# Recombination Coefficients for Complex Ions of Na, $\mathrm{Al}, \mathbf{P}, \mathrm{Cl}, \mathrm{Ar}$ and Ca 

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Radiative and dielectronic recombination coefficients are calculated for all non-hydrogenic ions of $\mathrm{Na}, \mathrm{Al}, \mathrm{P}, \mathrm{Cl}$ and Ar and also for CaI and CaII . The results for the radiative recombination coefficients are given in the range of temperature $30^{\circ} \mathrm{K} \leq T_{\mathrm{c}} \leq 10^{8}{ }^{\prime \prime} \mathrm{K}$. The dielectronic recombination coefficients are presented in a four-parameter form.

Calculam-se os coeficientes de recombinação radiativa e dieletrônica para os íons, com mais de um eletron, dos seguintes elementos: $\mathrm{Na}, \mathrm{Al}, \mathrm{P}, \mathrm{Cl}, \mathrm{Ar}$, como também para CaI e CaII. Apresentam-se resultados para os coeficientes de recombinação radiativa no intervalo de temperaturas $30^{\circ} \mathrm{K} \leq T_{\mathrm{e}} \leq 10^{8}{ }^{\circ} \mathrm{K}$.

## 1. Introduction

New observational techniques by rockets and satelites increased considerably the spectral range accessible to astronomers. As a consequence, spectral lines of low abundance elements are more and more observed and will be intensively used in the future to test theoretical models. For example, the ultraviolet observations from Copernicus satelite ${ }^{1,2}$ seem to indicate that abundances of heavy elements in the interstellar medium are less than in the sun and give information about the degree of ionization of the interstellar gas.

The knowledge of recombination rates for many ions of heavy elements is, therefore, necessary to theoretical calculations. The radiative and dielectronic recombination coefficients have been published previously ${ }^{3}$ (Paper I) for all non-hydrogenic ions of the most abundant elements, namely, $\mathrm{He}, \mathrm{C}, \mathrm{N}, \mathrm{O}, \mathrm{Ne}, \mathrm{Mg}, \mathrm{Si}$ and S. In this paper, we give the same information for all ions of $\mathrm{Na}, \mathrm{Al}, \mathrm{P}, \mathrm{Cl}$ and Ar and for CaI and CaII as well.

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## 2. Basic Formulas

In a gas, of electron-number density $N_{\mathrm{e}}$ and temperature $T_{\mathrm{e}}$, the number, $\mathscr{N}$, of recombinations per second per cubic centimeter, of an ion $\mathrm{X}_{i+1}$ of an element X , is

$$
\begin{equation*}
\mathscr{N}=N_{\mathrm{e}} N\left(\mathrm{X}_{i}+1\right) \alpha\left(\mathrm{X}_{i}\right), \tag{1}
\end{equation*}
$$

where $N\left(\mathrm{X}_{i+1}\right)$ is the number density of $\mathrm{X}_{i+1}$ and, $\alpha\left(\mathrm{X}_{i}\right)$, the recombination coefficient given by

$$
\begin{equation*}
\alpha\left(\mathrm{X}_{i}\right)=\alpha_{\mathrm{rad}}\left(\mathrm{X}_{i}\right)+\alpha_{\mathrm{di}}\left(\mathrm{X}_{i}\right), \tag{2}
\end{equation*}
$$

$\alpha_{\mathrm{rad}}\left(\mathrm{X}_{i}\right)$ and $\alpha_{\mathrm{di}}\left(\mathrm{X}_{i}\right)$ being the radiative and dielectronic recombination coefficients, respectively.

## 2a. Radiative Recombination Coefficient

The radiative recombination coefficient can be written as

$$
\begin{equation*}
\alpha_{\mathrm{rad}}\left(\mathrm{X}_{i}\right)=\alpha_{n_{v}}\left(\mathrm{X}_{i}\right)+\sum_{n>n_{v}} \alpha_{n}\left(\mathrm{X}_{i}\right), \tag{3}
\end{equation*}
$$

where $\alpha_{n}\left(\mathrm{X}_{i}\right)$ is the radiative recombination coefticient to level n and, n , the principal quantum number of the grourid state of $\mathrm{X}_{i}$.

Since the excited states, with increasing $n$, approach the hydrogenic states, the $\alpha_{n}\left(\mathrm{X}_{i}\right)$, for $\mathrm{n}>\mathrm{n}$, are calculated in the hydrogen-like approximation ${ }^{4}$.

For all ions, for which photoionization cross-sections are available, the radiative recombination coefficients to the ground state is computed according to the detailed balance formula ${ }^{4}$

$$
\begin{align*}
\alpha_{n_{v}}\left(\mathrm{X}_{i}\right)= & (2 / \pi)^{1 / 2} C^{-2}\left(m k T_{\mathrm{e}}\right)^{-3 / 2} g_{i} / g_{i+1} \\
& \exp \left[I\left(\mathrm{X}_{i}\right) / k T_{\mathrm{e}}\right] \int_{I\left(\mathrm{X}_{\mathrm{i}}\right)}^{\infty} E^{2} a_{E}\left(X_{i}\right) \exp \left(-E / k T_{\mathrm{e}}\right) d E, \tag{4}
\end{align*}
$$

where $g_{i}$ and $g_{i+1}$ are the statistical weights of $X_{i}$ and $X_{i+1}, I\left(\mathrm{X}_{i}\right)$ the ionization potential of the ion $\mathrm{X}_{i}$ (Ref. 5) and, $a_{E}\left(\mathrm{X}_{i}\right)$, the photoionization cross-section of the $X_{i}$ ground state.

The photoionization cross-sections are taken from: (a) Brown ${ }^{6}$ for NaI, CaI and CaII, (b) Brown ${ }^{7}$ for He-like ions, (c:) Seaton ${ }^{8}$ and Henry ${ }^{9}$
for $2 p^{q}$ configurations, (d) Flower ${ }^{10}$ for $2 s$ and $2 s^{2}$ configurations, (e) Chapman and Henry ${ }^{11,12}$ for $3 p^{q}$ configurations. For ions of the MgI isoelectronic sequence, the He-like approximation is assumed. For ions with a $3 s$ configuration, a $E^{-3}$ dependence of the photoionization cross-section is taken and the threshold value is obtained by extrapolation parallel to 2 s sequence. For PI, II, III and, ClI to ClV, no information is available and it was not possible to obtain the cross-sections by interpolation, so the H -like approximation was assumed. The ions for which the photoionization cross-section was obtained, by extrapolation along isoelectronic sequence, are indicated in Table 1 by (*).

## 2b. Dielectronic Recombination Coefficient

The dielectronic recombination coefficient is calculated by the formula given by Burgess ${ }^{13}$

$$
\begin{align*}
& \alpha_{\mathrm{di}}\left(\mathrm{X}_{i}\right)=3.03 \times 10^{-3} T_{\mathrm{e}}^{-3 / 2} B(z) \sum_{k} f_{0 k \times} \\
& \quad \times A(x) \exp \left\{-1.44 \times 10^{8}\left(T_{\mathrm{e}} \lambda_{0 k}\right)^{-1}\left[1+0.015 z^{3} /(z+1)^{2}\right]^{-1}\right\}, \tag{5}
\end{align*}
$$

where

$$
\begin{align*}
B(z) & =\left\{z(z+1)^{5} /\left(z^{2}+13.4\right)\right\}^{1 / 2} \\
A(x) & =x^{1 / 2}\left(1+0.105 x+0.015 x^{2}\right)^{-1}, \\
x & =9.133 \times 10^{2}(z+1)^{-1} \lambda_{0 k}^{-1}, \tag{6}
\end{align*}
$$

z being the effective charge of the recombining ion $\mathrm{X}_{i+1}, f_{o k}$ the oscillator strength of the transition between level k and ground state, and $\lambda_{0 k}$ the wavelength (in $\AA$ ) for the same transition. These quantities are taken from Wiese et $a l^{14}$.

It must be noted that Eq. (5)is valid a) at high temperatures ( $T_{\mathrm{e}} \gtrsim 10^{4} \mathrm{~K}$ ), but the dielectronic recombination coefficients are negligible $\left(\alpha_{d i} i\left(X_{i}\right)<\right.$ $<0.1 \alpha_{\text {rad }}\left(\mathrm{X}_{i}\right)$ )for $T_{\mathrm{e}}<10^{4}{ }^{\circ} \mathrm{K} ;$ b) at low particle density, $N_{\mathrm{e}} \lesssim 10^{8} \mathrm{~cm}^{-3}$ (Ref. 15).

For ions of the HeI sequence, the values are corrected by a factor given by Shore ${ }^{16}$.

## 3. Results

Values of the radiative recombination coefficients in the range $30^{\prime \prime} \leq$ $\leq T_{\mathrm{e}} \leq 10^{8} \mathrm{~K}$ are given in Table 1, for ions of $\mathrm{Na}, \mathrm{Al}, \mathrm{P}, \mathrm{Cl}, \mathrm{Ar}$ and Ca .
 PHOSPHORUS







 Table 1. Radiative Recombination Coefficients
 ARGON












$\begin{array}{llll}1 & .110 E=10 & .505 E-11 \\ 2 & .537 E=10 & .255 E=10 & .25=11 \\ & & \end{array}$ Table 1. (Continwed)

In Paper I, the results were fitted to an analytical expression but we think more useful to give values at different ternperatures in a larger range.

The radiative recombination coefficients to the ground state are, in general, a factor $0.01-0.3$ less than the total ones in that range of temperature, but, if we consider the hydrogenic approximation for the ground state, the factor is of the order of $0.2-0.8$ in the same range of temperatures.

For the dielectronic recombination coefficients, a four-term parametric fit is adopted (see Paper I):

$$
\begin{equation*}
\alpha_{\mathrm{di}}\left(\mathrm{X}_{i}\right)=A_{\mathrm{di}} T_{\mathrm{e}}^{-3 / 2} \exp \left(-T_{\mathrm{o}} / T_{\mathrm{e}}\right)\left[1+B_{\mathrm{di}} \exp \left(T_{1} / T_{\mathrm{e}}\right)\right] . \tag{7}
\end{equation*}
$$

The parameters $A_{\mathrm{d}}, B_{\mathrm{d}}, T_{0}$ and $T_{1}$ are given in Table 2, as well as $T_{\text {crit }}$ defined as the temperature above which the dielectronic recombinations are not negligible. For $T_{\mathrm{e}} \gtrsim T_{\text {crit }}, \alpha_{\mathrm{di}}\left(\mathrm{X}_{i}\right) \gtrsim 0.2 \alpha_{\mathrm{rad}}\left(\mathrm{X}_{i}\right)$. For PI, $T_{\text {crit }}$ corresponds to $\alpha_{d i} \simeq 0.1 \alpha_{\text {rad }}$.

The numerical results are fitted to better than $10 \%$, except for ions of CI and NI isoelectronic sequences, for which the accuracy is better than $15 \%$ and $25 \%$, respectively.

We take the opportunity to correct a misprint in Table 1 of Paper I: $T_{n}$ for CII is 1.5 (5) instead of 1.5 (4).

| ion | $T_{\text {crii }}$ | $A_{\mathrm{di}}$ | $T_{0}$ | $B_{\mathrm{di}}$ | $T_{\mathrm{i}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| NaI | $4.0(4)$ | $1.0(-3)$ | $3.6(5)$ | 0.0 |  |
| NaII | $5.0(4)$ | $2.6(-3)$ | $3.8(5)$ | 2.0 | $1.3(5)$ |
| NaIII | $4.4(4)$ | $6.0(-3)$ | $3.4(5)$ | 1.6 | $4.4(5)$ |
| NaIV | $4.0(4)$ | $1.1(-2)$ | $3.0(5)$ | 2.0 | $7.2(4)$ |
| NaV | $4.0(4)$ | $8.0(-3)$ | $2.7(5)$ | 5.8 | $2.8(5)$ |
| NaVI | $4.0(4)$ | $1.0(-2)$ | $2.8(5)$ | 5.2 | $4.0(5)$ |
| NaVII | $4.4(4)$ | $3.2(-2)$ | $3.2(5)$ | 2.8 | $1.3(6)$ |
| NaVIII | $3.0(4)$ | $1.2(-2)$ | $1.9(5)$ | 7.2 | $1.6(6)$ |
| NaIX | $1.9(6)$ | $2.2(-1)$ | $1.2(7)$ | 0.2 | $1.9(6)$ |
| NaX | $3.8(6)$ | $1.8(-1)$ | $1.3(7)$ | 0.2 | $3.0(6)$ |
|  |  |  |  |  |  |
| AlI | $8.0(3)$ | $5.5(-3)$ | $9.0(4)$ | 0.0 |  |
| AlII | $7.0(3)$ | $6.0(-3)$ | $7.5(4)$ | 0.0 |  |
| AlIII | $1.2(5)$ | $7.5(-3)$ | $8.5(5)$ | 0.0 |  |
| AlIV | $7.0(4)$ | $5.6(-3)$ | $5.0(5)$ | 5.0 | $7.0(5)$ |
| AIV | $6.5(4)$ | $1.2(-2)$ | $4.4(5)$ | 3.8 | $1.0(6)$ |

Tahle 2

| ion | $T_{\text {crit }}$ | $A_{\text {di }}$ | $T_{0}$ | $B_{\text {di }}$ | $T_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AlVI | 5.5 (4) | $1.9(-2)$ | 3.8 (5) | 4.8 | 1.3 (6) |
| AlVII | 5.5 (4) | $1.6(-2)$ | 3.4 (5) | 6.3 | 6.0 (5) |
| AlVIII | 5.5 (4) | $1.7(-2)$ | 3.4 (5) | 5.4 | 7.6 (5) |
| AlIX | 5.8 (4) | 4.4 (-2) | 3.9 (5) | 3.6 | 2.0 (6) |
| AlX | 4.0 (4) | $1.7(-2)$ | 2.3 (5) | 9.5 | 2.3 (6) |
| AIXI | 2.9 (6) | $3.0(-1)$ | 1.6 (7) | 0.2 | 2.6 (6) |
| AlXII | 5.5 (6) | $3.4(-0)$ | 1.7 (7) | 0.2 | 3.8 (6) |
| PI | 2.0 (4) | 6.8 (-6) | 9.3 (4) | 0.0 |  |
| PII | 1.3 (4) | 8.0 (-3) | 1.2 (5) | 0.0 |  |
| PIII | 1.3 (4) | $2.5(-2)$ | 1.5 (5) | 0.0 |  |
| PIV | 1.3 (4) | 1.8 (-2) | 1.3 (5) | 0.0 |  |
| PV | 2.2 (5) | 2.1 (-2) | 1.5 (6) | 0.0 |  |
| PVI | 1.1 (5) | $1.0(-2)$ | 6.0 (5) | 15. | 1.4 (6) |
| PVII | 9.5 (4) | $1.9(-2)$ | 5.4 (5) | 5.8 | 2.2 (6) |
| PVIII | 8.0 (4) | 2.8 (-2) | 4.8 (5) | 9.5 | 2.0 (6) |
| PIX | 8.0 (4) | $2.7(-2)$ | 4.1 (5) | 6.5 | 1.0 (6) |
| PX | 8.0 (4) | $2.6(-2)$ | 4.0 (5) | 5.5 | 1.2 (6) |
| PXI | 7.5 (4) | $5.5(-2)$ | 4.6 (5) | 4.4 | 2.9 (6) |
| PXII | 5.3 (4) | 2.1 (-2) | 2.8 (5) | 11. | 3.0 (6) |
| PXIII | 4.0 (6) | 3.8 (-1) | 2.2 (7) | 0.2 | 3.6 (6) |
| PXIV | 8.0 (6) | 6.4 ( 0) | 2.3 (7) | 0.2 | 5.4 (6) |
| ClI | 1.5 (4) | $5.5(-5)$ | 1.3 (5) | 0.0 |  |
| CIII | 1.5 (4) | $1.0(-2)$ | 1.4 (5) | 0.0 |  |
| CIIII | 1.5 (4) | $1.1(-2)$ | 1.4 (5) | 0.0 |  |
| CIIV | 1.6 (4) | $1.0(-2)$ | 1.4 (5) | 0.0 |  |
| CIV | 1.6 (4) | $5.0(-2)$ | 2.0 (5) | 0.0 |  |
| ClVI | 1.9 (4) | $3.2(-2)$ | 1.8 (5) | 0.0 |  |
| ClVII | 4.0 (5) | 3.4 (-2) | 2.3 (6) | 0.0 |  |
| CIVIII | 1.4 (5) | $1.6(-2)$ | 7.2 (5) | 30. | 2.2 (6) |
| CIIX | 1.3 (5) | 2.4 (-2) | 6.4 (5) | 7.8 | 2.5 (6) |
| CIX | 1.2 (5) | $4.0(-2)$ | 6.0 (5) | 15. | 2.7 (6) |
| CIXI | 1.1 (5) | $4.0(-2)$ | 4.9 (5) | 6.6 | 1.5 (6) |
| CIXII | 1.1 (5) | $3.8(-2)$ | 4.6 (5) | 5.5 | 1.8 (6) |
| CIXIII | 1.0 (5) | $6.8(-2)$ | 5.3 (5) | 5.0 | 3.7 (6) |
| CIXIV | 6.7 (4) | 2.6 (-2) | 3.2 (5) | 14. | 3.8 (6) |
| CIXV | 5.5 (6) | 4.6 (-1) | 2.6 (7) | 0.2 | 5.0 (6) |
| CIXVI | 1.2 (7) | 11. ( 0) | 2.8 (7) | 0.2 | 6.5 (6) |
| ArI | 2.5 (4) | 7.6 (-4) | 2.0 (5) | 0.0 |  |
| ArII | 3.0 (4) | 3.0 (-4) | 1.6 (5) | 0.0 |  |
| ArIII | 2.5 (4) | $1.7(-3)$ | 1.7 (5) | 0.0 |  |
| ArIV | 2.5 (4) | $1.5(-2)$ | 1.7 (5) | 0.0 |  |
| ArV | 1.8 (4) | $1.2(-2)$ | 1.6 (5) | 0.0 |  |
| ArVI | 1.8 (4) | $6.5(-2)$ | 2.3 (5) | 0.0 |  |
| ArVII | 2.2 (4) | $4.0(-2)$ | 2.1 (5) | 0.0 |  |

Table 2 (continued)

| ion | $T_{\text {crit }}$ | $A_{\mathrm{di}}$ | $T_{0}$ | $B_{\mathrm{di}}$ | $T_{1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| ArVIII | $5.0(5)$ | $6.0(-2)$ | $2.8(6)$ | 0.0 |  |
| ArIX | $1.6(5)$ | $2.0(-2)$ | $7.7(5)$ | 40. | $2.5(6)$ |
| ArX | $1.5(5)$ | $2.6(-2)$ | $6.8(5)$ | 8.6 | $3.0(6)$ |
| ArXI | $1.5(5)$ | $4.6(-2)$ | $6.6(5)$ | 20. | $3.1(6)$ |
| ArXII | $1.3(5)$ | $5.0(-2)$ | $5.4(5)$ | 6.6 | $1.8(6)$ |
| ArXIII | $1.3(5)$ | $4.4(-2)$ | $4.8(5)$ | 5.5 | $2.1(6)$ |
| ArXIV | $1.1(5)$ | $7.5(-2)$ | $6.0(5)$ | 5.2 | $3.2(6)$ |
| ArXV | $7.6(4)$ | $2.8(-2)$ | $3.4(5)$ | 15. | $4.1(6)$ |
| ArXVI | $6.5(6)$ | $5.5(-1)$ | $3.0(7)$ | 0.2 | $6.0(6)$ |
| ArXVII | $1.4(7)$ | $14 .(0)$ | $3.2(7)$ | 0.2 | $7.8(6)$ |
| CaI | $2.0(3)$ | $1.6(-3)$ | $3.6(4)$ | 0.0 |  |

Table 2 (continued)

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