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Germanium Thermometers in the Temperature Range .100°K to 4.2°K

S. Y. HSIEH* and D. H. SÁNCHEZ**

Department of Physics, Rutgers University, New Brunswick, NJ 08903

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This work describes the sensitivity characteristics of two germanium thermometers that proved to be convenient sensors in the temperature range from. 100 °K to 4.2°K. Their resistances change from about 8 x 10^5 ohms at. 100 °K to about 100 ohms at at 4.2°K. The calibration curves were fitted to natural spline functions of order 3 in the whole range of temperatures. These functions give less than half millidegree standard dispersion against 15 millidegree standard dispersion when usual polynomial interpolations are used. We discuss what spline functions are, and compare the goodness of spline interpolation with polynomial methods. Reference 4 gives the FORTRAN IV computer programs in subroutine form that permit to evaluate the splines.

Descrevem-se as características de sensibilidade de dois termômetros de germânio que demonstram ser sensores de utilidade, na faixa de temperatura de 0,100°K a 4,2°K. Suas resistências variam de cerca de 8 x 10^5 ohms, a 0,100°K, para cerca de 100 ohms, a 4,2°K. As curvas de calibração foram ajustadas utilizando-se *splines* naturais de 3.° grau, em todo o domínio de temperaturas. Essas funções dão um desvio padrão de menos de meio miligrau, contra 15 miligraus no caso em que se utilizam polinômios. Discute-se o que sejam as funções de *spline,* como também se compara, no processo de interpolação, a eficácia dessas funções relativamente aos polinômios. A Ref. 4 fornece subrotinas, em FORTRAN IV, que permitem escolher os *splines* que interpolam os pontos experimentais.

1. Introduction

It is known[1] how difficult it is to find germanium thermometers that do not have extremely high resistance at temperatures below .400°K and, at the same time, enough sensitivity above 1.0 °K. The germanium thermometers made by Solitron Devices, Inc. [2], that have the manufacturer's specifications shown in Table I, proved to be convenient sensors in, the temperature range .100°K to 4.2 °K.

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*Present address: Brookhaven National Laboratory, Building 911 B, Upton, NY 11973.
**Present address: *Departamento de Fisica, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires,* Buenos Aires, R. Argentina.

THERMOMETER #		1	2
Unit #		1816	1288
LHe(°T)	(volts/10μA)	103.30	97.562Ω
LN_2(°T)	(volts/100μA)	6.51Ω	2.993Ω
Amb(°T)	(volts/100μA)	3.86Ω	2.2537Ω

Table I. Manufacturer's specifications for the two germanium thermnmeters.

To fit the thermometer resistance (R_{Ge}) vs. temperature (T) curves, we use natural spline functions of order 3. The use of these functions as interpolation formulae for germanium thermometers was first suggested by Wepner[3]. They give less than half inillidegree standard dispersion in the temperature range .100 °K to 4.2 °K. Considering the problems found using other interpolation procedures[1,2,3], we concluded that the natural spline function of order 3 provides the simplest interpolation for germanium thermometers when the change of resistance'is drastically huge.

In this work, we give a brief description of spline functions, a discussion of some of the problems found with other interpolation methods, a brief description of the experimental methods used and the results of the two thermometers calibration, and the application of natural spline functions of order 3 to fit the R_{Ge} vs. T curves.

2. Spline Functions

Spline functions have been extensively discussed by Greville[S]. In what follows, we will briefly define them and give some of their properties.

The problem of finding a function $S(x)$, that fits n given data points (x_i, y_i) (i = 1, 2, ..., n) and has continuous derivatives of orders (1, 2, ..., k), has the following solutions: (a) For k > n, there is an infinity of polynomials of degree (k−1) that fit the data points exactly; (b) for k = n, the Lagrangian interpolation polynomial of degree (k−1) is the unique solution; (c) for k < n, the solution is a unique function and is given in any interval, (x_i, x_{i+1}), in general, by a different polynomial of degree at most (2k−1). This function has the property that in each of the intervals $(-\infty, x_1)$ and (x,, ∞) it reduces to a polynomial of degree (k − 1).

The functions $S(x)$ described in (c) belong to a class of functions known as spline functions.

A spline function $S(x)$, of degree (k), that fits the data points (called "knots" or "nodes") $x_1 < x_2 < \ldots <$ x„ is characterized by the following properties: (a) $S(x)$ is given in (x_i, x_{i+1}), (i = 0, 1, ..., n) where $(x_0 = -\infty)$ and $(x_{n+1} = \infty)$, by some polynomial of degree at most (k); (b) $S(x)$ and its derivatives of order 1, 2, ..., k – 1 are continuous in the interval (– ∞, co).

A spline function $S(x)$ of odd degree (2k – 1) is called natural [and is written $s(x)$] if it satisfies the further condition: (c) $s(x)$ reduces to a polynomial of degree (k – 1) in each of the intervals (– co, x_1) and (x„ ∞) (in general different polynomials in the two intervals).

The natural spline of degree (2k–1) has a unique representation in the form:

$$s(x) = P_{k-1}(x) + \sum_{i=1}^{n} c_i \, (x - x_i)_+^{2k-1},$$

where P_{k-1}(x) is a polynomial of degree (k – 1), c_i a set of constants and

$$(x - x_i)_+^{2k-1} = \begin{cases} (x - x_i)^{2k-1}, & [(x - x_i) > 0], \\ 0 & [(x - x_i) \le 0]. \end{cases}$$

It can be shown[3,5] that, for a natural spline, the following conditions are also satisfied

$$\sum_{i=1}^{n} c_i \, x_i = 0; \quad (i = 0,\ 1,\ \ldots,\ k-1),$$

$$\sum_{i=1}^{n} c_i = 0.$$

Natural spline functions of degree k = 3 are the ones most commonly used. They are easy to calculate and give satisfactory results. These functions, their derivatives $s'(x)$ and their second derivatives $s''(x)$ are continuous in the interval (x,, x,).

3. Interpolation of Germanium Thermometer Measurements

A calibration of resistance versus temperature of a germanium thermometer gives a set of points $[(R_{Ge})_i, T_i]_{CALB}$, where $(R_{Ge})_i$ are the thermometer resistances and T_i the corresponding temperatures.

One usually fits these calibration points to some polynomial that relates R_{Ge} to T. Some of the most common polynomials used are

$$(1/T) = \sum_{j=1}^{m} A_j \, (R_{Ge})^{\alpha_j} \, ,$$

$$\log(T) = \sum_{j=1}^{m} A_j \, [\log(R_{Ge})]^{\alpha_j} \, ,$$

where A_j is a set of constants and α_j some function of j.

Usually, the number m of terms has to be of the order of the number n of calibration points in order to obtain a fit within one millidegree.

Once a polynomial is found, it is used to interpolate between calibration points to calculate the temperature of a given value of the resistance.

A typical germanium thermometer changes from 10^2 to 10^6 ohms in the temperature range where it is used as a thermometer. It is usually not possible to find a single polynomial of order less than 10 that fits the calibration points in the whole range of temperatures within one millidegree.

Two methods are commonly used to solve this problem, at least partially: the use of polynomials of higher order or the division of the calibration curve into intervals, each of which is a fit to a different polynomial.

We will show, in Section 5, that spline functions give a better fit than the usual polynomial interpolation methods.

4. Experimental. Methods and Results

Two thermometers were calibrated from .080 °K to 4.2 °K. We will call them R_{Ge} # 1 (unit # 1816) and R_{Ge} # 2 (unit # 1288). We have measured 54 calibration points for R_{Ge} # 1 and 72 for R_{Ge} # 2.

The calibration was done in a $He^3 - He^4$ dilution refrigerator[6]. The temperature was measured using the magnetic susceptibility of CMN[7] below 1.0 °K, and the vapor pressure of liquid He^4 using the 1958 scale[8] above 1.0 °K. The procedure followed for the calibration is described in Ref. 9.

We show, in Figure 1, the resistance *(R_{Ge})* of the thermometers as a function of the temperature. The resistances change from about 8 x 10^5 ohms, at .100 °K, to about 100 ohms, at 4.2 °K.

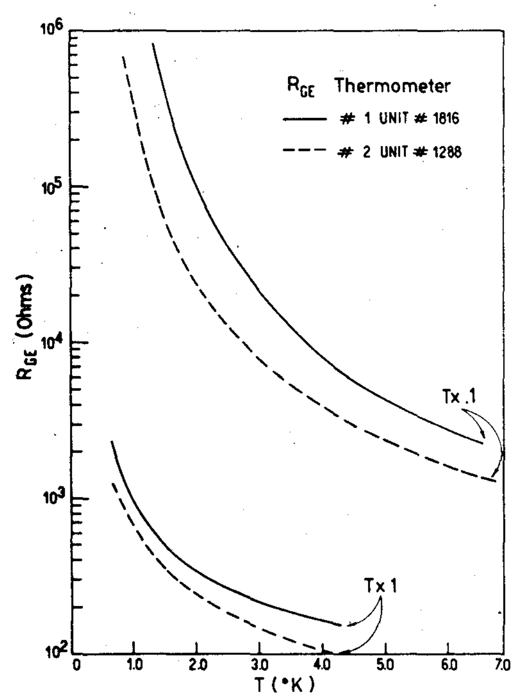

Fig. 1 - Resistance versus temperature for the two germanium thermometers. $T \times .1$ and $T \times 1$ mean that the temperature scale has to be multiplied by .1 and 1, respectively.

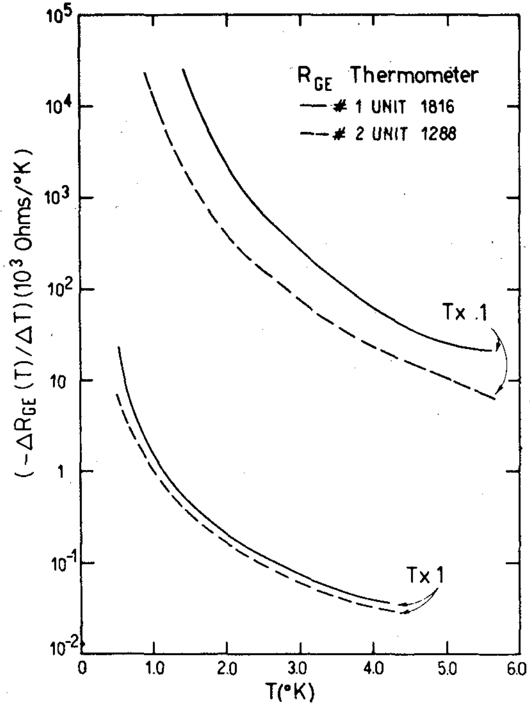

Fig. 2 - Sensitivity $[-(\Delta R_{Ge}(T))/(\Delta T)]$ versus temperature for the two thermometers. $T \times .1$ and $T \times 1$ mean that the temperature scale has to be multiplied by .1 and 1, respectively.

Using the data shown in Figure 1, we calculated the sensitivity $[-(\Delta R_{Ge}(T))/(\Delta T)]$ by using the values of R_{Ge} out of consecutive points. The results are shown in Figure 2. The sensitivity changes from about 10^8 ohms/°K at .100 "K to about 10 ohms/°K at 4.2 °K.

5. Fit to Spline Functions of Order 3

We use as variables (x,, y,) for the splines the set (log R_{Ge}, log T). From the 54 calibration points, for R_{Ge} # 1, and the 72 points, for R_{Ge} # 2, we have chosen 9 points at approximately half degree intervals for each thermometer. These points were used as "knots" of the spline functions. The computer programs, given in Reference 4, were used to evaluate the splines that go exactly through the knots. The same program interpolates, in between the knots, the other 45 and 63 points for each thermometer. For each point, we compared the measured and calculated values. In Figure 3, we plotted some of the AT= T_{CALC} – – T_{MEAS} points as a function of T_{MEAS}, where T_{MEAS} is the calibration temperature and T_{CALC} the temperature calculated using the spline functions of order 3 defined by the 9 knots. It can be graphically seen that the points are reproduced remarkably well within less than a millidegree difference.

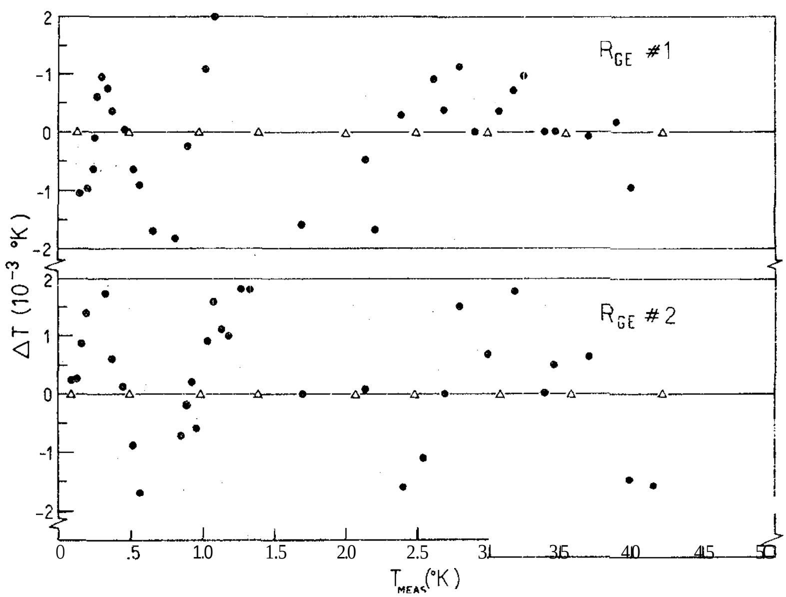

Fig. 3 - The differences $\Delta T = (T_{CALC} - T_{MEAS})$ versus T_{MEAS}. The triangles are the points selected to define the splines. The full dots are some of the other measured points. Obviously, the splines reproduced the knots (triangies) exactly.

It should be noted that we have not discarded any measurements in the data and the 9 knots were chosen arbitrarily. If one selects the knots more carefully or discards some "bad" points the fit can be greatly improved.

For the sake of comparison, we show, in Figure 4, AT as a function of T_{MEAS}, when all the calibration points are least mean square fitted to a polynomial of order 10 of the type

$$(1/T) = \sum_{j=0}^{M} A_j (R_{Ge})^{j/2}$$

One way to estimate the fit is to compute the standard dispersion, defined by

$$\sigma = \sum_{i=1}^{N} \left[\frac{(T_{CALC} - T_{MEAS})_i^2}{N(N-1)} \right]^{1/2},$$

where N is the number of points to be interpolated.

In Table II, we show the differences in the standard dispersion between the two interpolation methods.

	σ (Spline)	σ (Polynomial)
R_{Ge} # 1	0.00035 °K	0.00688 °K
R_{Ge} # 2	0.00023 °K	0.01571 °K

Table II. Comparison of the standard dispersions for the spline and polynomial fits for the two germanium thermometers.

6. Conclusions

In this work, we have shown that the germanium thermometers made by Solitron Devices, Inc., that have the specifications shown in Table I, can be used for precision thermometry between .100 °K and 4.2 "K.

We have also shown that natural spline functions of order 3 provide a simple and accurate interpolation method for the $R_{Ge}(T)$ vs. Tcurve. For crude work, it is only needed to calibrate the thermometers at a few points spaced at half degree intervals instead of many points spaced at 20 millidegrees as reqiiired using polynomial interpolation methods.

Recently, Ward[10] did a similar work at higher temperature ranges. This may give experimentalists a single interpolation formulae for germanium thermometers, valid from .100 °K to ambient temperature, free of the uncertainties of polynomial interpolations.

We are grateful to professor Peter Lindenfeld for many suggestions and helpful discussions.

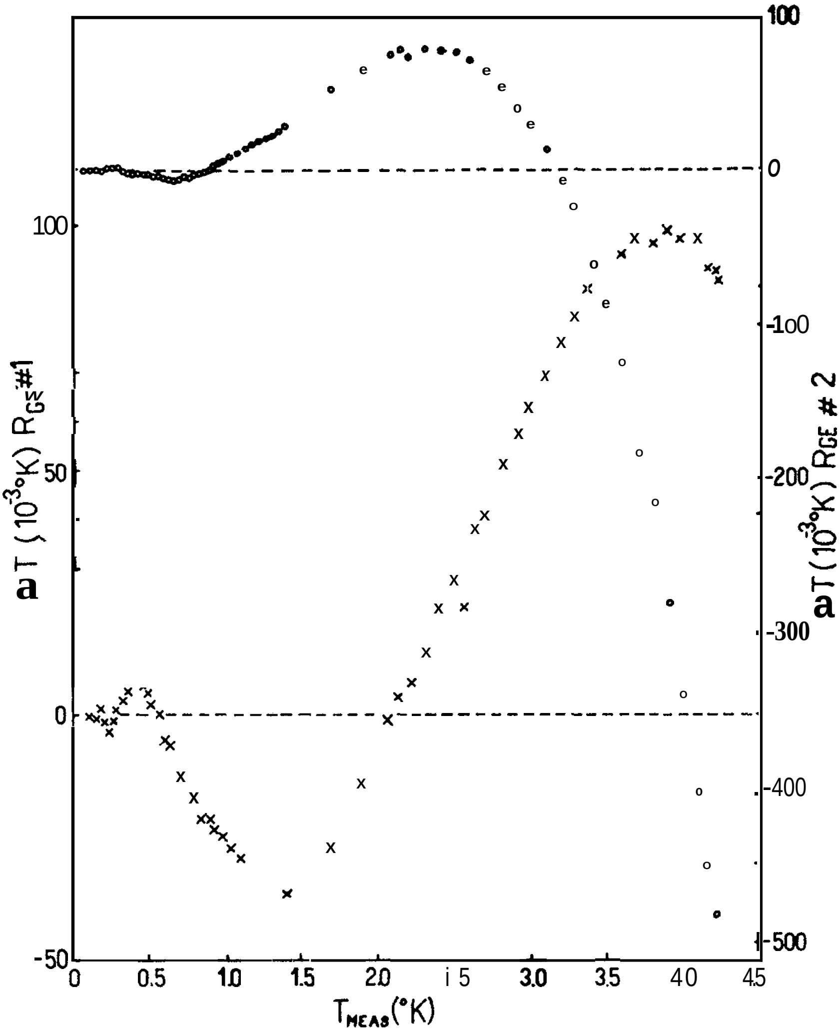

Fig. 4 - The differences $\Delta T = (T_{CALC} - T_{MEAS})$ versus T_{MEAS}. The crosses are the R_{Ge} # 1 and the circles the R_{Ge} # 2 differences when a polynomial fit of order 10 is used. Note the huge scale difference in *AT* as compared with Figure 3.

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