some particle of zero mass, the Goldstone boson.

In the case of dilational transformations, the current $j^a_\mu(x)$ is given by

$$\mathscr{D}_\mu(x) = x_\nu \theta^\nu_\mu(x),$$

and the generator of the transformation, Q^a, is given by

$$D = \int d^3x \mathscr{D}_0(x).$$

Then, in order to have σ_i as a Goldstone boson, we must have $\langle 0 | [D, \phi_{\sigma i}] | 0\rangle \neq 0$. In discussing PCDC, we shall take as an illustration the simple model where we identify the fields of scalar mesons with the scalar currents u_i. We, then, have

$$\langle 0 | [D, u_i] | 0\rangle \neq 0.$$

We know that

$$[D, u_i] = -i u_i d.$$

Then, to have σ_i as a Goldstone boson, we must have

$$\langle 0 | u_i | 0\rangle \neq 0,$$

even when $\partial^\mu \mathscr{D}_\mu = \theta^\mu_\mu = 0$.

It is an accepted idea that $(0 \mid u_0(0) \neq 0$ (this condition ensures, for example, a Goldstone status for the pion). Then, σ_0 will be a Goldstone particle.

However, $(0 \mid u_8 \mid O)$, in the real world, is smaller than $(0 \mid u_0 \mid 0\rangle$ and, its behaviour in the limit of the symmetry may be questionable. If $(0 \mid u_8 \mid 0\rangle$ remains different from zero in this limit, we must consider o, as a Goldstone boson, but, otherwise, not.

Let us return to the discussion of the cases which may occur with two dilatons. In case a), when both dilatons are Goldstone bosons, we can repeat Table 1 for each of the dilatons. We show next that this is really possible.

The order of magnitude of f_π and m_π^2 do not change when one introduces a second Goldstone particle. Both f_{σ_0} and f_{σ_8} must be of order $O(1)$, because of the hypothesis that σ_0 and σ_8 are Goldstone bosons (cf. Goldstone's theorem). $g_{\sigma_0\pi\pi}$ and $g_{\sigma_8\pi\pi}$, each of them, separately, must be of order $O(\lambda, \varepsilon)$. This can be seen by taking Eq. (2-12) for $q^2 = 0$, obtaining thus

$$f_\pi\, g_{\sigma i\pi\pi} + \bar{\partial}_{\pi\sigma i}(0) = -(m_{\sigma i}^2 - m_\pi^2) F_{\pi\sigma i}(0).$$

For simplicity, we assume PCAC realized in an exact way. This implies that $\bar{\partial}_{\pi\sigma i} = 0$. It is expected that $F_{\pi\sigma i}$ is of order $O(1)$. Then, as π and o, are Goldstone bosons ($m_\pi^2 \sim O(\lambda, \varepsilon)$ and $m_{\sigma i}^2 \sim O(\lambda, \varepsilon)$), it follows that $g_{\sigma i\pi\pi} \sim O(\lambda, \varepsilon)$. As a consequence, we can also conclude that $\bar{\partial}_{\pi\sigma i}$, if different from zero, is of order $O(\lambda, \varepsilon)$. On the other hand, $m_{\sigma 0}^2$ and $m_{\sigma 8}^2$ are both of order $O(\lambda, \varepsilon)$ by the very hypothesis that they are Goldstone bosons. In order to show that $F_2^R(k^2) \sim O(\lambda, \varepsilon)$ and $\bar{\theta}_\pi(k^2) \sim O(\lambda, \varepsilon)$, we must consider Eq. (2-9) generalized for the case of two poles:

$$\langle \pi(p) \mid \theta(0) \mid \pi(q) \rangle$$

$$= -\frac{f_{\sigma_0} m_{\sigma_0}^2 g_{\sigma_0\pi\pi}}{k^2 - m_{\sigma_0}^2} - \frac{f_{\sigma_8} m_{\sigma_8}^2 g_{\sigma_8\pi\pi}}{k^2 - m_{\sigma_8}^2} + \bar{\theta}_\pi(k^2)$$

$$= 2m_\pi^2 F_1(k^2) - \frac{k^2 f_{\sigma_0} g_{\sigma_0\pi\pi}}{k^2 - m_{\sigma_0}^2} - \frac{k^2 f_{\sigma_8} g_{\sigma_8\pi\pi}}{k^2 - m_{\sigma_8}^2} - 3k^2 F_2^R(k^2).$$

As we have already determined the orders of magnitude of $g_{\sigma_0\pi\pi}$ and $g_{\sigma_8\pi\pi}$, we can apply the same procedure that was developed in the case of one Goldstone boson, obtaining the expected result, i.e., $F_2^R(k^2) \sim O(\lambda, \varepsilon)$ and $\bar{\theta}_\pi(k^2) \sim O(\lambda, \varepsilon)$.

Finally, as we have already mentioned, it is assumed that $F_{\pi\sigma i} \sim O(1)$. In this case, analizing Eq. (2-12), generalized for two dilatons, and using a srnoothness hypothesis, we get $G^R(q^2) \sim O(\lambda, \varepsilon)$.

Now, in case b), when σ_8 is not a Goldstone boson, we have:

f_π	$\sim$	$O(1)$,			
f_{σ_0}	$\sim$	$O(1)$,	f_{σ_8}	$\sim$	$O(\lambda, \varepsilon)$,
$g_{\sigma_0\pi\pi}$	$\sim$	$O(\lambda, \varepsilon)$,	$g_{\sigma_8\pi\pi}$	$\sim$	$O(1)$,
m_π^2	$\sim$	$O(\lambda, \varepsilon)$,			
m_σ^2	$\sim$	$O(\lambda, \varepsilon)$,	$m_{\sigma_8}^2$	$\sim$	$O(1)$,
$F_2^R(k^2)$	$\sim$	$O(\lambda, \varepsilon)$			
${}^0G^R$	$\sim$	$O(\lambda, \varepsilon)$,	${}^8G^R$	$\sim$	$O(\lambda, \varepsilon)$,
$F_{\pi\sigma_0}$	$\sim$	$O(1)$,	$F_{\pi\sigma_8}$	$\sim$	$O(1)$,
$\theta_\pi(k^2)$	$\sim$	$O(\lambda, \varepsilon)$,			
$\bar{\partial}_{\pi\sigma_0}$	$\sim$	$O(\lambda, \varepsilon)$,	$\bar{\partial}_{\pi\sigma_8}$	$\sim$	$O(\lambda, \varepsilon)$.

Table 2.

It is easy to see why there appear differences between Tables 1 and 2. Let us take

$$\langle 0 | \theta | \sigma_8 \rangle = -f_{\sigma_8} m_{\sigma_8}^2.$$

By the virial theorem, we have $\theta \sim O(\lambda, \varepsilon)$. Then, $f_{\sigma_8} m_{\sigma_8}^2 \sim O(\lambda, \varepsilon)$. But, by hypothesis, σ_8 is not a Goldstone particle. This means that $m_{\sigma_8}^2 \sim O(1)$. We can, then, conclude that $f_{\sigma_8} \sim O(\lambda, \varepsilon)$.

On the other hand, we have

$$2m_\pi^2 = f_{\sigma_0}\, g_{\sigma_0\pi\pi} + f_{\sigma_8}\, g_{\sigma_8\pi\pi} + \dots$$

We have, however, accepted, in the beginning, the fact that $m_\pi^2 \sim O(\lambda, \varepsilon)$. This imposes the constraint that $f_{\sigma_8}\, g_{\sigma_8\pi\pi} \sim O(\lambda, \varepsilon)$. But we have shown that $f_{\sigma_8} \sim O(\lambda, \varepsilon)$. Then, $g_{\sigma_8\pi\pi} \sim O(1)$.

Now, taking Eq. (2-12), for σ_8, we shall have the relation:

$$f_\pi\, g_{\sigma_8\pi\pi} - \bar{\partial}_{\pi\sigma_8}(q^2) = (m_{\sigma_8}^2 - m_\pi^2)\; F_{\pi\sigma_8}(q^2) + q^2\; {}^8G^R(q^2). \qquad (2\text{-}16)$$

In this last equation, $\bar{\partial}_{\pi\sigma_8}$ is the regular part of the matrix element $\langle \sigma_8 | \partial^\mu A_\mu | \pi \rangle$. It can, then, be assumed that its order of magnitude is the same as the one of $\partial^\mu A_\mu$, which is $O(\lambda, \varepsilon)$. Then, we shall have $\bar{\partial}_{\pi\sigma_8} \sim O(\lambda, \varepsilon)$.

On the other hand, analyzing Eq. (2-16), in the light of the smoothness hypothesis, we expect to have ${}^{8}G^{R}(q^2) \sim O(\lambda,$ E). But, as $f_\pi g_{\sigma_8\pi\pi}$ and and $m^2_{\sigma_8}$ are of order one, it follows immediately, from Eq. (2-16), that $F_{\pi\sigma_8} \sim O(1)$. This completes our discussion of Table 1.

In general, we have to interpret the results of Tables 1 and 2 as upper bounds to the quantities there exhibited. For example, in case a), it may, as we shall see later, happen that F,,, $\sim O(\lambda,$ E).

In case a), as well in case b), the generalizations of Eqs. (2-13) and (2-14) are:

$$f_{\sigma_0} F_{\pi\sigma_0} + f_{\sigma_8} F_{\pi\sigma_8} = f_\pi + O(\lambda, \varepsilon) \qquad (2\text{-}17)$$

and

$$f_\pi f_{\sigma_0} g_{\sigma_0\pi\pi} + f_\pi f_{\sigma_8} g_{\sigma_8\pi\pi} - f_{\sigma_0} m^2_{\sigma_0} F_{\pi\sigma_0} - f_{\sigma_8} m^2_{\sigma_8} F_{\pi\sigma_8}$$
$$= (\mathrm{d}-2) m^2_\pi f_\pi + O(\lambda^2, \text{ÂE}, \varepsilon^2). \qquad (2\text{-}18)$$

This can easily be verified if we observe that in the Ward identities, the terms which originate from dilaton poles, appear linearly. In introducing a new dilaton ***(o,)***, we have only to add a corresponding term in Eqs. (2-13) – (2-14), which themselves follow from the Ward identities.

In case b), however, it can be seen from Table 2 that $f_{\sigma_8} F_{\pi\sigma_8} \sim O(\lambda,$ E). Eq. (2-17) takes, then, the form of Eq. (2-13).

Combining Eqs. (2-13) and (2-18), we shall have

$$f_\pi[f_{\sigma_0} g_{\sigma_0\pi\pi} - m^2_{\sigma_0} - (d-2) m^2_\pi]$$
$$= f_{\sigma_8}[m^2_{\sigma_8} F_{\pi\sigma_8} - f_\pi g_{\sigma_8\pi\pi}] + O(\lambda^2, \lambda\varepsilon, \varepsilon^2). \qquad (2\text{-}19)$$

By using Eq. (2-16) and the results of Table 2, we obtain

$$m^2_{\sigma_8} F_{\pi\sigma_8} - f_\pi g_{\sigma_8\pi\pi} \sim O(\lambda, \varepsilon). \qquad (2\text{-}20)$$

Since f_{σ_8} is $O(\lambda,$ E), then the right hand side of (2-19) will be 0(Â^2, $\lambda\varepsilon, \varepsilon^2$). This in mind, we obtain the Kleinert-Weisz relation, Eq. (2-15), for σ_0:

$$f_{\sigma_0} g_{\sigma_0 \mathrm{MM}} = m^2_{\sigma_0} + (d-2) m^2_{\mathrm{M}}. \qquad (2\text{-}21)$$

In case a), it can be seen that it is impossible to obtain, the Kleinert-Weisz relation, starting from Eqs. (2-17)–(2-18).

3. PCDC

We shall, here, make some considerations about the PCDC hypothesis with the dilational current dominated by two poles. We shall separately analize cases a) and b) of last section.

Let us first consider case a), where σ_0 and σ_8 are, both, Goldstone bosons. We want to show that, if we assume

$$f_\pi - f_K \sim O(\lambda, \varepsilon) \tag{3-1}$$

and

$$F_{\pi\sigma_0} - F_{K\sigma_0} \sim O(\lambda, \varepsilon), \tag{3-2}$$

then PCDC will get in trouble (for case a)). First of all, we must show that the hypothesis made in (3-1)– (3-2) are plausible ones. Condition (3-1) is not obvious for the case of vacuum broken by $SU(3)$. We shall, however, show that this condition is not incompatible with a broken vacuum and, consequently, is compatible with the existence of the σ_8 dilaton as a Goldstone boson. To show that this is so, let us consider the expression

$$\langle 0 \,|\, \theta \,|\, \sigma_8 \rangle = -f_{\sigma_8} m_{\sigma_8}^2. \tag{3-3}$$

By using the virial theorem, we can write

$$\begin{aligned} \langle 0 \,|\, \theta \,|\, \sigma_8 \rangle &= \langle 0 \,|\, 4\delta \,|\, \sigma_8 \rangle + (4-d) \, \langle 0 \,|\, \varepsilon_0 u_0 + \varepsilon_8 u_8 \,|\, \sigma_8 \rangle \\ &= -f_{\sigma_8} m_{\sigma_8}^2. \end{aligned} \tag{3-4}$$

From (3-4), one can see that, if $m_{\sigma_8}^2 \longrightarrow 0$, then it must follow that $\varepsilon_0 \longrightarrow 0$ and, simultaneously, $\varepsilon_8 \longrightarrow 0$. This is so because the vacuum has an 8^{th}-component of the $SU(3)$ octet, and the matrix element $\langle 0 \,|\, u_0 \,|\, \sigma_8 \rangle$ can be different from zero.

Following standard calculations and using the PCAC hypothesis (only for simplifying the calculations), we obtain:

$$(f_\pi/f_K)^2 \, (m_\pi^2/m_K^2) = \frac{(\sqrt{2} + c) \, [\sqrt{2}\, \lambda_0 + \lambda_8]}{[\sqrt{2} - (c/2)] \, [\sqrt{2}\, \lambda_0 - (1/2)\lambda_8]}, \tag{3-5}$$

where

$$\lambda_i = \langle 0 \,|\, u_i \,|\, 0 \rangle \tag{3-6}$$

and

$$c = \varepsilon_8/\varepsilon_0. \tag{3-7}$$

As an illustration, let us consider the case where σ_8 is not a Goldstone boson. We can, then, perform the symmetry limit by letting, first, $\varepsilon_8 \longrightarrow 0$ and, after (not simultaneously), $\varepsilon_0 \longrightarrow 0$. The limit $\varepsilon_8 \longrightarrow 0$ will imply that

$$c \longrightarrow 0, \tag{3-8}$$

$$m_\pi^2 + m_K^2 \longrightarrow 0. \tag{3-9}$$

In this limit, Eq. (3-5) turns into

$$(f_\pi/f_K)^2 = \frac{\sqrt{2}\,\lambda_0 + \lambda_8}{\sqrt{2}\,\lambda_0 - (1/2)\,\lambda_8}$$

Yet, as σ_8 is not a Goldstone boson, we expect the vacuum to be invariant under $SU(3)$ and this means that $\lambda_8 = 0$. In this way, we obtain the familiar result

$$f_\pi/f_K = 1. \tag{3-11}$$

The same procedure cannot, however, be applied to the case when σ_8 is a Goldstone boson, just becairse of the simultaneous limit in ε_0 and ε_8, which we mentioned a while ago. The "simultaneous limit" can be performed in a somewhat arbitrary fashion. The essential point of it is that $c = \varepsilon_8/\varepsilon_0$ must not go to zero. ãhis allows us to have an $SU(3)$ broken vacuum (with $\lambda_8 \neq 0$) and, at the same time, to have the equality $f_\pi = f_K$.

A conclusion that we reach, from the above discussion, is that the assumption that $f_\pi - f_K \sim O(\lambda, \varepsilon)$ does not entail $\lambda_8 = 0$; in other words, such an assumption does not contradict the idea of two Goldstone bosons.

On the other hand, the similarity between f_π and f_K in the real world and, also, the search for maximum symmetry, motivated our acceptance of (3-1) as a working hypothesis.

As to Eq. (3-2), we expect, using group theoretic argurnents, ta have

$$F_{\sigma_0\pi} = \mathscr{K} d_{033} + O(\lambda, \varepsilon) \tag{3-12}$$

and

$$F_{\sigma_0 K} = \mathscr{K}\, d_{044} + O(\lambda, \varepsilon), \tag{3-13}$$

thus obtaining Eq. (3-2).

It we now take Eq. (2-17) for pions and kaons, and subtract one from the other, we obtain

$$f_{\sigma_0}[F_{\pi\sigma_0} - F_{K\sigma_0}] + f_{\sigma_8}[F_{\pi\sigma_8} - F_{K\sigma_8}] = f_\pi - f_K + O(\lambda, \varepsilon).$$

Using Eqs. (3-1) – (3-2), we conclude that

$$f_{\sigma_8}(F_{\pi\sigma_8} - F_{K\sigma_8}) \sim O(\lambda, \varepsilon), \tag{3-14}$$

and, therefore,

$$(F_{\pi\sigma_8} - F_{K\sigma_8}) \sim O(\lambda, \varepsilon). \tag{3-15}$$

From group theoretic arguments, we expect to have something like

$$F_{\pi\sigma_8} \sim d_{833}, \; F_{K\sigma_8} \sim d_{844}. \tag{3-16}$$

From Eqs. (3-15) – (3-16), we conclude that

$$F_{\pi\sigma_8} \sim O(\lambda, \varepsilon), \; F_{K\sigma_8} \sim O(\lambda, \varepsilon). \tag{3-17}$$

Once we know that, we can take Eq. (2-18) for pions and kaons. Subtracting one from the other, we get

$$\begin{aligned} &f_\pi[f_{\sigma_0} g_{\sigma_0\pi\pi} + f_{\sigma_8} g_{\sigma_8\pi\pi}] - f_K[f_{\sigma_0} g_{\sigma_0 KK} + f_{\sigma_8} g_{8KK}] \\ &- f_{\sigma_0} m_{\sigma_0}^2 [F_{\pi\sigma_0} - F_{K\sigma_0}] - f_{\sigma_8} m_{\sigma_8}^2 [F_{\pi\sigma_8} - F_{K\sigma_8}] \\ &\qquad = (d-2)\,[f_\pi m_\pi^2 - f_K m_K^2]. \end{aligned} \tag{3-18}$$

Using the results obtained so far, we can simplify Eq. (3-18) and write

$$\begin{aligned} &f_{\sigma_0}[g_{\sigma_0\pi\pi} - g_{\sigma_0 KK}] + f_{\sigma_8}\,[g_{\sigma_8\pi\pi} - g_{\sigma_8 KK}] \\ &\qquad = (d-2)\,(m_\pi^2 - m_K^2) + O(\lambda^2, \lambda\varepsilon, \varepsilon^2). \end{aligned} \tag{3-19}$$

In this equation, the term with $g_{\sigma_8 MM}$ may be of greater order in (λ, ε). This can be seen by considering Eq. (2-12) for σ_8 and taking $q^2 \longrightarrow 0$, the following relation being obtained:

$$-f_\pi\, g_{\sigma_8\pi\pi} + \vec{\partial}_{\pi\sigma_8}(0) = -\,(m_{\sigma_8}^2 - m_\pi^2)\, F_{\pi\sigma_8}(0). \tag{3-20}$$

If PCAC is exact, then $\vec{\partial}_{\sigma_8\pi\pi}(0) = 0$ and, consequently,

$$g_{\sigma_8\pi\pi} \sim O(\lambda^2, \; \lambda\varepsilon, \; \varepsilon^2). \tag{3-21}$$

In this case, Eq. (3-19) takes a simpler form.

Let us turn to the PCDC hypothesis. It can be summarized by the equation

$$\theta_\mu^\mu = \partial^\mu \mathscr{D}_\mu = f_{\sigma_0}\, m_{\sigma_0}^2\, \phi_{\sigma_0} + f_{\sigma_8}^2\, m_{\sigma_8}^2\, \phi_{\sigma_8}. \tag{3-22}$$

In this case we shall have

$$2m_\pi^2 = f_{\sigma_0}\, g_{\sigma_0\pi\pi} + f_{\sigma_8}\, g_{\sigma_8\pi\pi},$$
$$2m_K^2 = f_{\sigma_0}\, g_{\sigma_0 KK} + f_{\sigma_8}\, g_{\sigma_8 KK}, \qquad (3\text{-}23)$$

from which we can write

$$f_{\sigma_0}(g_{\sigma_0\pi\pi} - g_{\sigma_0 KK}) + f_{\sigma_8}(g_{\sigma_8\pi\pi} - g_{\sigma_8 KK}) = 2(m_\pi^2 - m_K^2). \qquad (3\text{-}24)$$

By comparing Eqs. (3-24) and (3-19), we conclude that $d = 4$, a result which cannot be accepted. The discussion presented here cannot be considered as raising a serious difficulty for the PCDC hypothesis with the dilational current density dominated by two Goldstone bosons, but it should be taken into account.

We discuss now PCDC for case b) of last Section, the case in which σ_0 is a Goldstone boson but σ_8 is not. This situation was studied in the paper of Eliezer and Dutt[2]. It is interesting to observe that, in that paper, the authors were forced to assume $d = 1$. This is due to the fact they used the PCAC hypothesis realized in an exact way. This is not, however, necessary, as we have already seen in the last Section [see, e.g., Eq. (2-21), which is the same as the one derived by Eliezer and Dutt].

Let us consider Eq. (2-21), assume PCDC [cf. Eqs. (3-22) and (3-23)] and make the reasonable assumption[2] that

$$g_{\sigma_8 M_i M_j} = g_8\, d_{8ij}. \qquad (3\text{-}25)$$

We obtain

$$m_{\sigma_0}^2 = (1/3)\,(2m_K^2 + m_\pi^2)\,(4 - d), \qquad (3\text{-}26)$$

which coincides with Eliezer's and Dutt's result, with an arbitrary d, however.

We return, now, to the analysis of Eq. (2-21). The dilaton σ_0 is, by hypothesis, an $SU(3)$ singlet. This suggests that the simplest form for the $g_{\sigma_0 M_i M_j}$ coupling, in terms of $SU(3)$ parameters, will be

$$g_{\sigma_0 M_i M_j} = g_0 \delta_{ij} \qquad (3\text{-}27)$$

This relation, however, is incompatible with Eq. (2-21) for $d \neq 2$. This fact forces us to accept an explicit breaking term in the $\sigma_0 M_i M_j$ coupling. Then, instead of (3-27), we must have

$$g_{\sigma_0 M_i M_j} = g_0 \delta_{ij} + h\, d_{8ij}. \qquad (3\text{-}28)$$

This hypothesis is accepted by Eliezer and Dutt. Let us first show that (3-28) is a plausible hypothesis. In spite of the fact that $SU(3)$ is realized a *la* Wigner, it is still an explicitly broken symmetry. We can, therefore, expect that the pure $SU(3)$ parametrization will not be directly applicable to the case of the coupling constant. Moreover, as we are treating here the case where σ_8 is not a Goldstone boson, we can take $\varepsilon_8 \longrightarrow 0$, keeping $\varepsilon_0 = \text{const.}$. In this case, if $h \sim O(\varepsilon_8)$, Eq. (3-28) will turn into (3-27). On the other hand, we know that foi- this kind of limit, the octet of pseudoscalar mesons displays the same mass. Eq. (2-21) becomes then compatible with (3-28), for any d.

Our objection to (3-28) is related to the order of magnitude of the breaking term. We would like it to be of greater an order in (λ, ε), our wish being motivated by the spurion mechanism of the breaking.

There is yet one more problem related to the difficulty in explaining, physically, the mechanism of the dependence of the parameter h [cf. Eq. (3-28)] on the dimension d. We, however, postpone the discussiori of the problem to next Section, where we shall consider an alternativí: physical hypothesis that retains the basic characteristics of PCDC, leads to analogous results and does not need the assumption of Eq. (3-28).

4. Dominance of the Dilaton σ in Processes Involving the Scalar Current u_0

We assume now that instead of two dilatons (σ_0 and σ_8), there is only one, we call it σ, which is relevant for the pole part (singular part) of the matrix elements. The σ dilaton may be a pure $SU(3)$ singlet or a mixture of an $SU(3)$ singlet and the eighth component of an $SU(3)$ octet. As a dynamical hypothesis, we assume that u_0 acts only through the dilaton σ, in this region of energy. We also assume that u_8 couples to σ, but without exclusiveness. The matrix elements which involve u_8 will, therefore, contain the pole terms and regular terms as well, which we suppose $SU(3)$ parametrizable. The same does not, however, happen with the matrix elements which involve the current u_0 which will contain only terms due to the σ pole.

The parametrization of regular and connected matrix elements is given by

$$[\langle M_k \mid u_i \mid M_j \rangle^{REG}]^{CONN} = \beta\, d_{ijk}, \tag{4-1}$$

for $i, j, k = 1, \ldots, 8$.

To simplify the discussions that follow, we introduce a few linguistic conventions. We shall speak of the *dominant aspect* of some field when this field acts, with exclusiveness, in some process. So, for PCDC, the dominant aspect of σ_0 is that it couples to u_0 so that the field ϕ_{σ_0} can be directly related to u_0 through some convenient constant. In an analogous way, we can speak about σ_8.

When making the hypothesis of *dominance*, the dominant aspect of the o dilaton is similar to that of the σ_0 for the PCDC hypothesis. All processes, in which the current u_0 participates, can be described by the field of the σ dilaton. The difference here is that we cannot identify u_0 with the field of o, because ϕ_σ has more degrees of freedom than u_0, a consequence of its $SU(3)$ transformation properties.

As to the u_8 current, this current is not, strictly speaking, dominated by any field, which was the case in PCDC. u_8 can also couple to the G dilaton. We, besides, believe that there exist other intermediate states that can couple to this current. We expect to meet, among them, a second dilaton which is another independent linear combination of the non physical σ_0 and σ_8 constituents from which our physical o dilaton is composed.

In writing the matrix elements, we include, in the $f_\sigma g_{\sigma M_i M_j}$ term, the contributions of o to the currents u_0 and u_8. The part which is due to other intermediat states is described by a regular term that is expressed by $SU(3)$ parameters (cf., e.g., Eq. (4-I)]. For $d = 2$, the dilaton is an $SU(3)$ singlet because of the Kleinert-Weisz relation, Eq. (2-21). In this case, we can identify our model with the PCDG hypothesis in which the second particle (not a Goldstone boson) is represented by an effective field that describes all intermediate states that couple to u_8. Such an effective field will not be of the Coidstone type.

For $d \neq 2$, our model will not be so similar to PCDC, but the results will still be the same. To show this, we consider Eq. (4-1). Proceeding in the spirit of the *dominance* model, we obtain

$$2m_\pi^2 = f_\sigma g_{\sigma\pi\pi} + (4-d)\ (1/\sqrt{3})\, \varepsilon_8 \beta, \tag{4-2}$$

$$2m_K^2 = f_\sigma g_{\sigma KK} - (4-d)\ (1/2\sqrt{3})\, \varepsilon_8 \beta. \tag{4-3}$$

These equations, together with the Kleinert-Weisz relation, (2-15), written for pions and kaons, give

$$m_\sigma^2 = (1/3)\,(2m_K^2 + m_\pi^2)\,(4-d). \tag{4-4}$$

The numerical values for the mass of the o dilaton are:

$$m_\sigma = \begin{cases} 717 \text{ MeV, for } d = 1, \\ 585 \text{ MeV, for } d = 2, \\ 414 \text{ MeV, for } d = 3. \end{cases}$$

With the dominance hypothesis, the following widths are obtained:

for $c = -1.25$ and $f_K/f_\pi = 1$,

$$\Gamma_\sigma = \begin{cases} 653 \text{ MeV, for } d = 1, \\ 363 \text{ MeV, for } d = 2, \\ 135 \text{ MeV, for } d = 3; \end{cases}$$

and, for $c = -1.29$, $f_K/f_\pi = 1.28$,

$$\Gamma_\sigma = \begin{cases} 427 \text{ MeV, for } d = 1, \\ 239 \text{ MeV, for } d = 2, \\ 88 \text{ MeV, for } d = 3. \end{cases}$$

Since, experimentally, there is no observed resonance with masses close to those predicted by our formula, **Eq.** (4-4) (experimentally, one observes resonances near 1BeV), we prefer to interpret the o dilatoni as an effective particle that describes an average interaction in the energy region considered.

It is interesting to observe that Le Guillou, Morel and Navelet[4] has found, for the dilaton, the values

$$m_\sigma = 420 \text{ MeV}, \ \Gamma_\sigma = 380 \text{ MeV},$$

values which are consistent with the calculations of the model dependent s-wave amplitudes, of $\pi - \pi$ scattering, based on analyticity, unitarity and crossing. The above values are slightly dependent on the variation of the parameters involved. The phase shift does noi. pass through 90° but over a nearby value.

We would, of course, be interested on the possibility of formulating some different kind of PCDC, with two poles, which would avoid the: necessity of introducing the explicit breaking in the coupling constant, **Eq.** (3-28). This is, however, an impossible task. It is necessary, to avoid **Eq.** (3-28), to propose some sort of PCDC, with two particles, only one of then being a Goldstone boson. The first would be a mixture of an $SU(3)$ singlet and an eighth component of an $SU(3)$ octet. The existence of such a Goldstone boson would imply the existence of another Goldstone boson, orthogonal to the first one, which, however, cannot be the second relevant particle in PCDC. We shall, therefore, go to a theory with there particles.

Before finishing our paper, let us return to the problem raised in Sec. **3,** which concerns the physical meaning of the breaking term in the $\sigma_{0M_iM_j}$ coupling. In PCDC, we must admit, in order to have consistency between (3-21) and (3-28), that h [see (3-28)] depends strongly on d, with the condition $h = 0$ for $d = 2$.

None of the fundamental physical properties of this model change explicitly with the dimension d. The group-theoretic nature of σ_0 and of the vacuum stays the same, with some variation in d. In order to explain the breaking mechanism in the coupling constants it will, then, be necessary to imagine a rather involved mechanism.

In the dominance model, however, the properties of the dilaton depend directly on the parameter d. Then, for $d = 2$, we deduce, using the Kleinert-Weisz relation that o must be a singlet. From the same relation, we obtain an invariant coupling constant and so on, for other dimensions.

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