Revista Brasileira de Física, Vol. 3, N.° 3, 1973

Piou-Deuteron Scattering near the (3,3) Resonance

M. EL AFIONI and J. L. ACIOLI
Departamento de Física, Instituto de Ciências Exatas, Universidade de Brasilia, Brasilia DF*

Recebido em 18 de Julho de 1973

The scattering of pions by deuterons has been studied under the Impulse Approximation for incident pions with a kinetic energy of 225 MeV. The differential cross-sections for the elastic and elastic plus inelastic processes have been obtained by the usual scattering theory and also by means of the Chew-Low amplitude for the pion-nucleon scattering. Numerical calculations were carried out and comparison has been made with the experimental results. The validity of the Impulse Approximation is discussed for this range of energy.

O espalhamento de pions por deuterons é estudado usando a aproximação de impulso para pions incidentes com uma energia cinética de 225 MeV. As seções de choque diferenciais para os processos elásticos e elásticos mais inelásticos foram obtidas pela teoria de espalhamento usual e também por meio da amplitude de Chew-Low para o espalhamento pion-nucleon. Cálculos numéricos foram efetuados e é feita comparação com os resultados experimentais. A validade da aproximação de impulso é discutida na região da ressonância (3,3) de pion-nucleon.

1. Introduction

In this paper, we study the scattering of pions by deuterons using the so-called Impulse Approximation for incident pions with a kinetic energy of 225 MeV. The Impulse Approximation (IA), originally formulated by Chew and Wick[1], is essentially based on the assumptions that the incident particle does not interact strongly with two components of the target-system at the same time and that during the time of interaction, supposed very short, the binding forces of the target-system are negligible. However, when one considers incident pions with an energy close to the energy of the pion nucleon resonance (nearly 200 MeV), the pion and one of the nucleons can form a quasi-bound state and therefore the time of interaction will be longer. This has been used as an argument contrary to the validity of the IA for this range of energy[2].

In order to define the range in which the IA is a reliable one, many expe-

*Postal address: 70000 - Brasilia DF.

riments at different pion energies have been made and the results so obtained have been compared with the corresponding theoretical values. In a general way, the predictions of the IA for the differential scattering cross-sections are in agreement with the experimental results at 85 MeV (Refs. 3, 4) and 300 MeV (Refs. 2, 5, 6), but there is a disagreement at 140 MeV (Refs. 7, 8), mainly for Iarge scattering angles. On the other hand, the numerical results of the IA differ among themselves for pion energies near 200 MeV (Refs. 7, 9, 10), depending on the particular procedure adopted for the calculations.

We shall obtain the differential cross-sections for the elastic and elastic plus inelastic scatterings and compare them with the experimental results''. We will not consider charge exchange processes:

2. Scattering Cross-Sections

We now apply the "pure" IA to calculate the differential cross-sections for π^- – D scattering, neglecting multiple scatterings. The matrix elements of the total reaction operator T are written as

$$T_{fi} = (\psi_f \ , \ (t_p + t_n) \ \psi_i), \tag{1}$$

where the operators t_p and t_n describe single scatterings of a negative pion by a proton and by a neutron, respectively. We have initially an incoming pion which can be represented by a plane wave $\exp(i\mathbf{q}_0 \cdot \mathbf{r})$ and a deuteron described by a Hulthem wave function

$$\phi_D(r) = \left[\frac{\alpha\beta(\alpha+\beta)}{2\pi(\beta-\alpha)^2}\right]^{1/2} \frac{e^{-\alpha r} - e^{-\beta r}}{r}, \tag{2}$$

where r is the internucleonic distance, a = 45.7 MeV, and $\beta = 7\alpha$. In the final state ψ_f we can have a free pion and a deuteron, or a free pion plus two free nucleons, according to the process being studied.

For the differential cross-section, in the laboratory system, for the elastic scattering we find the expression

$$\frac{d\sigma^E}{d\Omega} = \frac{4v}{9k^4}\left[\frac{E' \, W'^2}{M \, W \, E \, E_0}\right] \rho(\theta) \left[\int_{U_{f'}} |I(Q/2)|^2 \, q^2 dq \, \delta(U_{f'} - U_{0'})\right] \cdot$$

$$\cdot \left\{ \left| 2\eta_3 + \cos\theta' \, (4\eta_{33} + 2\eta_{13} + 2\eta_{31} + \eta_{11}) + \frac{3\varepsilon}{4v \sin^2(\theta'/2)} \right|^2 + \right.$$

$$\left. \sin^2\theta' \, |2\eta_{33} + \eta_{13} - 2\eta_{31} - \eta_{11}|^2 \right\}, \tag{3}$$

where E, and E are the initial and final values of the pion energy, W is the nucleon energy, $U_{0'}$ and $U_{f'}$ are the initial and final energies of the pion-deuteron system, θ is the scattering angle, all of them in the laboratory system, and v is the pion-nucleon relative velocity, E', k, θ' are the energy, momentum, and scattering angle of the pion in the pion-nucleon center of mass system, M is the nucleon rest mass, and ε is the fine structure constant. $\rho(\theta)$ is defined as the ratio of

$$J' = \frac{(2\pi)^4}{v} \int_{U_{f'}} q^2\, dq\, \delta(U_{f'} - U_{0'})$$

to

$$J = \frac{(2\pi)^4}{v} \int_{U_{f'}} q^2\, dq\, \delta(U_f - U_0),$$

which are the phase-space factors for the pion-deuteron and the pion nucleon scatterings. The function $\rho(\theta) = J'/J$ is plotted against the scattering angle in Figure (1).

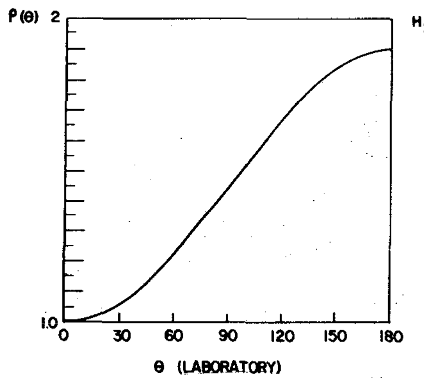

Fig. 1 - The phase-space ratio $\rho(\theta) = J'/J$ vs. scattering angle in the lab. system.

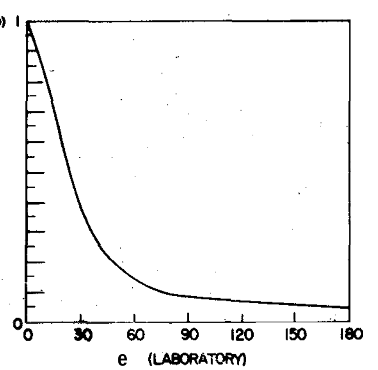

Fig. 2 - The atomic form factor $I(Q)$ at 225 MeV vs. scattering angle in the lab. system.

The quantity $I(Q)$, where $Q = q_0 - q$ is the momentum transfer, is given by

$$I(Q) = \frac{2\alpha\beta(\alpha+\beta)}{Q(\beta-\alpha)^2}\left\{\mathrm{arctg}\left(\frac{Q}{2\alpha}\right) - 2\,\mathrm{arctg}\left(\frac{Q}{\alpha+\beta}\right) + \mathrm{arctg}\left(\frac{Q}{2\beta}\right)\right\}. \quad (4)$$

$I(Q)$ is the atomic form factor which is traced at 225 MeV in Figure (2). $\eta_{TL} = \exp(i\delta_{TL}) \sin \delta_{TL}$ are the usual phase-shift functions, where T is the isotopic spin of a pion nucleon system and L is the orbital angular momentum of the pion. For ?-waves, the second subscript is omitted according to the notation introduced by Fermi.

For the differential cross sections for the elastic plus inelastic scatterings we obtain, in terms of pion-proton and pion-neutron cross sections[3],

$$\frac{d\sigma^{E+I}}{d\Omega} = \frac{d\sigma^p}{d\Omega} + \frac{d\sigma^n}{d\Omega} + 2I(Q)\left[\frac{d\sigma^p}{d\Omega}\frac{d\sigma^n}{d\Omega}\right]^{1/2}, \tag{5}$$

where[12]

$$\frac{d\sigma^{p,n}}{d\Omega} = J\,[|f_{p,n}|^2 + |g_{p,n}|^2],$$

$$f_p = \frac{S}{12\pi}\left[\eta_3 + 2\eta_1 + \cos\theta'(2\eta_{33} + 4\eta_{13} + \eta_{31} + 2\eta_{11}) + \frac{3\varepsilon}{2v\sin^2(\theta'/2)}\right],$$

$$f_n = \frac{S}{4\pi}[\eta_3 + \cos\theta'(2\eta_{33} + \eta_{31})],$$

$$g_p = \frac{S}{22\pi}\sin\theta'(\eta_{33} + 2\eta_{13} - \eta_{31} - 2\eta_{11}),$$

$$g_n = \frac{S}{4\pi}\sin\theta'(\eta_{33} - \eta_{31}),$$

$$S = -\left[\frac{E' W'^2}{MWEE_0}\right]^{1/2}\frac{v}{\pi k^2}.$$

A paralell calculation of these cross-sections has been made using the Static Model, according to which the pion-nucleon interaction occurs through the coupling of the pion field to an infinitely heavy nucleon. The pion nucleon scattering amplitude is the Chew-Low amplitude[13]

$$t(2,1) = \frac{2\pi}{\omega}\sum_{j=1}^{4} P_j(2,1)\,h_j \tag{6}$$

where

$$h_j = \frac{e^{i\delta_j}\sin\delta_j}{k^3} = \frac{\eta_{TL}}{k^3} \quad \text{and} \quad P_j(2,1) = \mathscr{I}_j(2,1)\,\mathscr{J}_j(2,1).$$

$\mathscr{I}_j(2,1)$ and $\mathscr{J}_j(2,1)$ are the operators of isotopic spin and angular momentum, collectively denoted by the index j, and ω and k are the pion energy and momentum, taken in convenient units, in the pion-nucleon center of mass system. The expressions for the cross-sections and the details of the calculation can be found in the paper by A. Ramakrishnan *et al.*[10].

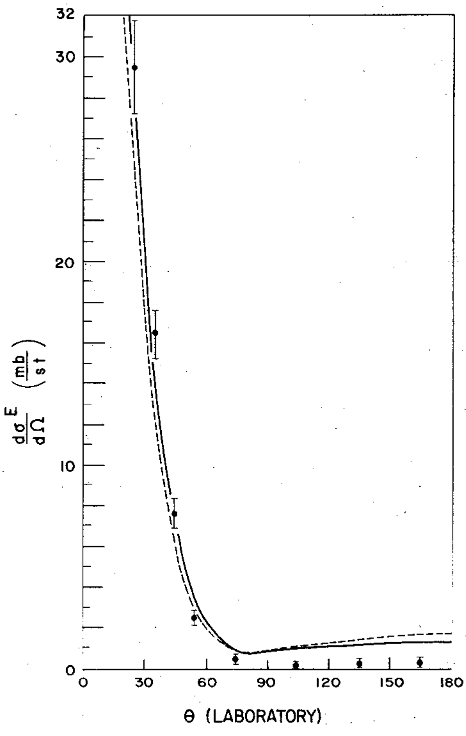

Fig. 3 - The elastic differential cross-section, Eq. (3). The solid **line is** obtained with $\delta_1 = .2583$, $\delta_3 = .2705$, $\delta_{11} = .1029$, $\delta_{13} = 0.0$, $\delta_{31} = -.0366$, and $\delta_{33} = 1.960$. ***For*** the **dashed line**, $\delta_1 = 0.0$, $\delta_3 = 0.0$, $\delta_{11} = .13089$, $\delta_{13} = -.4799$, $\delta_{31} = -.08726$, and $\delta_{33} = 1.96349$

3. Results and Conclusions

The differential cross-sections given by Equations (3) and (5), and the corresponding ones obtained by means of the Chew-Low amplitude (6) have been calculated at 225 **MeV** and compared with the experimental data obtained by one of **us**[11], who studied the π^-**-deuteron** interaction using a bubble chamber. The final sample of this experiment, used to calculate the cross-sections, consisted of 5387 elastic and inelastic events. The phase-shifts used in the calculations are those given by J. Deahl et *al*[15].

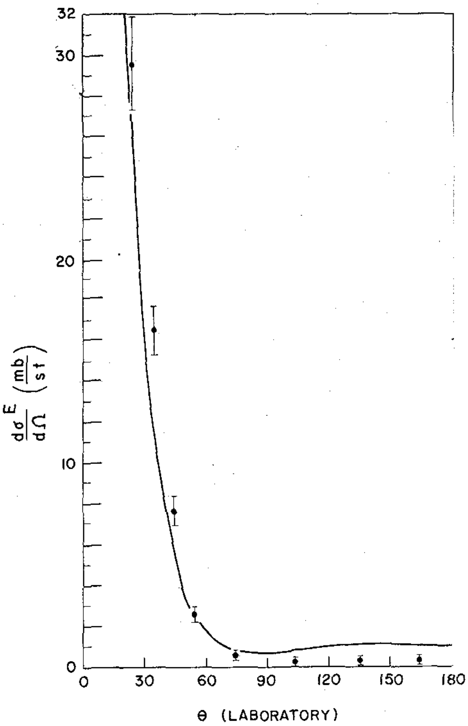

Fig. 4 - The elastic differential cross-section as predicted by the Static Model, The curve is obtained considering only the dominant phase-shift, $\delta_{33} = 1.96349$.

As can be seen from the figures, the fit is in general good, but there is a considerable deviation for large scattering angles, mainly in the elastic scattering. This suggests that possibly the greater deficiency in our calculations is the non-inclusion of the contributions from the double-scattering terms to the elastic cross section. As we know, the individual scattering amplitudes assume great values near a resonance and therefore the double scattering terms should contribute significantly to the cross-section. These terms interfere with those of simple scattering, and have their greater effect for large scattering angles. This is because the double-scattering form factor has a maximum value for backward scattering, decreasing to a very small value for forward scattering[7]. With regard to the double scattering contributions, the reader is referred to a recent calculation by J. M. Wallace[14], which also incorporates the nuclear Fermi motion.

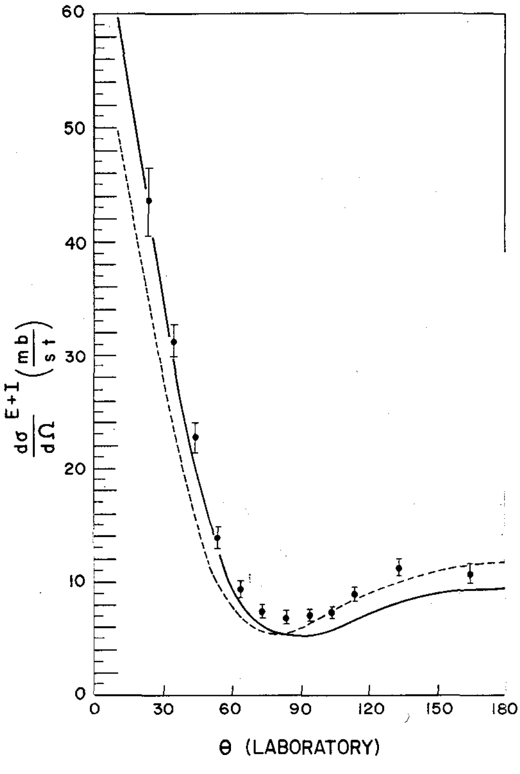

Fig. 5 - The differential cross-section for the sum elastic plus inelastic. The solid and dashed lines are obtained using the phase-shifts given in the caption for Fig. 3.

Figures (5) and (6) represent the differential cross section for the sum elastic plus inelastic, according to the static model and the usual scattering theory, for two sets of phase shifts. They show a good fit between the predictions of the IA and the experimental results. They also show that this agreement does not depend critícally on the adopted formalism, but it depends on our knowledge about some quantities such as the phase-shifts and the deuteron wave function. Our results indicate that a possible temporary pion-nucleon binding to produce a N* and other suggestions currently given for the non validity of the IA at this energy are not really significant. In conclusion, our view is that the Impulse Approximation is realiable to describe the pion-deuteron scattering in the neighbourhood of the (3,3) resonance and that the inclusions of the Fermi motion of the

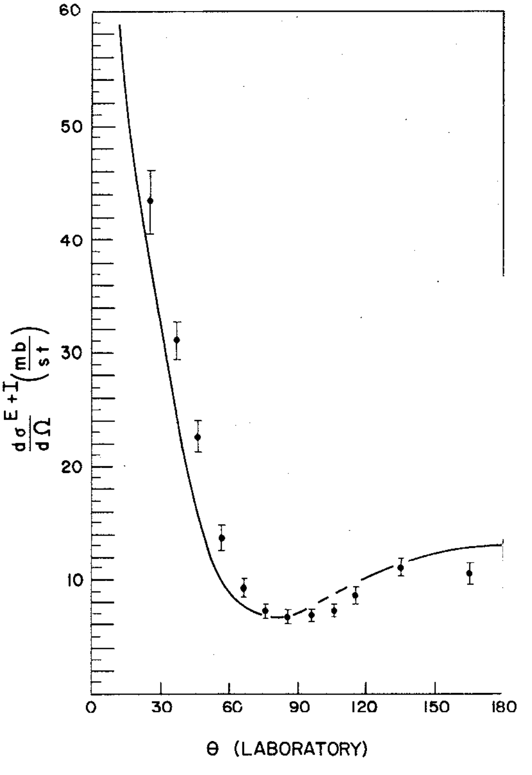

Fig. 6 - The differential cross-section for the sum elastic plus inelastic, as predicted by the Static Model. Thecurve is obtained by considering only the dominant phase-shif, $\delta_{33} = 1.96349$.

nucleons and of the multiple scattering terms can improve the fitting, but are not essential to assert the validity of the approximation.

References

1. *G.* F. Chew and *G.* C. Wick, Phys. Rev. **80,** 196 (1950).
2. V. Devanathan, **Nuc.** Phys. **43,** 684 (1963).
3. R. Rockmore, Phys. Rev. **105,** 256 (1957); **13,** 1696 (E) (1959).
4. K. C. Rogers and L. M. Lederman, Phys. Rev. **105,** 247 (1957).
5. *G.* M. Vagradov and I. B. Sokolov; JEPT Soviet Physics 9, 669 (1959).
6. L. S. Dul'kova, I. B. Sokolov, and M. *G.* Shafranova, JETP Soviet Physics **8,** 217 (1959).
7. H. N. Pendleton, Phys. Rev. 131, 1833 (1963).
8. E. *G.* Pewitt, H. H. Fields, G. B. Yodh, J. G. Petrovitch and M Derrick, Phys. Rev. **131,** 1826 (1963).

9. B. H. Bransden and R. G. Moorhouse, Nuc. Phys. 6, 310 (1959).
10. A. Ramakrishnan, V. Devanathan and K. Venkatesan, Nuc. Phys. 29, 680 (1962).
11. J. L. Acioli, Ph. D. Thesis, Department of Physics, The University of Chicago (1969).
12. S. Fernbach, T. A. Green and K. M. Watson, Phys. Rev. 84, 1084 (1951).
13. G. F. Chew and F. E. Low, Phys. Rev. 101, 1570 (1956).
14. J. M. Wallace, Phys. Rev. D5, 1840 (1972).
15. J. Deahl, M. Derrick et *al.*, Phys. Rev. 124, 1987 (1961).