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Spin Wave Amplification in Ferromagnetic Semiconductors — Plasma Effects*

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We have evaluated the damping coefficient of magnons interacting with drifting carriers in ferromagnetic semiconductors for both $kl > 1$ and $kl < 1$ regimes. It is shown that for $CdCr_2Se_4$ the amplification factor can be of the order or greater than the losses of spin waves due to other effects. It is also discussed the influence of plasma effects on the spin wave amplification problem.

Calculamos o coeficiente de amortecimento de magnons interagindo com portadores em semicondutores ferromagnéticos tanto para $kl > 1$ como para $kl < 1$. Mostra-se que, para o $CdCr_2Se_4$, o fator de amplificação pode ser de mesma ordem ou maior que as perdas de ondas de spin devidas a outros efeitos. Discute-se também a influência dos efeitos de plasma no problema da amplificação das ondas de spin.

1. Introduction

Akhiezer *et al*[1] and Vural[2] have investigated the possibility of amplifying spin waves (magnons) through their interaction with drifting carriers. According to their theory, the spin waves are coupled to a drift wave and there is a net gain for the spin wave when its frequency matches that of the drift wave. More recently, however, it has been suggested by Spector[3] that magnons can be amplified through the interaction with the drifting electrons themselves. This will occur when a d.c. electric field is applied such that the drift velocity of the electrons exceeds the phase velocity of the spin wave, in complete analogy with the amplification of sound waves in semiconductors[4]. This author also reached the conclusion that in the limit of no collisions (i.e. $\tau \to \infty$) the absorption coefficient goes to zero. This implies that in the absence of collisions there is no net amplification or absorption of the spin wave. He concludes further that if the condition $\omega\tau > 1$,

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where o is the magnon frequency, could be satisfíed in ferromagnetic semiconductors, then the magnon-conduction electron interaction would be considerably enhanced.

There is however another stronger motivation for looking at this problem, i.e., the influence of the interactions between the carriers and spin waves in the transport properties of magnetic semiconductors. In fact, it was suggested recently by Balberg and Pinch[5] that the observed positive electric-field dependent magnetoresistance of $CdCr_2Se_4$ could be explained by the amplification of spin waves due to carrier interaction.

The existing theories of spin wave amplification are based on phenomenological treatments[6,7]. This type of approach is valid only when the magnon wavelength is longer than the electron mean free path i.e.

$$kl < 1 \tag{1}$$

and the magnon frequency is smaller than the electron collision frequency, i.e. $\omega\tau < 1$. Condition (1) means that the carriers undergo many collisions while travelling a distance equal to the magnon wave--length. This entails, in turn, that the drift velocity is the only velocity to be compared with the spin wave phase velocity v_S. If, however

$$kl > 1, \tag{2}$$

single carriers can emit Cherenkov magnons, as the thermal velocity (or Fermi velocity) normally confortably exceeds the spin wave velocity. Even in the absence of an externa1 field, the carriers can in that case be in resonance with the wave field, that is, move with the same velocity and thus remain in phase and resonantly lose or gain energy of the wave.

In this paper we give a quantum mechanical treatment of the electron--magnon interaction and compare our results with the ones suggested by Spector[3]. The present treatment is valid wheri $kl > 1$. In particular, it is shown that in the collisionless regime the absorption coefficient does not go to zero, and the magnon-carrier effect was found to be of the same order or even less than that in the opposite regime.

We consider a magnetic semiconductor as composed by two magnetic sub-systems, namely, the ferromagnetic bound electrons, which in the phenomenological description is characterized by the magnetization, and the free carriers *(s-electrons)*. The carrier-magnon interaction considered is the interaction of the s-electron currents with the self-consistent field created by the oscillations of the magnetization (spin waves).

We have also derived the damping coefficient from the transverse a.c. conductivity previously obtained[8] by solving the Boltzmann equation for carriers in the presence of d.c. electric fields.

Finally, we shall also discuss the influence of plasma effects in the spin wave amplification problem.

2. Interaction Hamiltonian

The interaction Hamiltonian operator per carrier which we assume is[9]

$$\mathscr{H}(\mathbf{r}) = -\frac{e}{mc}\,\mathbf{A}(\mathbf{r})\,.\,\mathbf{P} \tag{3}$$

Here A (r) is the vector potential of the field created by the oscillations of the magnetization, and P and m are the momentum and mass of the carriers.
The vector potential A(r) due to the magnetization is given by

$$\mathbf{A}(\mathbf{r}) = \int \frac{\mathbf{M}(\mathbf{r}') \times (\mathbf{r}-\mathbf{r}')\,d^3r'}{|\mathbf{r}-\mathbf{r}'|^3} \tag{4}$$

We are not taking into account the screening due to carriers because in the present case of nondegenerate semiconductors the screening radius[10], $r_0 \frac{c}{\omega_p}$, is much greater than the lattice spacing.

Eq. (3) can then be written as

$$\mathscr{H}(\mathbf{r}) = -\frac{e}{mc}\int d^3r'\,\frac{\mathbf{M}(\mathbf{r}')\,.\,\mathbf{L}}{|\mathbf{r}-\mathbf{r}'|^3}, \tag{5}$$

where L is the angular momentum operator of the carrier relative to the point r', $\mathbf{L} = (\mathbf{r}-\mathbf{r}') \times \mathbf{P}$. In a microscopic point of view this is the interaction of the magnetic localized spins with the angular momentum of the carriers, a spin-orbit interaction.

The total interaction Harniltonian is given by

$$H = -\frac{e}{mc}\int\int d^3r\, d^3r'\, \frac{\psi^\dagger(\mathbf{r})\mathbf{M}(\mathbf{r}')\,.\,\mathbf{L}\psi(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|^3} \tag{6}$$

This interaction has been recently used[11] in discussing the problem of anomalous Hall effect in magnetic materials.

The interaction of spin waves with s-electron current does not cause s-electron spin transitions so that we may not consider the electron spin explicitly. The system of conduction electrons is treated as a quantized free electron fíeld defined by

$$\psi(\mathbf{r}) = V^{-1/2}\sum_p a_p\, e^{i\mathbf{p}.\,\mathbf{r}} \tag{7a}$$

$$\psi^\dagger(\mathbf{r}) = V^{-1/2}\sum_p a_p^\dagger\, e^{-\mathbf{p}\cdot\mathbf{r}} \tag{7b}$$

where a, and $a_p^\dagger$ are the annihilation and creation operators of conduction electrons wave-vector p, and V is the volume of the system. In the usual rnanner we have for the spin wave field[12]

$$\begin{aligned} M_{kz} &= M_o\delta_{k,o}, \\ M_k^\dagger &= M_{kx} + iM_{ky} = \left(\frac{2g\mu_B M_o}{V}\right)^{1/2} b_k^\dagger, \\ M_k^- &= M_{kx} - iM_{ky} = \left(\frac{2g\mu_B M_o}{V}\right)^{1/2} b_k, \end{aligned} \tag{8}$$

where b_k and $b_k^\dagger$ are the magnon annihilation arid creation operators, respectively, and μ_B is the Bohr magneton.

Eqs. (3)–(8) lead to the following expression for the interaction Hamiltonian H in the second quantization form

$$H = \sum_{\mathbf{p},\mathbf{k}} M_{e-m}\, a_{p-k}^\dagger\, a_p b_k^\dagger + \text{h.c.} \tag{9}$$

where M_{e-m} is the electron-magnon vertex fiinction for the current interaction.

$$M_{e-m}(\mathbf{p},\mathbf{k}) = i2\pi\, \frac{eh}{mc}\left[\frac{2g\mu_B M_o}{V}\right]^{1/2} \frac{(\mathbf{p}\times\mathbf{k})_-}{k^2}\,. \tag{10}$$

We have taken the z-coordinate axis as the quantization axis, and $(\mathbf{p}\times\mathbf{k})\pm$ means $(\mathbf{p}\times\mathbf{k})_x \pm (\mathbf{p}\times\mathbf{k})_y$.

3. Evaluation of the Damping Coefficient

The rate of change of the magnon distribution function due to the spin wave-current interaction is given by

$$\frac{dN_k}{dt} = \frac{2\pi}{\hbar} \sum_p |M_{e-m}|^2 \{f_p(1-f_{p-k})\ (N_k+1) - -f_{p-k}\ (1-f_p)\ N_k\}\ \delta(E_p - E_{p-k} - \hbar\omega_k), \tag{11}$$

where E, and $\hbar\omega_k$ are the energies of a carrier and a magnon with wave vectors **p** and k respectively. To get the equation of motion for the spin wave intensity $I_k = \hbar\omega_k N_k$, we multiply (11) by $\hbar\omega_k$. We then get

$$\frac{dI_k}{dt} = \gamma(k)\ I_k + \gamma'(k), \tag{12}$$

with

$$\gamma(k) = \frac{2\pi}{\hbar} \sum_p |M_{e-m}|^2\ (f_p - f_{p-k})\ \delta(E_p - E_{p-k} - \hbar\omega_k). \tag{13}$$

The spontaneous emission term $\gamma'(k)$ is not important for our problem, and therefore, we shall not consider it further. If the quantity $\gamma(k)$ in Eq. (12) is positive (negative) one has amplification (damping) of the spin waves.

In Eq. (13) f_p is the particle distribution function which we shall assume to be the equilibrium distribution function. The use of the equilibrium function for the carrier distribution is the main assumption of the theory. This corresponds to assume that there is a certain independent relaxation mechanism by which equilibrium in the sub-system is reached far more rapidly than between them and the spin waves. Also, the effect of the externa1 d.c. electric field is taken into account by using the drifted distribution function, $f(p - p_0)$, as the equilibrium carrier distribution[13]

Restricting ourselves to semiconductors which under the experimental conditions are non-degenerate we may assume the Maxwell-Boltzmann distribution function for the equilibrium distribution,

$$f(\mathbf{p}-\mathbf{p}_0) = n_0 \left(\frac{2\pi\hbar^2}{mk_BT}\right) \exp\left[-\frac{\hbar^2(\mathbf{p}-\mathbf{p}_0)^2}{2mk_BT}\right], \qquad (14)$$

with $p_0 = mv_d/\hbar$.

Now, since the carrier energy is usually much larger than that of the magnons we may allow for the approximate equation

$$f(\mathbf{p}-\mathbf{p}_0-\mathbf{k})-f(\mathbf{p}-\mathbf{p}_0) \simeq -\frac{\partial f}{\partial E(\mathbf{p}-\mathbf{p}_0)} \left[\frac{\hbar^2(\mathbf{p}-\mathbf{p}_0)\,.\,\mathbf{k}}{m} - \frac{\hbar^2k^2}{2m}\right]. \qquad (15)$$

Substituting Eqs. (10), (14) and (15) into Eq. (13), and changing the summation over p into an integral we then get

$$\gamma(k) = 2\pi^{1/2} \frac{\omega_M\,\omega_p^2}{v_{\rm th}^3c^2}\,\frac{1}{k^2}\left(v_d - \frac{\omega_k}{k}\right)\left\{v_{\rm th}^2 - \left[\frac{\omega_k}{k} + \frac{\hbar k}{2m} - v_d\right]^2\right\}, \qquad (16)$$

where $\omega_p = (4\pi e^2n_0/m)^{1/2}$ is the plasma frequency, $\omega_M = 4\pi\gamma M_0$, y is the gyromagnetic ratio ($\simeq 2.8\,MHzG^{-1}$); and $v_{\rm th} = (2k_BT/m)^{1/2}$. We have assumed for the sake of simplicity that the drift velocity $\mathbf{v}_d$ is parallel to k. As usually thermal velocity is much larger than both the drift and the spin wave phase velocities, and, therefore, for reasonable values of k and noticing that $k_0 \ll$ k in the region of interest, one may approximate (16) by

$$\gamma(k) = 2\pi^{1/2}\,\frac{\omega_M\omega_p^2}{c^2}\,\frac{v_S}{v_{\rm th}}\,\frac{1}{k^2}\,(X-1), \qquad (17)$$

where X $= v_d/v_S$ Hence, spin wave amplification nccurs when $v_d > v_S$

In connection with Eq. (16) one should notice that if k approaches $\bar{k}$ (where $\bar{k}$ is essentially related to the maximurn momentum transfer from the carrier system; $\bar{k} = 2mv_{\rm th}/\hbar$), $\gamma(k)$ approaches zero as expected.

Let us now compare the result (17) with the one obtained from the dispersion relation for the carrier-spin wave system[6]

$$\left(\frac{kc}{\omega}\right)^2 = \left[1 + \frac{\omega_M}{\omega_k \mp \omega}\right] \varepsilon_1. \qquad (18)$$

The term in the bracket in the right hand side of Eq. (18) is the effective permeability for right-(left) handed circularly polarized plane waves and ε_1 is the transverse dielectric constant. In writing Eq. (18) we have assumed that the magnetic field does not affect the properties of the plasma ($\hbar\omega_c \ll k_BT$), and that the applied electric field is parallel to the magnetic field. We may also notice that the left-handed circularly polarized plane wave will give the maximum permeability to the system and therefore in the following we shall only take it into account.

Looking for the solution of Eq. (18) in the form

$$\omega = \omega_k + \Gamma, \tag{19}$$

where $\mathrm{Re}\Gamma$ and $\mathrm{Im}\Gamma$ are much smaller than ω, one has

$$\mathrm{Im}\Gamma \approx -4\pi \frac{\omega_M \omega_k}{k^2 c^2} \mathrm{Re}\sigma_1. \tag{20}$$

The transverse conductivity was obtained by Spector by solving Boltzmann's equation for electrons interacting with transverse waves in the presence of an applied electric Field. The result is[5]

$$\sigma_1 = \frac{i\omega_p^2 (1 - X)}{4\pi[(\omega_k - kv_d) + 1/\tau]}, \qquad kl < 1 \tag{21a}$$

and

$$\sigma_1 = \frac{\pi^{1/2} \omega_p^2 (1 - X)}{4\pi k v_{\mathrm{th}}}, \qquad kl > 1. \tag{21b}$$

Combining Eqs. (21) and (20) we get

$$\gamma(k) = \frac{\omega_p^2 \omega_M v_S (X - 1)}{\tau k c^2 [(\omega_k - kv_d)^2 + (1/\tau)^2]}, \qquad kl < 1 \tag{22a}$$

$$\gamma(k) = \frac{\pi^{1/2} \omega_p^2 \omega_M v_S (X - 1)}{k^2 c^2 v_{\mathrm{th}}}, \qquad kl > 1. \tag{22b}$$

Eq. (22a) agrees with the result of Robinson *et al*[7] if ones makes $\omega_k \to \omega_0$ and neglects the effect of the magnetic field. Eq. (22b) differs from (17) only by a factor 2. This is due to the fact that we have considered the damping of the energy whereas (22b) expresses the damping of the Field. It is worth mentioning that Eq. (22b) does not contain all the information as Eq. (16) does. This is because the solution of Boltzmann's equation for the carriers was obtained in the limit $\hbar \to 0$.

Finally, we shall comment on the effect of $s-d$ interaction in nondegenerate semiconductors. This interaction has been extensively used by White and Woolsey[14] in studying ferromagnetic semiconductors. In the present paper we shall avoid discussing the $s-d$ interaction by assuming that one has just the up-moment conduction band[14] which means that either $J_{s-d} \gg k_B T$ (nondegenerate) or $J_{s-d} \gg \varepsilon_F$ (degenerate).

4. Plasma Effects

Except for very strong magnetic fields the plasmon frequency branch and the magnon branch do not intersect, so that we can neglect the direct process between magnons and plasmons. However, we can still have the indirect process

plasmon + charge carrier $\longrightarrow$ magnon + charge carrier.

A similar process was recently considered[15] to investigate plasma effects in sound amplification in piezoelectric semiconductors. It was shown[15] that under certain experimental conditions the sound amplification may be enhanced by processes involving plasmons.

The condition for plasma effects to be considered is that

$$\omega_p \tau > 1. \tag{24}$$

This condition means that we are dealing with a collisionless plasma in which we can have collective oscillations with frequencies satisfying the dispersion law

$$\omega = \omega_p + \mathbf{k}' \cdot \mathbf{v}_d, \tag{25}$$

where k' is the wavevector of the collective modes. If, however, $\omega_p \tau \ll 1$, plasmons oscillations will be strongly damped and we should not have any plasma effect.

Using the electron-plasmon interaction given by[16]

$$M_{e-p}(k') = \left(\frac{2\pi e^2 \hbar\omega_{k'}}{V k'^2}\right)^{1/2} \tag{26}$$

We obtain the following kinetic equation for the magnon distribution function

$$\frac{dN_k}{dt} = \frac{2\pi}{\hbar} \sum_{\mathbf{p},\mathbf{k}'} |M|^2 \{f_p \tilde{N}_{k'}(N_k + 1)\,(1 - f_{p+k'-k}) - - f_{p+k'-k} N_k (\tilde{N}_{k'} + 1)(1 - f_p)\} \times \delta(E_p + \hbar\omega_{k'} - E_{p+k'-k} - \hbar\omega_k), \tag{27}$$

where M, given by

$$M = M_{e-p}(k') \left[\frac{M_{e-M}(\mathbf{p}+\mathbf{k}', \mathbf{k})}{(E_p + \hbar\omega_{k'} - E_{p+k'})} + \frac{M_{e-M}(\mathbf{p}, \mathbf{k})}{(E_{p+k'-k} - \hbar\omega_{k'} - E_{p-k})}\right], \tag{28}$$

is the effective vertex for the interaction (23) and $\tilde{N}_{k'}$ the plasmon distribution function. If we retain only the lowest-order terms in A, we can approximate it by

$$M_{h\to 0} = M_{e-p}(k')\ M_{e-m}(\mathbf{p},\mathbf{k})\ \frac{(\mathbf{k}\,.\,\mathbf{k}')/m}{(\omega_{k'}-\mathbf{v}\,.\,\mathbf{k}')^2}, \tag{29}$$

with $\mathbf{v} = \frac{\hbar\mathbf{p}}{m}$.

Similarly to Eq. (12), one now has

$$\frac{dI_k}{dt} = [\gamma_1(k)-\gamma_2(k)]N_k + \gamma''(k), \tag{30}$$

where

$$\gamma_1(k) = A\int d^3k' \int d^3v\ \frac{\omega_{k'}}{k'^2}\ N_{k'}\ \frac{\hbar}{m}\left[(\mathbf{k}-\mathbf{k}')\ .\ \frac{\partial f}{\partial \mathbf{v}}\right][(\mathbf{v}\times\mathbf{k})_x^2+(\mathbf{v}\times\mathbf{k})_y^2]\times$$
$$\times\frac{(\mathbf{k}\,.\,\mathbf{k}')^2}{(\omega_{k'}-\mathbf{v}\,.\,\mathbf{k}')^4}\ \delta[\omega_{k'}-\omega_k+(\mathbf{k}-\mathbf{k}')\,.\,\mathbf{v}], \tag{31}$$

$$\gamma_2(k) = A\int d^3k' \int d^3v\ \frac{\omega_{k'}}{k'^2}\ f(\mathbf{v})\,[(\mathbf{v}\times\mathbf{k})_x^2+(\mathbf{v}\times\mathbf{k})_y^2]\ \frac{(\mathbf{k}\,.\,\mathbf{k}')^2}{(\omega_{k'}-\mathbf{v}\,.\,\mathbf{k}')^4}\times$$
$$\times\ \delta[\omega_{k'}-\omega_k+(\mathbf{k}-\mathbf{k}')\,.\,\mathbf{v}], \tag{32}$$

with

$$A = \frac{e^4\omega_M}{m^2c^2k^2}, \tag{33}$$

and γ'' is the spontaneous emission term.

We assume that the plasmons are in equilibrium with a distribution function,

$$\tilde{N}(k') = [e^{\hbar\omega_{k'}/k_BT} - 1]^{-1}. \tag{34}$$

This is consistent with Eq. (14) for $f(\mathbf{v})$.

At this point we may distinguish two limiting cases for this distribution function, namely,

$$\tilde{N}(k') \simeq \frac{k_BT}{\hbar\omega_{k'}}, \quad \text{for} \quad \hbar\omega_{k'} \ll k_BT, \tag{35a}$$

and

$$\tilde{N}(k') \simeq e^{-\hbar\omega_{k'}/k_BT}, \quad \text{for} \quad \hbar\omega_{k'} \gg k_BT. \tag{35b}$$

Considering first the case $\hbar\omega_{k'} \ll k_B T$ and remembering that plasmons are well-behaved excitations only when their phase velocity is larger than the thermal velocity, we can put $\omega_{k'} - \mathbf{v}\cdot\mathbf{k}' \simeq \omega_p$, and then get

$$\gamma_1 \simeq \tilde{\gamma}\,(kv_d - \omega_k + \omega_p), \tag{36}$$

and

$$\gamma_2 \simeq -\tilde{\gamma}\omega_p, \tag{37}$$

with

$$\tilde{\gamma} = \frac{e^2\omega_M\, v_{\rm th}}{8\, mc^2\, \omega_p^2} \int_0^{k_D} dk'\, k'^2 \int_{-1}^{1} \frac{x^2\, dx}{(k'^2 + k^2 - 2kk'x)^{1/2}}, \tag{38}$$

where for the evaluation of the integrals in v we have taken an average in the directions between $\mathbf{u} = \mathbf{v} - \mathbf{v}_d$ and k. This greatly simplifies the calculations without modifying the order of magnitude.

Evaluating the integrals in (38), by expanding the denominator in terms of Legendre polynomials and assuming $k_D > k$ which is the case of experimental interest one gets,

$$\gamma \simeq \frac{e^2\,\omega_M\, k\, v_S(X-1)}{24\, mc^2\, v_{\rm th}}, \quad \hbar\omega_p \ll k_B T. \tag{39}$$

Hence, a contribution to spin wave amplification occurs when $v_d > v_S$

When $\hbar\omega_{k'} \gg k_B T$, γ_1 in (31) is much less then γ_2 and we may approximate

$$\gamma \simeq -\frac{e^2\,\omega_p\,\omega_M}{24\, mc^2\, v_{\rm th}}, \quad \hbar\omega_p \gg k_B T \tag{40}$$

In this case one has a loss mechanism for the magnon system.

5. Discussion

The recent development of high mobility ferromagnetic semiconductors has enhanced the possibility of obtaining spin wave amplification by drifted carriers firstly suggested some time ago[1]. Despite this fact, no direct observation of amplification seems to have been made up to date. Recent measurements of magnetoresistance[5] and microwave transmission[17] only qualitatively indicate the existence of spin wave amplifícation. The lack of better experimental results may partly be due to the discouraging theoretical predictions. In

Ref. 7, for example, it is concluded that to obtain bulk amplification at microwave frequencies one would need a material with the product spin wave linewidth-resistivity 10^3 times smaller than in the most suitable material known, namely $CdCr_2Se_4$.

In the present work we have analized the question of spin wave-carrier interaction in the regime $kl > 1$. Contrary to the conclusion of Ref. 3, the spin wave damping coefficient does not vanish in the electron collisionless regime ($\tau \to \infty$). One may have amplification of spin waves in this regime as a consequence of the drift instability in the electron-magnon system, analogously to the acoustoélectric effect.

To get an estimate of the growth rate of spin waves we consider $CdCr_2Se_4$ dopped with Ag(p type) at 100°K. We use the following material parameters[18,19]: (%$= 6.5\,10^{10}$ sec^{-1}, $n_0 = 10^{17} - 10^{18}\,\text{cm}^{-3}$, $m = 10^{-28}$ gr, $\tau = 10^{-12}$ sec, $v_d = 10^6\text{cm sec}^{-1}$. For a maximum estimate we shall assume the values of k giving the maximum of $\gamma(k)$ both in the classical and quantum regimes, $k = 2\,\dfrac{\omega_k}{v_d}$ and $k = \dfrac{3}{2}\,\dfrac{\omega_k}{v_d}$, respectively. We shall also take $\omega_k = 10^9\,\text{sec}^{-1}$ and $\omega_k = 7 \times 10^{10}\,\text{sec}^{-1}$ for the clasical and quantum regimes. We then get

$$\gamma \simeq 6 \times 10^7 - 6 \times 10^8 \ \text{sec}^{-1}, \quad kl < 1 \tag{41a}$$

$$\gamma \simeq 3 \times 10^6 - 3 \times 10^7 \ \text{sec}^{-1}, \quad kl > 1. \tag{41b}$$

The growth rate has to be greater than the damping of the spin waves due to other effects to lead to net amplification. The magnon linewidth data available[18] for $CdCr_2Se_4$ from parallel pumping measurements give $\Delta H_k \simeq 3.0$ Oe for $k \simeq 10^5\text{cm}^{-1}$ which leads linear losses of the order of 10^7sec^{-1}. Hence, since the amplification factor may be of the order or greater than the damping coefficient, one might hope to achieve spin wave amplification. Note that this possibility seems to be more favorable in the regime $kl < 1$. This effect might possibly be observed in parallel pumping[20,21] experiments in which the sample is subjected to an additional electric field. In these experiments one can excite directly magnons with large wavenumbers and thus assure the work in the proper regime. The amplification would, in this case, be observed through a distortion of the "butterfly curve", indicating a decrease in the effective linewidth.

Finally, it should be mentioned that one might entirely neglect the plasma effects in the spin wave amplification in $CdCr_2Se_4$ under the experimental conditions described above. This is because for the present discussion it happens that $\hbar\omega_p \gg k_B T$ which makes it a small loss mechanism rather than enhances the spin wave amplification. However, if we could realize a magnetic semiconductor with high mobility and a relatively small carrier density (say of the order of 10^{13}cm^{-3}) then one could be in the regime $\hbar\omega_p \ll k_B T$ and as a result Eq. (39) would compete with Eq. (17) in analogy with the sound amplification[15].

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