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# On the Applicability of the "Rule-of-Mixtures" to the Strength Properties of Metal-Matrix Composites

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The "Rule-Of-Mixtures" as applied to the strength properties of continuous, uniaxial metal fiber reinforced metal-matrix composites is evaluated critically in the light of some recent experimental results and is shown to be, at best, a good rule-of-thumb. It is not strictly valid. There appears to be a case of synergism in the composite strength properties the explanation of which is sought in the fiber/matrix interaction during (a) the fabrication process leading to matrix structure alteration and (b) straining leading to plastic constraint on the soft matrix during the stage wherein the matrix is deforming plastically while the fiber is deforming elastically (i.e. Stage II).

A "regra das misturas", quando aplicada às propriedades de resistência de compostos contínuos de metal-matriz, reforçadas por fibras uniaxiais de metal, é avaliada criticamente a luz de resultados experimentais recentes e mostrada ser, no máximo, uma boa regra prática, não sendo rigorosamente válida. Parece existir aí, um caso de sinergismo nas propriedades de resistência do composto, a explicação da qual é procurada na interação fibra/matriz durante: a) o processo de fabricação, levando à alteração na estrutura da matriz; b) deformação levando a vínculos plásticos sobre a matriz mole durante o estágio de deformação quando a matriz se deforma plasticamente e a fibra, elasticamente.

## 1. Introduction

Most studies concerned with the evaluation of mechanical behavior of fiber reinforced composites use what is called a "Rule-Of-Mixtures" (hereafter designated as ROM) to predict and/or to compare the strength properties of the composites[1,2,3,4,5,6]. The ROM is nothing but an operational tool that uses weighted volume average of the component properties in isolation to obtain the magnitude of the property for the composite. Specifícally, in the case of a composite containing uniaxially aligned, continuous fibers, the composite stress is written as

$$\sigma_c = \sigma_f V_f + \sigma_m V_m, \tag{1}$$

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where o is the axial stress, Vis the volume fraction of the component and the subscripts c, f and m refer to the composite, fiber and matrix, respectively. It is to be noted that $V_f + V = 1$.

Under conditions of isostrain, i.e, the longitudinal strain in the components being equal, one may write another ROM relationship for the elastic moduli, viz.,

$$E, = E_f V_f + E, V, \tag{2}$$

where E is the elastic modulus and the subscripts represent the components as before. **Eq.** (2) neglects any transverse strain arising because of the different contractile tendencies of the components (i.e, $\nu_f \neq \nu$, where v is Poisson's ratio). However, for metallic systems, the difference in Poisson's ratio of the two components is generally insignificant and the ROM values are generally found to be within the limits of the experimental error[7,8,9].

Another example of a property for which ROM works very well is the density, ***p.*** One can write

$$\rho_c = \rho_f V_f + \rho_m V_m. \tag{3}$$

In short, one can say that ROM works quite well for properties that are relatively structure-insensitive. In this article, the stress-strain behavior of the composites, which is very structure sensitive, and the applicability of the ROM in predicting it, will be the main focus. We shall consider only the alterations in the in-situ behavior of the matrix, tacitly assuming that the in-situ fiber strength is equal to the average strength of a large number of similar fibers tested in isolation. This assumption is clearly not valid for brittle fibers where the strength parameter assumes a statistical nature and a distribution function needs to be used. This problem has been considered by Rosen[10].

The usual method of analysis of the composite behavior, applying the ROM, assumes that the components are non-interacting during straining and also that they have the same properties as those of the isolated fíbers and isolated matrix. Then, a series of composites of different fiber volume fractions would give o,, at a given strain, linear in $V_f$ according to Eq. (2). We disregard, here, any negative deviations from ROM that are due to fiber misalignment, degradation of fibers or formation of a reaction product between the fiber and the matrix.

## 2. Factors Affecting the Components

There are many factors that are brought into existence by the mere act of putting the two components together and bonding them to form a composite whole. For one, the matrix and fiber structure could very well be altered by method of fabrication. Secondly, fiber composite materials, consisting as they are, in most cases, of two components of widely varying thermomechanical properties, are likely to have residual stresses and/or structure alteration during fabrication. Differential contraction during cooling from fabrication temperatures can give rise to stresses large enough to make the soft matrix flow plastically[11]. It is worthwhile to point out here that the commonly used method of stress-relief-anneal would be of no avail, because any stresses relieved at the annealing temperature will be regenerated during cooling to room temperature[8]. Then, again, the mode of deformation of the two components might be affected by the rheological interaction between the two components. Plastic constraint on the matrix due to the large difference in Poisson's ratios of the fiber and the matrix during the stage wherein the matrix is deforming plastically while the fiber is deforming elastically, could change the state of stress in the composite. Or fibers could act as obstacles to dislocation motion leading to piling up of dislocations and interna1 stresses.

In view of all these factors, one would expect the ROM, as conventionally applied to be a good rule-of-thumb, at best. Some of the factors enumerated above are examined and the extent of validity of ROM in predicting the strength properties of fiber reinforced metal-matrix composites is evaluated below in the light of some recent experimental results.

## 3. Decomposition of the Composite Curve

Consider Eq. (1) given above. Under conditions of equal longitudinal strain, i.e., $e_f = e_m = e$ and within the elastic range of the fibers (i.e., $\sigma_f = E_f\, e$), one can obtain $\sigma_m(e)$ from measured $\sigma_c(e)$ as per expression given below:

$$\sigma_m(e) = (1/V_m)\,[\sigma_c(e) - E_f \,.\, e \,.\, V_f]. \tag{4}$$

The ROM says that this $\sigma_m(e)$ is the same as the flow stress of the matrix in isolation at strain e. One way to check whether this is so would be to deduce the $\sigma_m(e)$ by using Eq. (4) (assuming, of course, that the fiber behavior in the composite is the same as in isolation, which seems to be a fair enough assumption so long as the fibers remain elastic), and compare with the stress at strain *e* in the unreinforced matrix. Underlying this idea is the expectation that major changes would occur in the behavior of the relatively soft, ductile, metallic matrix when it goes plastic.

## 4. Experimental

To investigate this issue, composite specimens consisting of *single-crystal* copper matrix containing small number of continuous tungsten wires (228 $\mu$m diameter), aligned parallel to the specimen axis, were tested in tension to strains less than or equal to the strain at which the fibers yielded in isolation. The method of specimen preparation and the testing procedure are described elsewhere[12,13]. The strain sensitivity, using LVDT's, was better than 2.5 x $10^{-5}$mm. The high strain sensitivity was essential in evaluating the ROM critically.

## 5. Results and Discussion

Some typical derived matrix stress-strain curves, obtained as per method outlined above, are shown in Fig. 1. Clearly, copper matrix in the

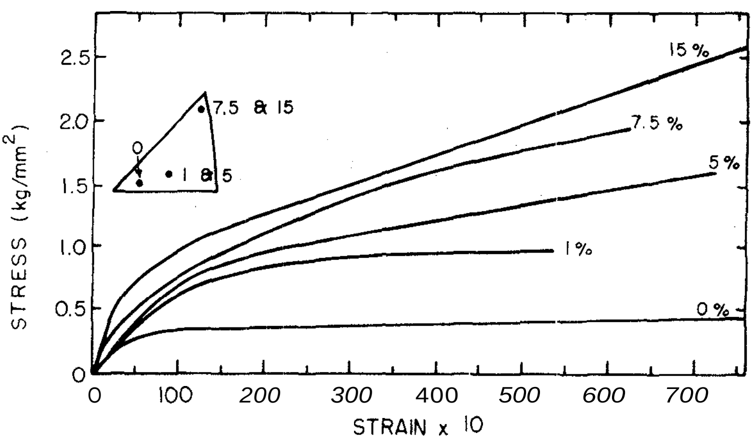


**Fig. 1** – Derived matrix stress-strain curves.

presence of tungsten wires is much stronger than copper matrix by itself (i.e., 0% $V_f$). The important point shown by these curves is that a copper matrix of a given orientation shows higher stress levels at any given strain for higher volume fractions. This is clear from the paired samples – 1% and 5% $V_f$ is one pair and 7.5% and 15% $V_f$ is another pair – their matrices were of the same orientation but the fiber volume fractions were different. The matrix behavior is dependent on $V_f$. Thus, the principal assumption underlying the ROM, viz., the components in the composite have the same properties as in isolation is not valid. Instead, there appeared to be a true case of synergism here, the explanation for which was found in the alteration of the matrix structure during cooling from the melting of copper to ambient temperature in the process of specimen preparation. To examine the matrix structure, specimens were sectioned along (111) planes and Livingston's dislocation etch[14] was used to find out the dislocation density and distribution. There existed[13] a zone of high dislocation density in the vicinity of the fiber which decreased rapidly with distance from the fiber/matrix interface up to about one fiber radius and then levelled off. The level-off or the plateau value of dislocation density also increased with the fiber volume fraction. Fig. 2 shows the dislocation etch pit density profiles for 2.5% $V_f$ and 0% $V_f$ before and after deformation. The effect of

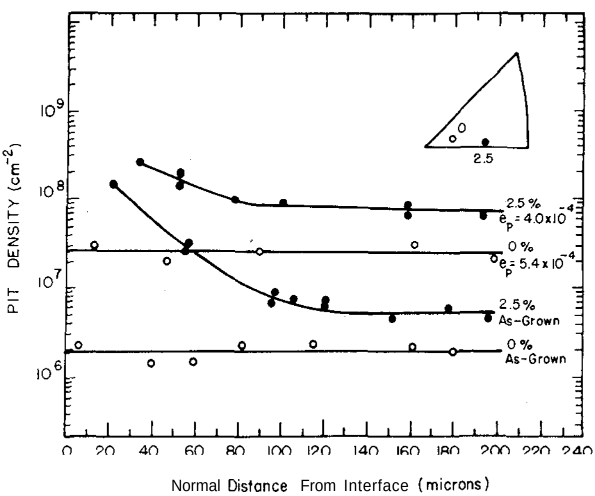


Fig. 2 – Dislocation etch pit density profiles.

incorporating tungsten fibers into copper is shown very clearly by the 2.5% $V_f$ curve which is representative of the general trend. Higher $V_f$ specimens showed higher curves. The increase in dislocation density on straining was mainly confined to the thickening of the cell walls and to the filling in of the pre-existing cell structure. This is shown in Fig. 3, a representative scanning micrograph of a 7.5% $V_f$ specimen

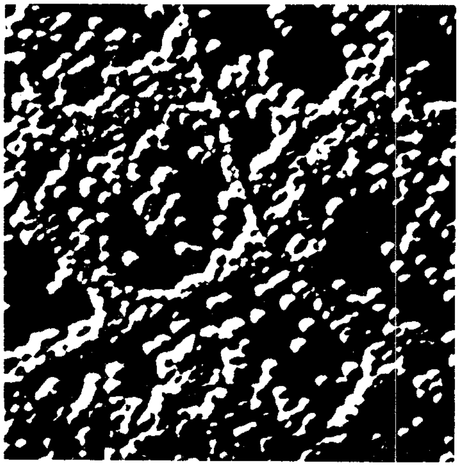

**Fig. 3** – Scanning electron micrograph of a 7.5% $V_f$ specimen after 2.6 x $10^{-4}$ plastic strain. 3000 ×.

after 2.6 x $10^{-4}$ plastic strain. The initial cell structure was well formed for $V_f$'s$>$ 0.5%. The cell size changed little on straining and a rough estimate of the mean free path from the increase in dislocation density, using the relationship $1 = (1/b)\,(da_p/d\rho)$, where $b$ is the Burgers vector, a, the plastic shear strain and $\rho$ the dislocation density, indicated that the mean free path was about the same as the cell size. Thus, interaction between components during fabrication, which introduced a higher dislocation density and cell-wall obstacles, was responsible for the positive deviations from ROM. No major interaction during straining was found to occur in these low $V_f$ composites.

## 6. Other Studies

Conditions maximizing interaction during straining the components would be expected to occur when the interfiber spacing is rather small. This aspect was examined by Kelly & Lilholt[7]. Small diameter tungsten wires (10$\mu$m and 20$\mu$m) and the relatively high $V_f$'s were employed in their work. They derived matrix stress-strain curves as per procedure described above. The copper matrix in the composite exhibited much higher stress levels for a given strain than unreinforced copper. However the stress dropped beyond a strain $\sim$ 4 x $10^{-3}$ which was the fiber yield strain. Kelly & Lilholt envisaged the effect to be due to the constraint on the matrix constituent in Stage II wherein the fibers deform elastically while the matrix deforms plastically. The constraint arises because of the large difference in the Poisson's ratios of the matrix and the fiber. When the matrix goes plastic, its Poisson's ratio attains a value of 0.5 in the ideal case while the tungsten fibers are elastic and their Poisson's ratio remains at 0.28. The Poisson's ratio difference becomes zero when the fibers also become plastic and the lateral contractions become equal (v, $= \nu_f = 0.5$). They proposed a model consisting of an elastic hollow cylinder of tungsten containing copper and treated the latter as a fluid in the plastic stage. On the basis of this "tube under interna1 pressure" model, they deduced the slopes of the matrix stress-strain curves. However, this model could explain the extremely high stress levels only if it was assumed that a 2-3 micron thick copper layer at the surface remained elastic even in Stage II. It should be remarked here there has been no experimental evidence to support this hypothesis so far. In any case, there were definite positive deviations from ROM. No structural observations were made, so one cannot say as to what extent structure alteration during fabrication and to what extent the rheological interaction during straining was responsible for the enhanced stress values attained by the matrix. Garmong & Shepard[14] investigated copper matrix-iron fiber composites over a wide range of fiber diameters and fiber volume fractions. They reported a dependence of matrix yielding on fiber diameter and fiber spacing and large deviations from the ROM. The matrix grain size was also found to change with fiber diameter and interfiber spacing. For widely spaced large diameter fibers, the matrix grain size was smaller than the interfiber spacing and the matrix yielding in this case was interpreted as it would have been for the small grain-size matrix in isolation. But it is to be noted that the *raison d'être* for the small grain size was the presence of fiber of a particular size. Subsequent strain hardening was explained as due to the dislocations piling up against the fibers and the grain boundaries. For closely spaced small diameter fibers, on the other hand, matrix yielding was'explained as controlled by dislocation extrusion between the fibers as per Ashby's reformulation of the Orowan model, although their strain hardening rates were higher than those predicted by the model. In another note[15], Garmong *et al.* have described dependence of copper matrix structure on the size and volume fraction of iron fibers present. Using transmission electron microscopy, they found the dislocation present to be associated primarily with the fibers; the dislocation lines often extended radially from the fiber. This was explained as due to the thermal stresses arising during the processing of the composites.

## 7. Conclusions

It would appear from these studies that the ROM as applied conventionally to the strength properties of composites with metallic matrices is not valid. The whole is more than the sum of individual components in isolation, i.e, a true synergism is exhibited. The mistake is not in the averaging process, for, the load is certainly partitioned between the fiber and the matrix but in the lack of knowledge of the *in-situ* strengths of the individual components. The reasons for these enhanced strength levels might vary under different conditions. The enhancement has been explained to be due to (a) the interaction between the fibers and the matrix during fabrication leading to the alteration of the structure of the metallic matrix (b) the plastic constraint on the soft matrix during Stage II. Which one of these dominates depends upon the component properties, the relative volume fractions present and the fabrication process used.

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