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Deep Inelastic Electron-Nucleon Scattering in a Nonrelativistic Symmetric Quark Model

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Deep inelastic electron-nucleon scattering is studied in a resonance model derived from a symmetric quark model in the framework of non-relativistic dynamics. The coupling electromagnetic current-bound quarks leads to a cross section for inelastic electron-nucleon scattering which is similar to the one obtained by parton scattering.

Estuda-se o espalhamento inelástico eletron-nucleo na região inelástica profunda num modelo ressonante em um modelo quarks simétrico dentro de uma dinâmica não relativística. O acoplamento (corrente eletromagnética)-(quarks ligados) leva a uma seção de choque para o espalhamento inelástico eletron-nucleo semelhante àquela obtida no espalhamento por partons.

1. Introduction

We discuss deep inelastic electron-nucleon scattering in the framework of a non-relativistic, symmetric, Galilei invariant quark model[1]. In the quark model, the nucleon is a strong bound state of three quarks with spin 1/2. The electron is now scattered by bound quarks and not by free particles as in the parton model[2]. They cannot be ejected but they make transitions to excited states. We assume that the hadronic final state is completely expressible as a superposition of resonant states.

In the SU(6) symmetry scheme, the binding forces are independent of spin and F-spin. To construct the wave functions of the three spin 1/2 quarks with oscillator interactions and definite symmetry, we use a group theoretical method developed by R. F. Meyer[3].

The explicitly calculated form-factor of our oscillator model does not satisfy universality assumptions[4] and, therefore, we do not have approximate scaling for finite momentum and energy transfers, q^2 and ν. In the Bjorken limit, however, our structure functions approach a scaling function

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representing a quasielastic invariant peak. The Gottfield[5] and Callan-Gross[6] sum-rules hold in the Gell-Mann-Zweig quark model[7].

In Section 2, we construct the symmetric wave fimctions. The current matrix elements are calculated in Section 3 and the structure functions in Section 4 .

2. Symmetric Wave Functions

To calculate the total symmetric wave functions, it is advantageous to use a convention for the description of the permutation symmetry of the baryon states constructed from the three quark scates. We use the following standard forms for the irreducible represeritations S„ *A,* and M_i of the permutation group S_3:

S3: $(e = (1),\ a = (123),\ a^2 = (132),\ b = (12),\ ba = (23),\ ba^2 = (13)\}$,
S,: $e = 1,\ a = 1,\ b = 1$,
A,: $e = 1,\ a = 1,\ b = -1$,

$$M_i:\ e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ a = \begin{pmatrix} \varepsilon & 0 \\ 0 & \bar{\varepsilon} \end{pmatrix},\ b = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \varepsilon = \exp\left[i\,\frac{2}{3}\,\pi\right],\ \bar{\varepsilon} = \varepsilon^*.$$

Wave functions which transform according to these representations are symmetric, antisymmetric or of mixed symmetry, respectively.

To construct the radial wave functions of three quarks with the correct permutation symmetry, we use the following internal coordinates

$$\bar{\mathbf{Z}} = \sqrt{2/3}\ (\bar{\varepsilon}\,\mathbf{r}_1 + \varepsilon\,\mathbf{r}_2 + \mathbf{r}_3), \qquad \mathbf{Z} = \sqrt{2/3}\ (\varepsilon\,\mathbf{r}_1 + \bar{\varepsilon}\,\mathbf{r}_2 + \mathbf{r}_3), \quad (1)$$

which transform according to the components of the mixed representation, i.e.

$$(123)\ \bar{\mathbf{Z}} = \varepsilon\,\bar{\mathbf{Z}}, \qquad (123)\ \mathbf{Z} = \bar{\varepsilon}\,\mathbf{Z}, \qquad (12)\ \bar{\mathbf{Z}} = \mathbf{Z}. \qquad (2)$$

The symmetric combination describes the center of mass coordinates

$$\mathbf{X} = \sqrt{1/3}\ (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3). \qquad (3)$$

In our resonance model of inelastic scattering, we sum over all quantum numbers of the intermediate states and we do not need wave functions that are eigenvalues of angular momentum. For that reason, it is suitable to make the following coordinate transformation

$$\bar{Z}_i = \rho_i \exp i\varphi_i \tag{4}$$

and express the Hamiltonian operator as a sum of three harmonic oscillators in two dimensions

$$H = \frac{1}{6m} \mathbf{P}^2 + \frac{1}{2m} \sum_{i=1}^{3} \left[\frac{\partial^2}{\partial \rho_i^2} + \frac{1}{\rho_i} \frac{\partial}{\partial \rho_i} + \frac{1}{\rho_i^2} \frac{\partial^2}{\partial \varphi_i^2} + \frac{\kappa}{2} \rho_i^2 \right]. \tag{5}$$

The solutions of the harmonic oscillator in two dimensions are

$$\Psi_{ki,\, mi}(\rho_i, \varphi_i) = (\beta/\pi)^{1/2} \Lambda_{ki}^{|mi|}(\beta\rho_i^2) \exp[i m_i \varphi_i],$$

$$\Lambda_{ki}^{|mi|}(\beta\rho_i^2) = N_{ki}^{|mi|} [\beta \rho_i^2]^{|mi|/2} L_{ki}^{|mi|}(\beta \rho_i^2) \exp[-(1/2)\beta \rho_i^2],$$

$$N_{ki}^{|mi|} = \left[\Gamma(|m_i| + 1) \binom{k_i + |m_i|}{k_i} \right]^{-1/2}. \tag{6}$$

The total wave functions are

$$\Psi_{k,\, m}(\rho, \varphi) = (\beta/\pi)^{-3/2} \prod_{i=1}^{3} \Lambda_{ki}^{|mi|}(\beta \rho_i^2) \exp[i m_i \varphi_i]. \tag{7}$$

Since, in the coordinates (ρ, φ), particle permutations change only the phase φ,

$$(123)\varphi = \varphi + \frac{2}{3}\pi, \quad (12)\varphi = -\varphi, \tag{8}$$

we obtain eigenfunctions with defmite symmetry by means of some special linear combinations of the phase factors. The radial wave functions with defmite symmetry are obtained if we apply to Eq. (7) the projection operator of S_3 (Ref. 3):

$$\mathscr{P}^{s} = (1/6) \sum_{\sigma} T(\sigma),$$

$$\mathscr{P}^{A_n} = (1/6) \sum_{\sigma} \varepsilon(\sigma)\, T(\sigma), \tag{9}$$

$$\mathscr{P}^{\bar{M}_i} = (1/3)\, [T(e) + \bar{\varepsilon}\, T(a) + \varepsilon T(a^{-1})],$$
$$\mathscr{P}^{\underline{M}_i} = (1/3)\, [T(e) + \varepsilon T(a) + \bar{\varepsilon}\, T(a^{-1})],$$

where $\sigma \in S_3$, $T(\sigma)$ is a representation, and $\varepsilon(\sigma)$ is a permutation sign. The radial wave functions are in Table I.

$\left[\frac{\Omega}{\pi}\right]^{3/2} \prod_{i=1}^{3} \Lambda_{ki}^{\lvert m_i \rvert} (\Omega \rho_i^2)$			
$m = \sum_{i=1}^{3} m_i$	$m \equiv 0$ (mod. 3)	$m \equiv 2$ (mod. 3)	$m \equiv 1$ (mod. 3)
$\Psi^S_{k,m}$	$\frac{1}{2}[\exp(im.\varphi) + \exp(-im.\varphi)]$	0	0
$\Psi^A_{k,m}$	$\frac{1}{2}[\exp(im.\varphi) - \exp(-im.\varphi)]$	0	0
$\Psi^{\overline{M}}_{k,m}$	0	$\exp(im.\varphi)$	$\exp(-im.\varphi)$
$\Psi^{M}_{k,m}$	0	$\exp(-im.\varphi)$	$\exp(im.\varphi)$

Table I. Radial Wave Functions.

In the same way, we can obtain the spin wave functions: the symmetric $\left|\begin{matrix}3/2\\ S_3\end{matrix}\right\rangle$ and the two components of mixed symmetry, $\left|\begin{matrix}\overline{1/2}\\ S_3\end{matrix}\right\rangle$, $\left|\begin{matrix}1/2\\ \underline{S_3}\end{matrix}\right\rangle$. And also the $SU(3)$ wave functions: the symmetric $\left|\begin{matrix}10\\ B_{10}\end{matrix}\right\rangle$, the mixed symmetric, $\left|\begin{matrix}\overline{8}\\ B_8\end{matrix}\right\rangle$, $\left|\begin{matrix}8\\ B_8\end{matrix}\right\rangle$, and the antisymmetric $\left|\begin{matrix}1\\ B_1\end{matrix}\right\rangle$.

In the following, we have to combine spin wave functions with $SU(3)$ wave functions to produce $SU(6)$ wave functions with definite symmetry, and combine the latter with the radial functions. For this purpose, the Clebsh-Gordan rules for the reduction of the direct products of S_3 representations are useful and the total symmetric wave functions are exhibited in Table II.

3. The Current Matrix Elements

The electromagnetic current of pointlike particles, in the Schrodinger picture, at $t = 0$, is

$$j_0(x) = \sum_{i=1}^{3} \varrho_i \ \delta(\mathbf{x}-\mathbf{r}_i),$$

$$\boldsymbol{j}(x) = \sum_{i=1}^{3} \varrho_i \ \{\boldsymbol{r}_i\, \delta(x-r_i)\}_{Sym} + (\nabla_{\mathbf{X}} \times \sum_{i=1}^{3} \mu_i\, \sigma_i)\, \delta(\mathbf{x}-\mathbf{r}_i), \qquad (10)$$

where $\boldsymbol{r}_i$ and $\boldsymbol{r}_i = \boldsymbol{P}_i/m$ are the position and velocity of each particle.

$SU(6)$	$SU(3)$	S	$\left\lvert \begin{matrix} S & \langle SU(6)\rangle \\ r & B \end{matrix} (\rho,\ \varphi) \right\rangle = \frac{(2m)^{1/2}}{(2\sqrt{3}\pi)^{3/2}} \exp\,[iP.X/\sqrt{3}]$
56	10	3/2	$\lvert B_{10}\rangle \left\lvert \begin{matrix} 3/2 \\ r \end{matrix} \right\rangle \Psi^{S}_{k,m}(\rho,\ \varphi)$
56	8	1/2	$\frac{1}{\sqrt{2}}\left[\lvert \overline{B}_8\rangle \left\lvert \begin{matrix} 1/2 \\ \underline{r} \end{matrix} \right\rangle + \lvert \underline{B}_8\rangle \left\lvert \begin{matrix} \overline{1/2} \\ r \end{matrix} \right\rangle\right] \Psi^{S}_{k,m}(\rho,\ \varphi)$
70	10	1/2	$\frac{1}{\sqrt{2}} \lvert B_{10}\rangle \left[\left\lvert \begin{matrix} \overline{1/2} \\ r \end{matrix} \right\rangle \Psi^{M}_{k,m}(\rho,\ \varphi) + \left\lvert \begin{matrix} 1/2 \\ \underline{r} \end{matrix} \right\rangle \Psi^{\overline{M}}_{k,m}(\rho,\ \varphi)\right]$
70	8	3/2	$\frac{1}{\sqrt{2}} \left\lvert \begin{matrix} 3/2 \\ r \end{matrix} \right\rangle \left[\lvert \overline{B}_8\rangle \Psi^{M}_{k,m}(\rho,\ \varphi) + \lvert \overline{B}_8\rangle \Psi^{\overline{M}}_{k,m}(\rho,\ \varphi)\right]$
70	8	1/2	$\frac{1}{\sqrt{2}}\left[\lvert \overline{B}_8\rangle \left\lvert \begin{matrix} \overline{1/2} \\ r \end{matrix} \right\rangle \Psi^{\overline{M}}_{k,m}(\rho,\ \varphi) + \lvert \underline{B}_8\rangle \left\lvert \begin{matrix} 1/2 \\ \underline{r} \end{matrix} \right\rangle \Psi^{M}_{k,m}(\rho,\ \varphi)\right]$
70	1	1/2	$\frac{1}{\sqrt{2}} \lvert B_1\rangle \left[\left\lvert \begin{matrix} \overline{1/2} \\ r \end{matrix} \right\rangle \Psi^{M}_{k,m}(\rho,\ \varphi) + \left\lvert \begin{matrix} 1/2 \\ \underline{r} \end{matrix} \right\rangle \Psi^{\overline{M}}_{k,m}(\rho,\ \varphi)\right]$

Table II. Total Symmetric Wave Functions.

In order to calculate the e.m. current matrix element,

$$\langle j_\mu(0)\rangle = \left\langle \begin{matrix} M & S' & \langle SU(6)\rangle \\ P' & S_3' & B_8' \end{matrix}\; N \middle| j_\mu(0) \middle| \begin{matrix} M & 1/2 & \langle 56\rangle \\ P & S_3 & B_8 \end{matrix}\; N_0 \right\rangle \tag{11}$$

between baryon states, we make full use of our staadardized treatment of permutation symmetry. Since the current operators consist of symmetric sums of products $\sum_i a_i b_i$ or $\sum_i a_i b_i c_i$, the following formulas allow the decomposition of the currents in sums of products of factors with definite symmetry,

$$\sum_{i=1}^{3} a_i b_i = \frac{1}{3}(ab + \bar{a}\bar{b} + \underline{a}\underline{b}), \quad \sum_{i=1}^{3} a_i b_i c_i = \frac{1}{3}(dc + \bar{d}\underline{c} + \underline{d}\bar{c}), \tag{12}$$

where

$$a = a_1 + a_2 + a_3, \quad \bar{a} = \bar{\varepsilon} a_1 + \varepsilon a_2 + a_3, \quad \underline{a} = \varepsilon a_1 + \bar{\varepsilon} a_2 + a_3,$$

$$d = \frac{1}{3}(ab + \bar{a}\underline{b} + \underline{a}\bar{b}), \quad \underline{d} = \frac{1}{3}(a\underline{b} + \bar{a}\bar{b} + \underline{a}b), \quad \bar{d} = \frac{1}{3}(\bar{a}b + a\bar{b} + \underline{a}\underline{b}), \quad \text{etc.}$$

We see that, with respect to the $SU(3)$ variables, besides the charges $Q(B_8) = \langle \overline{B_8}|Q|\overline{B_8}\rangle = \langle \underline{B}_8|Q|\underline{B}_8\rangle$, there are also mixed charges $\overline{Q}(B_8) = \langle \overline{B_8}|\underline{Q}|\underline{B}_8\rangle = \langle \underline{B}_8|\overline{Q}|\overline{B_8}\rangle$, $\overline{Q}(B_8/B_{10}) = \langle B_{10}|\underline{Q}|\overline{B_8}\rangle = \langle B_{10}||\overline{Q}|\underline{B}_8\rangle \cdot$

They are given in Table III.

It is straight fonvard to calculate the current matrix elements $\langle j_\mu(0)\rangle$. They are given in Table IV.

4. The Nucleon Inelastic Structure Functions

In Ref. 1, we have discussed the current tensor $W_{\mu\nu}$ in a Galilei invariant model and showed that $W_{\mu\nu}$ is completely expressible by two invariant functions $W_1(p.q, q^2)$ and $W_2(p.q, q^2)$, as in the relativistic case[8]. The structure function, $W_2(p.q, q^2)$, is given by the W_{00} component of W:

$$Q(B_8) = \langle \overline{B_8} | Q | \overline{B_8} \rangle = \langle \underline{B}_8 | Q | \underline{B}_8 \rangle,$$

$$\overline{Q}(B_8) = \langle \overline{B}_8 | \underline{Q} | \underline{B}_8 \rangle = \langle \underline{B}_8 | \overline{Q} | \overline{B_8} \rangle,$$

$$\overline{Q}(B_8/B_{10}) = \langle B_{10} | \overline{Q} | \overline{B_8} \rangle = \langle B_{10} | Q | \underline{B}_8 \rangle,$$

B_8	p	Σ^+	n	Σ^0	Λ	Λ/Σ^0	Ξ^0	Σ^-	Ξ^-
$\overline{Q}(B_8)$	−1	−1	1	−1/2	1/2	$\sqrt{3}/2$	1	0	0
B_8/B_{10}	Δ^+/p	Δ^0/n	Σ^{*+}/Σ^+	Σ^{*0}/Σ^0	Σ^{*0}/Λ	Ξ^{*0}/Ξ^0	Σ^{*-}/Σ^-	Ξ^{*-}/Ξ^-	
$\overline{Q}(B_8/B_{10})$	−1	−1	−1	−1/2	$-\sqrt{3}/2$	−1	0	0	

Table III. Mixed Charges

$$W_2 = W_{00} = \frac{1}{(2\pi)^6} \sum_N \langle p | j_0(0) | p', N \rangle \langle p', N | j_0(0) | p \rangle$$

$$= \sum_{k_i} \sum_{\substack{m \equiv 0 \ (\text{mod} \ 3) \\ m \equiv 1 \ (\text{mod} \ 3) \\ m \equiv 2 \ (\text{mod} \ 3)}} \delta[(p+q)^2 - M(2Ep' + M)] \, | \langle j_0(0) \rangle |^2, \tag{13}$$

with $2ME_{p'} = 6 \sum_{i=1}^{3} [2k_i + |m_i| + 3] \, \beta$ and $m = m_1 + m_2 + m_3$. We calculate the contribution of $m \equiv 0$ (mod. 3):

$$W_2(m \equiv 0) = 2M^2 Q^2(B_8) \exp[-(1/3)q^2/\beta] \, .$$

$$\cdot \sum_{k_i=0}^{\infty} \sum_{m_i=-\infty}^{\infty} \delta[2p.q + q^2 - 6(\sum_i 2k_i + |m_i| + 3)\beta]$$

$$\cdot \prod_{i=1}^{3} [k_i!(k_i + |m_i|)!]^{-1} [q_i^2/6\beta]^{2k_i + |m_i|}. \tag{14}$$

We introduce a δ-function, $\delta(N_i, 2k_i + |m_i|)$, for each two-dimensional harmonic oscillator and use the integral representation,

$$\delta(N_i, 2k_i + |m_i|) = \frac{1}{2\pi i} \oint_{\lambda_i=0} d\lambda_i \frac{\lambda_i^{2k_i + |m_i|}}{\lambda_i^{N_i+1}},$$

$$\langle 56 \;\; 8' \;\; 1/2 \, |j_0| \, 56 \;\; 8 \;\; 1/2 \rangle = 2M \, Q(B_8) \, \delta_{S'_3 S_3} \, I_q ,$$

$$\langle 56 \;\; 8' \;\; 1/2 \, |\boldsymbol{j}| \, 56 \;\; 8 \;\; 1/2 \rangle = 2M \, Q(B_8) \, \delta_{S'_3 S_3} \frac{1}{6m} [p + p' + 6\Omega \nabla_q + 2q] \, I_q +$$

$$+ \frac{2M}{3} I_q \left[\mu(B_8) \left\langle {1/2 \atop s'_3} \right| i(\sigma \times q)_S \left| {1/2 \atop s_3} \right\rangle + \right.$$

$$\left. + \bar{\mu}(B_8) \left\langle {1/2 \atop s'_3} \right| i(\sigma \times q) \left| {1/2 \atop s_3} \right\rangle \right],$$

$$\langle 56 \;\; 10' \;\; 3/2 \, |j_0| \, 56 \;\; 8 \;\; 1/2 \rangle = 0,$$

$$\langle 56 \;\; 10' \;\; 3/2 \, |\boldsymbol{j}| \, 56 \;\; 8 \;\; 1/2 \rangle = 2M \frac{\sqrt{2}}{3} \bar{\mu}(B_8/B_{10}) \left\langle {3/2 \atop s'_3} \right| i(\bar{\sigma} \times q) \left| {1/2 \atop s_3} \right\rangle I_q \, .$$

$$\langle 70 \;\; 10' \;\; 1/2 \, |j_0| \, 56 \;\; 8 \;\; 1/2 \rangle = \sqrt{2} \, M \, \bar{Q}(B_8/B_{10}) \, \delta_{S'_3 S_3} \, I_q ,$$

$$\langle 70 \;\; 10' \;\; 1/2 \, |\boldsymbol{j}| \, 56 \;\; 8 \;\; 1/2 \rangle = \sqrt{2} \, M \, \bar{Q}(B_8/B_{10}) \, \delta_{S'_3 S_3} \frac{1}{6m} [p + p' + 6\Omega \nabla_q + 2q] \, I_q +$$

$$+ \frac{\sqrt{2} \, M}{3} \bar{\mu}(B_8/B_{10}) \left\langle {1/2 \atop s'_3} \right| i(\bar{\sigma} \times q) \left| {1/2 \atop s_3} \right\rangle I_q \, .$$

$$\langle 70 \;\; 8' \;\; 3/2 \, |j_0| \, 56 \;\; 8 \;\; 1/2 \rangle = 0,$$

$$\langle 70 \;\; 8' \;\; 3/2 \, |\boldsymbol{j}| \, 56 \;\; 8 \;\; 1/2 \rangle = \frac{\sqrt{2} \, M}{3} [\mu(B_8) + \bar{\mu}(B_8)] \left\langle {3/2 \atop s'_3} \right| i(\bar{\sigma} \times q) \left| {1/2 \atop s_3} \right\rangle I_q ,$$

$$\langle 70 \;\; 8' \;\; 1/2 \, |j_0| \, 56 \;\; 8 \;\; 1/2 \rangle = \sqrt{2} \, M \, \bar{Q}(B_8) \, \delta_{S'_3 S_3} \, I_q ,$$

$$\langle 70 \;\; 8' \;\; 1/2 \, |\boldsymbol{j}| \, 56 \;\; 8 \;\; 1/2 \rangle = \sqrt{2} \, M \, \bar{Q}(B_8) \, \delta_{S'_3 S_3} \frac{1}{6m} [p + p' + 6\Omega \nabla_q + 2q] \, I_q +$$

$$+ \frac{\sqrt{2} \, M}{3} I_q \left[\bar{\mu}(B_8) \left\langle {1/2 \atop s'_3} \right| i(\sigma \times q)_S \left| {1/2 \atop s_3} \right\rangle + \right.$$

$$\left. + \mu(B_8) \left\langle {1/2 \atop s'_3} \right| i(\bar{\sigma} \times q) \left| {1/2 \atop s_3} \right\rangle \right],$$

$$I_q = i^m \exp\left[- \frac{q^2}{6\Omega} \right] \cdot \prod_{i=1}^{3} [k_i ! \, (k_i + |m_i|) !]^{-1/2} \left(\frac{q_i}{6\Omega} \right)^{2k_i + |m_i|}$$

Table IV — Current matrix-elements.

to write

$$W_2(m \equiv 0) = 2M^2 . 3Q^2(B_8) \exp\left[-\frac{1}{3} q^2/\beta\right] .$$

$$\cdot \sum_{N_i=0}^{\infty} \delta[2p.q + q^2 - 6(\sum_i N_i + 3)\beta] \bar{I}_q ,$$

where

$$\bar{I}_q = \prod_{i=1}^{3} \sum_{m_i=-\infty}^{+\infty} \frac{1}{2\pi i} \oint_{\lambda_i=0} \frac{d\lambda_i}{\lambda_i^{N_i+1}} \sum_{k_i=0}^{\infty} [k_i!(k_i+|m_i|!)]^{-1} \cdot \left(\frac{1}{2} \frac{\lambda_i q_i^2}{3\beta}\right)^{2k_i+|m_i|}$$

$$= \prod_{i}^{3} \sum_{m_i=-\infty}^{+\infty} \frac{1}{2\pi i} \int_{\lambda_i=0} \frac{d\lambda_i}{\lambda_i^{N_i+1}} I_{|m_i|} \left(\frac{\lambda_i q_i^2}{3\beta}\right),$$ where $I_{|m_i|}\left(\frac{\lambda_i q_i^2}{3\beta}\right)$ is the modified Bessel function and $\bar{I}_q = \prod_{i=1}^{3} \frac{1}{3} \frac{1}{N_i!} (q_i^2/3\beta)^{N_i}$.

Equation (13) then becomes

$$W_2(m \equiv 0) = 2M^2 \cdot \frac{1}{9} Q^2(B_8) \exp[-q^2/3\beta]$$

$$\cdot \sum_{N_i=0}^{\infty} \delta(2p.q + q^2 - \sum_i N_i\beta) \prod_{i=1}^{3} \frac{1}{N_i!} \left(\frac{q_i^2}{3\beta}\right)^{N_i} \quad (15)$$

and we proceed as in Ref. 1 to obtain

$$W_2(m \equiv 0) = 2M^2 \cdot \frac{1}{9} Q^2(B_8) \frac{1}{6\beta} \cdot \frac{1}{\sqrt{2\pi}} (q^2/3\beta)^{-1/2}.$$

$$. \exp\left[-\frac{1}{2}\left(\frac{2p.q + q^2}{6\beta} \quad \frac{q^2}{3\beta}\right)^2 \Big/ (q^2/3\beta)\right] \cdot \quad (16)$$

In a similar way, we calculate the contributions $m \equiv 2$ (mod. 3) and $m \equiv 1$ (mod. 3), obtaining

$$\check{M} W_2(p.q, q^2) = \frac{1}{3} [Q^2(B_8) + \bar{Q}^2(B_8) + \bar{Q}^2(B_8/B_{10})] \cdot$$

$$\frac{\omega}{2\sqrt{\pi}} (q^2/3\beta)^{1/2} \exp\left[-\frac{1}{4} \cdot \frac{q^2}{6\beta} (\omega - 3)^2\right] \quad (17)$$

We do not have approxirnate scaling for finite momentum and energy transfers but, in the Bjorken limit $q^2 \longrightarrow \infty$,

$$B_j \lim \frac{v}{M} W_2(p.q,\ q^2) = F_2(\omega),$$

$$F_2(\omega) = \frac{1}{3}\left[Q^2(B_8) + \overline{Q}^2(B_8) + \overline{Q}^2(B_8/B_{10})\right] \omega\, \delta(\omega - 3), \qquad (18)$$

and we have a quasi-elastic, scale invariant peak.

If we use the followingrelations[3] between the spin operator matrix elements,

$$\left\langle \begin{matrix}\overline{1/2} \\ S_{3'}\end{matrix} \middle| \sigma^m \middle| \begin{matrix}1/2 \\ S_3\end{matrix} \right\rangle = \left\langle \begin{matrix}1/2 \\ S_{3'}\end{matrix} \middle| \overline{\sigma}^m \middle| \begin{matrix}\overline{1/2} \\ S_3\end{matrix} \right\rangle = -2\left\langle \begin{matrix}\overline{1/2} \\ S_{3'}\end{matrix} \middle| \sigma^m \middle| \begin{matrix}\overline{1/2} \\ S_3\end{matrix} \right\rangle =$$

$$= \sqrt{3}\ \langle 1/2,\ S_{3'} | 1,\ m;\ 1/2,\ S_3 \rangle,$$

$$\left\langle \begin{matrix}3/2 \\ S_{3'}\end{matrix} \middle| \underline{\sigma}^m \middle| \begin{matrix}\overline{1/2} \\ S_3\end{matrix} \right\rangle = \left\langle \begin{matrix}3/2 \\ S_{3'}\end{matrix} \middle| \overline{\sigma}^m \middle| \begin{matrix}1/2 \\ S_3\end{matrix} \right\rangle = -\sqrt{6}\langle 3/2, S_{3'} | 1, m;\ 1/2, S_3\rangle, \qquad (19)$$

with $\sigma^{m=\pm 1} = \mp \frac{1}{\sqrt{2}}(\sigma_1 \pm i\sigma_2)$; $\sigma^{m=0} = \sigma_3$ and, if the quark magnetic momentum is given by $\mu_q = Q_q/m$, we obtain for the Bjorken limit

$$B_j \lim W_1(p.q,\ q^2) = F_1(\omega),$$

$$F_1(\omega) = \frac{3}{2}\left[Q^2(B_8) + \overline{Q}^2(B_8) + \overline{Q}^2(B_8/B_{10})\right] \delta(\omega - 3) \cdot \qquad (20)$$

Equa 'Sons (18) and (20) give the structure functions in a dynamical resonance model in the framework of the nonrelativistic symmetric quark model. For these structure functions, the following properties hold:

a) the Callan-Gross relation,

$$F_1(\omega) = \frac{\omega}{2} F_2(\omega);$$

b) the Gottfíeld sum rule,

$$\int \frac{d\omega}{\omega} F_2(\omega) = \frac{1}{3}\left[Q^2(B_8) + \overline{Q}^2(B_8) + \overline{Q}^2(B_8/B_{10})\right] = \begin{cases} 1 \text{ for the proton,} \\ (2/3) \text{ for the neutron,} \end{cases}$$

$$\int \frac{d\omega}{\omega} \left[F_2^{ep}(\omega) - F_2^{en}(\omega)\right] = 1/3;$$

c) Callan-Gross sum rule,

$$\int \frac{d\omega}{\omega^2} F_2(\omega) = \begin{cases} 1/3 & \text{for the proton,} \\ 2/9 & \text{for the neutron;} \end{cases}$$

d) they have the same form as in elastic scattering on quasifree partons with an effective mass 1/3 of the nucleon mass.

We see that the coupling electromagnetic current-constituents leads to electron-resonance scattering which is similar to parton scattering.

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